1. Biomaterials – Stress-strain behavior

Conversion factor	1
Length	8
Width	5
Thickness	6
Area	30

Notes

Stress = Force / Area
Strain = Displacement/L
Area = w x L
Elastic modulus is slope of stress vs. strain curve

Displacement (mm)	Force (N)	Stress (N/mm^2)	Strain (%)
0	0	0	0
0.05	21	0.7	0.625
0.1	49	1.63333333	1.25
0.15	118	3.93333333	1.875
0.2	190	6.33333333	2.5
0.25	301	10.0333333	3.125
0.3	399	13.3	3.75
0.35	503	16.7666667	4.375
0.4	602	20.0666667	5
0.45	700	23.3333333	5.625
0.5	810	27	6.25
0.55	960	32	6.875
0.6	1090	36.3333333	7.5
0.65	1200	40	8.125
0.7	1298	43.2666667	8.75
0.75	1390	46.3333333	9.375
0.8	1487	49.5666667	10
0.85	1523	50.7666667	10.625
0.9	1490	49.6666667	11.25
0.95	1350	45	11.875

Plotting the stress vs. strain for all data points we get figure A (below). The ultimate tensile stress is the maximum stress a material can withstand; this a strain of 10.625%. Thus, the ultimate tensile stress is **~51 N/mm^2**. To find the elastic modulus, E, we find the slope of the stress/strain curve in the elastic region, which seems to occurs when the strain is between 1.875 and 9.375 %. We use Excel's linear regression solver to find the slope (figure B). Thus, we find that the elastic modulus is 5.85 ~ **5.9 N/mm^2**. To relate the elastic, E, and shear, G, moduli, we use an estimate of Poisson's ratio (typically $\nu \sim 0.33$) and the relationship K (bulk modulus) = $E/[3(1-2\nu)] = 2G(1+\nu)/[3(1-2\nu)]$.

Re-arranging, $G = E/[2(1+\nu)] = (5.85\ \text{N/mm}^2)/[2(1+0.33)] = 2.203$ ~ **2.2 N/mm^2**

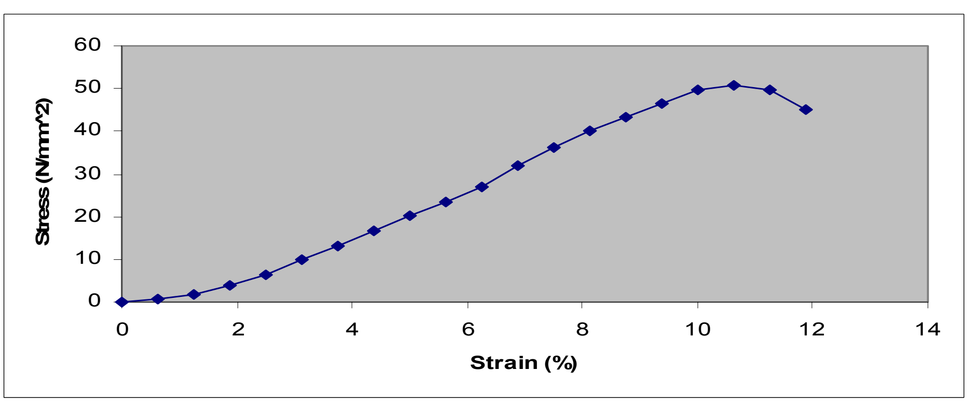


Figure A

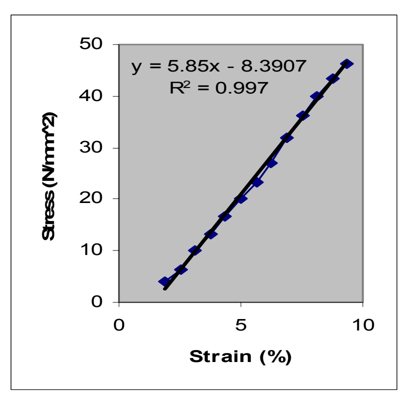
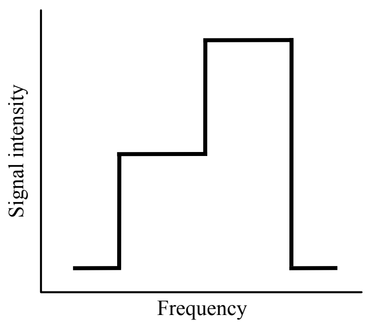
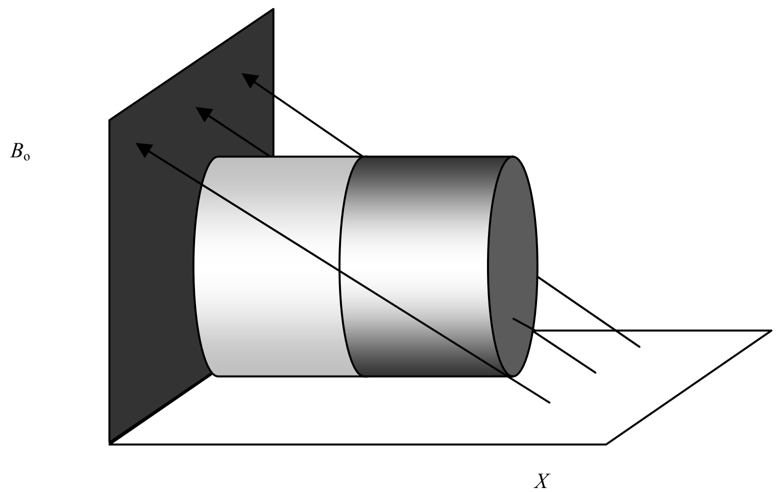


