Fall 2002 (Przybycien) **42-101 Intro to BME** **NAME: ____________________**

FINAL EXAM SOLUTIONS and GRADING SCHEME

Format: Open text, notes, homework and mind; closed neighbor.

Part I: Short Answer Questions (72 pts)

Short written-answer and short calculation questions - *No more than two or three sentences in answer to any question in this section please.*

1. (6 pts) Blood is said to have a "yield stress". What is this and why does it occur in blood?

   A yield stress is the smallest stress that will initiate fluid (blood) motion; applied stresses less than the yield stress will not result in the deformation of the fluid (blood). (+3 pts) is a concentrated (~45 vol%) suspension of interacting particles (cells); some stress must be applied to disrupt these interactions before blood will begin to flow. (+3 pts)

2. (6 pts) The average capillary in the microcirculation has an inner diameter of 0.0008 mm, smaller than the diameter of a red blood cell, which you might think would result in a huge pressure drop. Give two reasons why the pressure drop across the microcirculation is relatively small.

   - The flow is divided among many (~10^9) capillaries.
   - The capillaries are short (~0.05 cm).
   - The red blood cells are able to deform.
   - The walls of the capillaries are elastic.

   (+3 pts each for any two viable answers)

3. (6 pts) Give three reasons why it is much preferred to use a sphygmomanometer on the upper arm rather than on the big toe.

   - There is a significant hydrostatic pressure difference between the heart and the toe while the upper arm is at roughly the same height as the heart.
   - The toe is a long way from the heart and will have a much less obvious pulse than that in the upper arm.
   - The toe will have much less blood flow and will have a much less obvious pulse than in the upper arm.
   - The toe is much smaller than the upper arm making placement of both a pressure cuff and a stethoscope more difficult/less convenient.

   (+2 pts each for any three viable answers)

4. (6 pts) Suppose a mathematical expression was available that described the power expenditure, $-\dot{W}$, of an individual swimming in terms of the swimming velocity, $\vec{V}$, the body density, ρ, and the arm stroke length, l; e.g. an equation of the form $-\dot{W} = f(\vec{V}, \rho, l)$ was available. How would you determine from this function, given no other information, if there is an arm stroke length that minimizes the power expenditure?

   See if there is a length l* such that $\left(\frac{\partial(-\dot{W})}{\partial l}\right)_{l=l*} = 0$ (+3 pts) and $\left(\frac{\partial^2(-\dot{W})}{\partial l^2}\right)_{l=l*} > 0$

   (+3 pts)

5. (6 pts) Using our "walking box" model as a basis, describe two different scenarios (in terms of magnitudes of model variables and what they would mean in terms of the characteristics of how someone would walk) where the total rate of energy expended by walking would be dominated by kinetic energy effects.

   The kinetic energy term for the walking box model has a cubic dependence on velocity while the potential energy term has a linear dependence on velocity; at high walking speeds (large values of $\vec{V}$) the kinetic term should dominate. (+ 3 pts)

   Likewise the kinetic energy term has an inverse dependence on stride length while the potential energy term has a linear dependence on stride length; at short stride lengths (small values of a) the kinetic tem should dominate. (+3 pts)

6. (6 pts) What information does an "ergonomic analysis", say as applied to our "walking box" model, provide?

   An ergonomic analysis allows the performance of individuals with different body types who perform the same task to be compared. In terms of our walking box model, performance may be reckoned via the rate of energy expenditure on walking as a function of differences in body mass, leg mass, and leg length. (+6 pts)

7. (6 pts) Describe why "allostery" can be considered a form of discrete control in some situations.

