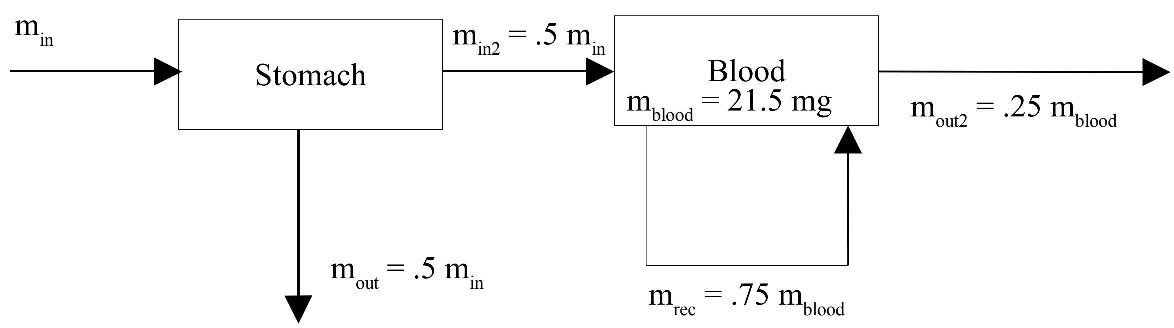
1. Mass Balancing – Human Iron Inventory

The first step is to draw a block diagram of the system. In solving this problem we assume that all iron in the blood that is not recycled exits the blood.