Figure B

2. MRI

The signal intensity depends upon the strength of the magnetic field and the amount of material in the field at each position. Since the signal shows a step-like formation, we can assume that at each position *X*, the magnetic field passes slices through the same amount of material. The dark material resonates at a lower frequency; therefore, the dark material experiences a lower magnetic field strength B_o. Thus, our field gradient is:





3. MRI

To solve this problem, we use two equations: $E = h * \gamma * \mathrm{B}$ and $N^-/N^+ = \exp\{-E/k_BT\}$

$h = 6.626\times10^{-34}$ J s
$\gamma = 42.58$ MHz / T = 42,580,000 Hz / T (for hydrogen)
$B = 0.75$ T
$k_B = 1.3805\times10^{-23}$ J/K
$T = 298$ K (assume room temperature)

$E = (6.626\times10^{-34}$ J•s)(42,580,000 Hz/T)(0.75 T) = 2.116×10^{-26} ~ **2.1×10^{-26} J**

$N^-/N^+ = \exp\{(-2.116\times10^{-26})/((1.3805\times10^{-23}\text{ J/K})(298\text{ K}))\} = \exp\{-5.144\times10^{-6}\}$ (or ~ 1.0)

$(N^+-N^-)/N^+ = 1 - N^-/N+ = 1 - \exp\{-5.144\times10^{-6}\}$ (or ~ 0.0)

MRI is extremely sensitive - able to operate with very small differences in the spin state populations.

Anything that will make E/k_BT larger will improve the signal: **we could increase the strength of the magnet (*B*) or we could decrease the temperature (*T*)**; everything else that impacts *E* – the constants (h, γ, k_B) - is set by Nature.

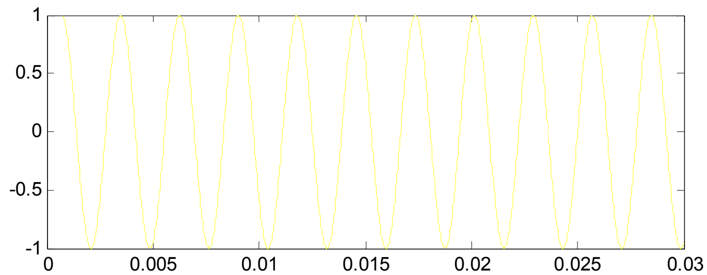
4. Signal processing

Using the aliasing formula, every signal with frequencies given by $(f - kf_{sample})$ will look the same:

Thus, signals with frequencies:
360 – 1*200 = 160
360 – 2*200 = -40
360 – 3*200 = 240 etc

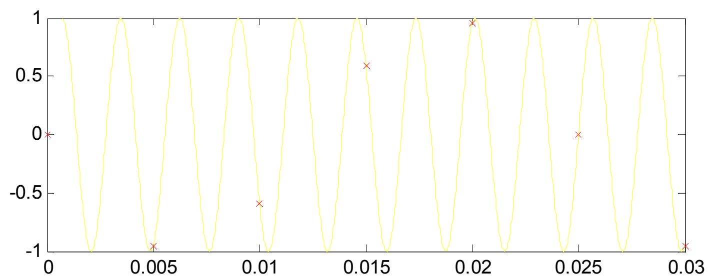
Will all look the same when sampled at 200 Hz; they will all look like a **40 Hz** signal. The maximum signal frequency that a 200 Hz sampling frequency can accurately represent is 100 Hz (recall the definition of the Nyquist frequency). So, a 160 Hz signal would also look like a 40 Hz signal when sampled at 200 Hz, as would a 240 Hz signal.

We can also show this graphically. We assume that our signal is a simple sine function. Thus, our signal is $f(x) = \sin(360 \cdot 2\pi \cdot x)$. Notice that this function describes a continuous signal, which looks like:

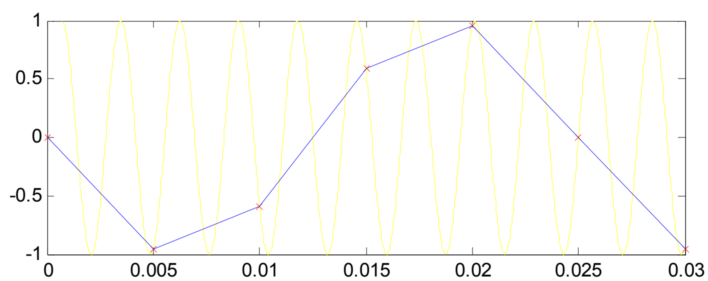


Where the x axis is time in seconds, and the y axis is the signal intensity.

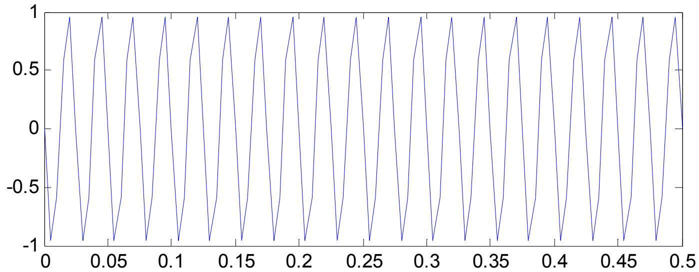
However, we are only sampling at a frequency of 200 Hz, or 200 times a second. Our samples are shown below as the red x's:



So connecting the x's we get:



Note that were we to sample for more than 0.03 seconds, say for 1 second, we would find that our aliased signal would repeat itself:



We find that the period of the aliased signal is 0.025 seconds, and thus the frequency is **40 Hz**.