   The binding/activity profile for an allosteric protein/enzyme as a function of ligand/substrate concentration is "S-shaped". At large values of the cooperativity coefficient, n, the "S" becomes sharper and begins to resemble a step function. At ligand/substrate concentrations smaller than $K_m^{1/n}$, the binding/activity will be very small (OFF); at ligand/substrate concentrations greater than $K_m^{1/n}$, the binding/activity will be large (ON). (+6 pts)

8. (6 pts) Several measurements are made of the activity, ν, of a given enzyme as a function of the substrate concentration, S, at a specific temperature and pH; the data are shown at the right. *Just by looking at the data, without doing a calculation*, make a good guess at values for the constants in the Michaelis-Menten equation that might describe the catalytic activity of this enzyme at these conditions. Include units with your guesses and indicate how you arrived at the values of your guesses.

S (mM)	ν (mM product/min)
0.1	0.040
0.5	0.181
1.0	0.322
5.0	1.014
10.0	1.3
50	1.852
100	1.945
500	2.03
1000	2.025

   For the largest three substrate concentrations, the activity does not change very much, i.e. the activity must be maxed out; ν_{max} must be in the neighborhood of 2 mM product/min. (+3 pts) At a substrate concentration of 5 mM, the activity of the enzyme is about half of the maximum activity; K_M must be in the neighborhood of 5 mM. (+3 pts)

9. (6 pts) From which material would you likely derive the most energy, in the form of ATP, if eaten: 1g of phosphatidyglycerol, 1g of α-maltose, or 1g of hemoglobin? Why?

   We produce energy via aerobic respiration (oxidative phosphorylation) which involves the oxidation of nutrients (removal of electrons and eventual transfer to oxygen) to produce ATP. Phosphatidylglycerol is a lipid; energy released on complete oxidation of 1g of a lipid is ~9.3 kcal. a-Maltose is a carbohydrate; energy released on complete oxidation of 1g of carbohydrate is ~4.1 kcal. Hemoglobin is a protein; energy released on complete oxidation of 1g of protein is ~4.2 kcal. We would derive the most energy from 1g of the lipid, phosphatidylglycerol. In nutrition terms, lipids (fats) have the most calories per gram. (+6 pts) {We would derive energy more *quickly* from the carbohydrate….}

10. (6 pts) A chemical reaction has ΔG_{rxn} = +4.5 kcal/mol. What would the addition of a catalyst, such as an enzyme, to a solution of the reactants do for this reaction?

    Nothing. The reaction is not spontaneous. Enzymes (catalysts) only lower the height of activation energy barriers, speeding the rate of reaction (kinetics); they do not change the thermodynamics of the reaction. (+6 pts) {The only way to make this reaction go would be to use energy coupling, such as coupling the hydrolysis of ATP to the reaction.}

11. (6 pts) Celebrex (Searle) and Vioxx (Merck) are nonsteroidal anti-inflammatory drugs (NSAIDs) that are used to treat pain, particularly that associated with rheumatoid arthritis. These NSAIDs work by competitively inhibiting cyclooxygenase-2 (COX-2), a membrane-associated enzyme that catalyzes the formation of prostaglandin H_2 from arachidonate. Given that the standard dose of Celebrex is 200 mg and that of Vioxx is 25 mg, which drug likely has the largest K_I value? Why?

    Since the dosage for Celebrex is greater than that for Vioxx, it must take greater concentrations of Celebrex to bind to a significant extent to the COX-2 active site than it does for Vioxx. Therefore, Celebrex must be a weaker binder than Vioxx and must have a larger value of the inhibition dissociation constant, K_I. (+ 6 pts)

12. (6 pts) Describe three differences between procaryotes and eucaryotes.

    - Eucaryotes have a membrane-enclosed nucleus; procaryotes do not.
    - Eucaryotes tend to be larger than procaryotes.
    - Eucaryotes have more complex internal structure/organelles than procaryotes.
    - Procaryotes are single-celled organisms; eucaryotes may be multicellular.
    - Eucaryotes grow more slowly than procaryotes.