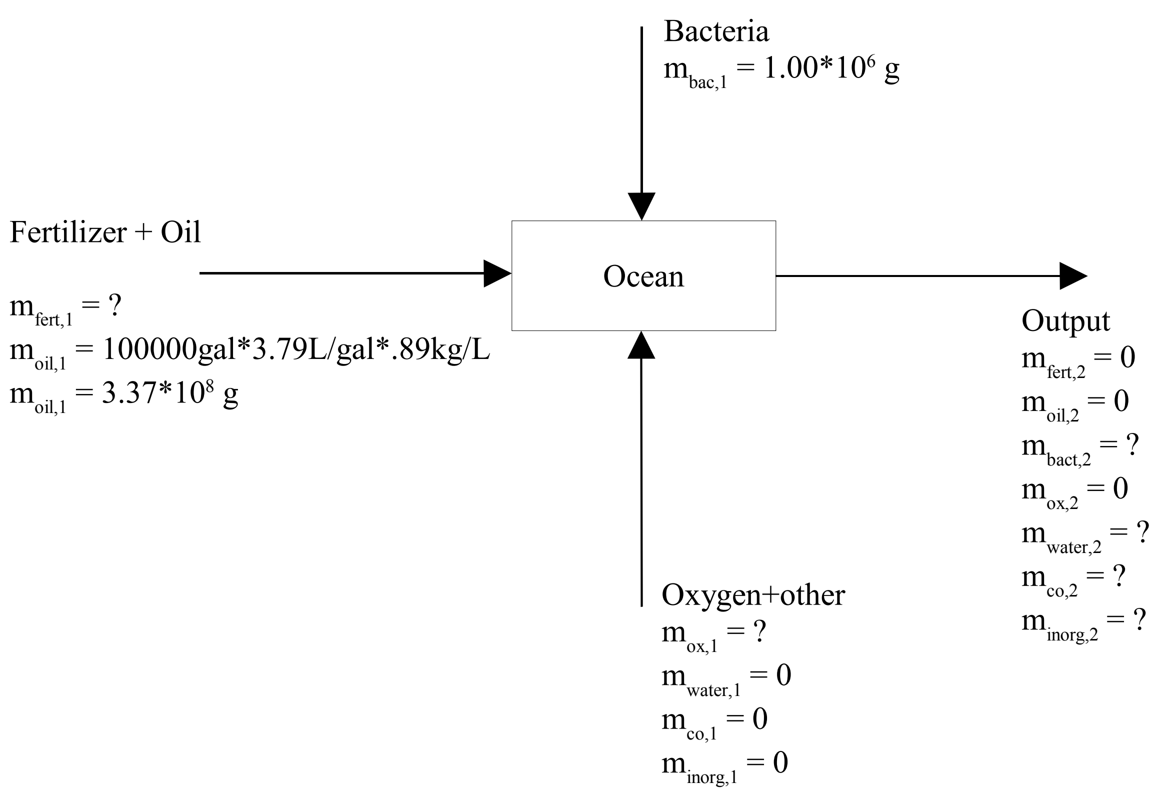


Another assumption we make is that the amount of iron in the blood stays constant, so $m_{in2} = m_{out2} = .5\ m_{in}$. Thus, solving for m_{out2} we can find m_{in}:

$$m_{out2} = .25*21.5 = 5.375 = m_{in2}$$
$$m_{in} = 2*m_{in2} = \mathbf{10.75\ mg}$$

2. Mass Balancing

Again, we start by drawing our system in box diagram form. We assume that all oil, fertilizer, and oxygen are consumed in the reaction.



We next perform a mass balance on each of the chemicals shown above. Our system is a batch reactor, so we can model the balance using the following equation,

$$m_{x,1}-m_{x,2}+\int r_x\,dt=m_{x,1}-m_{x,2}+mw_x*R_x=0$$

where the mass of species x at position one is the input, the mass of x at position two is the output, and R is the change in moles due to the reaction (and mw is the molecular weight of x).

Fertilizer: $$R_{fert}=\frac{-m_{fert,1}}{mw_{fert}} \quad (1)$$

Oil: $$R_{oil}=\frac{-m_{oil,1}}{mw_{oil}}=-3.37*10^8\,g\,\frac{1\,mol}{348.88\,g} \quad (2)$$

Bacteria: $$R_{bact}=\frac{m_{bact,2}-m_{bact,1}}{mw_{bact}}=\left(m_{bact,2}-1.00*10^6\,g\right)\frac{1\,mol}{23.82\,g} \quad (3)$$

Oxygen: $$R_{ox}=\frac{-m_{ox,1}}{mw_{ox}} \quad (4)$$

Water: $$R_{fert}=\frac{m_{water,2}}{mw_{water}} \quad (5)$$

CO_2: $$R_{co}=\frac{m_{co,2}}{mw_{co}} \quad (6)$$

Inorganic: $$R_{inorg}=\frac{m_{inorg,2}}{mw_{inorg}} \quad (7)$$

Total mass balance: $m_{fert,1} + m_{oil,1} + m_{bact,1} + m_{ox,1} - m_{bact,2} - m_{water,2} - m_{co,2} - m_{inorg,2} = 0$ (8)

Note that we are using chemicals and molecular weights calculated in problem 2 of PS3.

From the mass balance we have eight equations and thirteen unknowns. To solve for all of the R_x, we need to perform a mole balance, which we already did in problem 2 of PS3. The coefficients that we calculated then should be sufficient in relating the different R_x to each other. Note that if the reaction is involved in the consumption of a species, then its sign is negative.

a*$C_{24.7}H_{43.2}N_{0.31}O_{0.27}$ + b*$H_{2.65}NO_{1.62}P_{0.13}$ + c*O_2 -->
--> d*$CH_{1.66}N_{0.14}O_{0.50}P_{0.0057}$ + e*H_2O + f*CO_2 + g*$H_2PO^-_4$

a = 0.096
b = 0.110
c = 2.241
d = 1.000
e = 1.389
f = 1.381
g = 0.0086

$$-\frac{R_{fert}}{.110}=-\frac{R_{oil}}{.096}=\frac{R_{bact}}{1}=-\frac{R_{ox}}{2.241}=\frac{R_{water}}{1.389}=\frac{R_{co}}{1.381}=\frac{R_{inorg}}{.0086}$$

From our mass balance we know R_{oil}, so we can calculate the other R_x as:

$R_{fert} = -9.67*10^5$
$R_{oil} = -1.11*10^6$
$R_{bact} = 1.01*10^7$
$R_{ox} = -2.26*10^7$
$R_{water} = 1.40*10^7$
$R_{co} = 1.39*10^7$
$R_{inorg} = 8.66*10^4$

Plugging these values into equations (1)-(7) we can solve the species' mass:
$m_{fert,1} = 9.67*10^5 * 46.62 =$ **$4.51*10^7$ g**
$m_{ox,1} = 2.26*10^7 * 32.00 =$ **$7.22*10^8$ g**
$m_{bact,2} = 1.01*10^7 * 23.82 + 1.00*10^6 =$ **$2.41*10^8$ g**
$m_{water,2} = 1.40*10^7 * 18.01 =$ **$2.52*10^8$ g**
$m_{co,2} = 1.39*10^7 * 44.01 =$ **$6.12*10^8$ g**
$m_{inorg,2} = 8.66*10^4 * 96.99 =$ **$8.40*10^6$ g**

With this we can calculate the percent composition of each stream. For example, for the percent composition of the fertilizer + oil:

$$\frac{m_{oil,1}}{m_{oil,1}+m_{fert,1}}=\frac{3.37*10^8}{3.37*10^8+4.51*10^7}=.882$$
$$\frac{m_{fert,1}}{m_{oil,1}+m_{fert,1}}=\frac{4.51*10^7}{3.37*10^8+4.51*10^7}=.118$$

Thus, the percent composition of this stream is 88.2% oil and 11.8 % fertilizer.

3. Microbial Growth Kinetics

$$N=N_o e^{\mu_m t}$$
$$t=\frac{\ln\left(\frac{N}{N_o}\right)}{\mu_m}$$

$\mu_m = 1.35\ hr^{-1}$

$$\frac{N}{N_o}=\frac{m_{bact,2}}{m_{bact,1}}=\frac{241000\,kg}{1000\,kg}=241\,kg$$
$$t=\frac{\ln(241)}{1.35}=4.06\,hr$$

4. Microbial Growth Kinetics

We are given that

$$\frac{N}{N_o}=10$$

$\mu_m = 1.29 \pm 0.37\ hr^{-1}$

So we can solve:

$$t = \frac{\ln(10)}{1.29/60} = 107\, minutes$$

$$S_{\mu_m} = 0.37$$
$$S_t^2 = \left(\frac{\partial t}{\partial \mu_m}\right)^2 * S_{\mu_m}^2$$
$$\left(\frac{\partial t}{\partial \mu_m}\right)^2 = -\frac{\ln(10)}{\mu_m^2}$$
$$S_t = .52\, hr$$

So,

t = 107 ± 31.2 min

The mean concentration of cells at 5 hours is:

$$N = (5.00 * 10^6\, cells/mL)\, e^{1.29*5}$$
$$N = 3.16 * 10^9\, cells/mL$$

$$S_t^2 = \left(\frac{\partial t}{\partial \mu_m}\right)^2 * S_{\mu_m}^2$$
$$S_N^2 = \left(\frac{\partial N}{\partial N_o}\right)^2 * S_{N_o}^2 + \left(\frac{\partial N}{\partial \mu_m}\right)^2 * S_{\mu_m}^2$$
$$S_N^2 = \left(\frac{\partial N}{\partial N_o}\right)^2 * S_{N_o}^2 + \left(\frac{\partial N}{\partial \mu_m}\right)^2 * S_{\mu_m}^2$$
$$\frac{\partial N}{\partial N_o} = e^{\mu_m t} \quad \text{and} \quad \frac{\partial N}{\partial \mu_m} = N_o\, t\, e^{\mu_m t}$$
$$S_N^2 = \left(e^{1.29*5}\right)^2 * 490000^2 + \left(5000000 * 5 * e^{1.29*5}\right)^2 * .37^2$$
$$S_N = 5.86 * 10^9$$

N = (3.16 ± 5.86) * 10^9 cells/mL