    (+2 pts each for any three viable answers)

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Part II: Detailed Questions (78 points)

1. (26 pts) Phashun Plait, a student at a neighboring university, has taken to wearing 6.0 in high platform sneakers that weigh 2.7 lbm each, heedless of the implications with respect to optimum stride length and energy expenditure during walking; assume that "regular" shoes have 1.0 in heels and weigh 1.0 lbm each. You decide to help Phashun out with a few calculations. Before slipping into any shoes, Phashun weighs 135 lbm with 36% of that accounted for by total leg mass and is 3.2 ft from heel to hip. Phashun walks 4.0 miles per hour when late for class.
   a. (13 pts) By what fraction does Phashun's optimum stride length change when going from regular shoes to platform sneakers (and when late for class)?
   b. (13 pts) By what fraction does Phashun's minimum rate of energy expenditure change when going from regular shoes to platform sneakers (and when late for class)?

① a. $a^* = 2v\sqrt{\frac{\alpha L}{g}}$ ⇒ need α, L for 1" and 6" shoes
+3 pts

1" shoes: $L_1 = 3.2\,ft + 1.0\,in\left\langle\frac{1\,ft}{12\,in}\right\rangle = 3.28333\,ft$ +1

$\alpha_1 = \frac{(0.18)(135\,lbm) + (1.0\,lbm)}{(135\,lbm) + 2(1.0\,lbm)} = 0.1847$ +1

6" shoes: $L_6 = 3.2\,ft + 0.5\,ft = 3.7\,ft$ +1

$\alpha_6 = \frac{(0.18)(135\,lbm) + (2.7\,lbm)}{(135\,lbm) + 2(2.7\,lbm)} = 0.1923$ +1

EITHER: short cut: $\frac{a_6^*}{a_1^*} = \frac{2v\sqrt{\alpha_6 L_6/g}}{2v\sqrt{\alpha_1 L_1/g}} = \left(\frac{\alpha_6 L_6}{\alpha_1 L_1}\right)^{0.5}$ +4

$= \left(\frac{(0.1923)(3.7\,ft)}{(0.1847)(3.28333\,ft)}\right)^{0.5} = 1.0832$

OR: long-cut: $a_6^* = 2v\sqrt{\alpha_6 L_6/g}$

$= 2\left(4.0\,\frac{mi}{hr}\right)\sqrt{\frac{(0.1923)(3.7\,ft)}{(32.174\,ft/s^2)}}\left\langle\frac{5280\,ft}{mi}\right\rangle\left\langle\frac{hr}{3600\,s}\right\rangle$

$= 1.7449\,ft$ +2 pts

$a_1^* = 2v\sqrt{\alpha_1 L_1/g}$

$= 2\left(4.0\,\frac{mi}{hr}\right)\sqrt{\frac{(0.1847)(3.28333)}{(32.174)}}\left\langle\frac{5280\,ft}{mi}\right\rangle\left\langle\frac{hr}{3600\,s}\right\rangle$

$= 1.6109\,ft$ +2 pts

$\frac{a_6^*}{a_1^*} = \frac{1.7449\,ft}{1.6109\,ft} = 1.0832$

∴ optimal stride length increases ~8.3 % on going from 1" to 6" heels +1 pt #, +1 sig figs

b. $(-\dot{W})_{opt} = \frac{\alpha M}{2}\frac{v^3}{a^*} + \frac{Mg\,v a^*}{8L}$ ⇒ need M for 1", 6" +3 pts

1" shoes: $M_1 = 135 + 2(1.0\ lb_m) = 137\ lb_m$ +1 pt
6" shoes: $M_6 = 135 + 2(2.7\ lb_m) = 140.4\ lb_m$ +1 pt

EITHER: short cut: plug in $2v\sqrt{\frac{\alpha L}{g}}$ for a^* in expression for $-\dot{W}$

$$(-\dot{W})_{opt} = \frac{\alpha M}{2}\frac{v^3}{(2v\sqrt{\alpha L/g})} + \frac{Mg}{8L}v\left(2v\sqrt{\alpha L/g}\right) = \frac{Mv^2}{2}\sqrt{\frac{\alpha g}{L}}$$

$$\frac{(-\dot{W})_{opt\,6}}{(-\dot{W})_{opt\,1}} = \frac{\frac{M_6 v^2}{2}\sqrt{\alpha_6 g/L_6}}{\frac{M_1 v^2}{2}\sqrt{\alpha_1 g/L_1}} = \frac{M_6\sqrt{\alpha_6/L_6}}{M_1\sqrt{\alpha_1/L_1}} \text{ +6 pts}$$

$$= \frac{(140.4\ lb_m)\sqrt{(0.1923)/(3.7\ ft)}}{(137\ lb_m)\sqrt{(0.1847)/(3.283)}} = 0.98505$$

OR: long-cut: let's first convert v to ft/s as we'll need to use this several times (and it's the only conversion we really need)

$$v = \left(4.0\ \frac{mi}{hr}\right)\left(\frac{5280\ ft}{mi}\right)\left(\frac{hr}{3600\ s}\right) = 5.867\ ft/s$$ +2 pts

$$(-\dot{W})_{opt_6} = \frac{\alpha_6 M_6}{2}\frac{v^3}{a^*_6} + \frac{M_6 g}{8L_6}v a^*_6 \text{ +2 pts}$$

$$= \frac{(.1923)(140.4\ lb_m)}{2}\frac{(5.867\ \frac{ft}{s})^3}{(1.7449\ ft)} + \frac{(140.4\ lb_m)(32.174\ \frac{ft}{s^2})(5.867\ \frac{ft}{s})(1.7449\ ft)}{8\ (3.7\ ft)}$$

$$= 3{,}186\ \frac{lb_m \cdot ft^2}{s^3}\left(\left(\frac{1\ lb_f}{32.174\ \frac{lb_m \cdot ft}{s^2}}\right)\left(\frac{1\ hp}{550\ ft \cdot lb_f/s}\right)\right)$$

$$= 0.17659\ hp$$

(6)

$$(-\dot{W})_{opt_1} = \frac{\alpha_1 M_1}{2}\frac{V^3}{q_1^*} + \frac{M_1 g}{8 L_1} V q_1^* \quad \text{+2 pts}$$

$$= \frac{(.1847)(137\ lbm)(5.567\ \frac{ft}{s})^3}{2\ (ft)} + \frac{(137\ lbm)(32.174\ ft/s^2)}{8(3.2833\ ft)}(5.567\ \frac{ft}{s})(1.6109\ ft)$$

$$= 3172.2\ \frac{lbm \cdot ft^2}{s^3}\left(\left\langle \frac{1\ lbf}{32.174\ \frac{lbm \cdot ft}{s^2}}\right\rangle \times \left\langle \frac{1\ hp}{550\ ft \cdot lbf/s}\right\rangle\right)$$

$$= 0.17927\ hp$$

$$\frac{(-\dot{W})_{opt_6}}{(-\dot{W})_{opt_1}} = \frac{3124.8\ \frac{lbm \cdot ft^2}{s^3}}{3172.2\ \frac{lbm \cdot ft^2}{s^3}} = \frac{0.17659\ hp}{0.17927\ hp} = 0.98506$$

$\therefore (-\dot{W})_{opt}$ decreases by ~1.5 % on going from 1" to 6" heels +1 pt #, +1 pt sig figs

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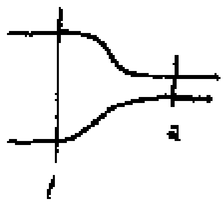
2. (26 pts) Blood flows through an aorta with a 1.0 cm inner diameter at an average velocity of 50.0 cm/s at an average pressure of 100 mmHg. This blood enters a region of stenosis (a narrowing, perhaps due to a sclerotic plaque) where the inner diameter is only 0.50 cm. The viscosity of the blood is 3.0 cP.
   a. (16 pts) Make a good estimate of the average pressure, in mmHg, in the narrow region.
   b. (10 pts) Is the flow in the narrowed region laminar or turbulent?

(2) a. 

inviscid flow : Bernoulli eqn ...

$$P_1 + \rho_{blood}\frac{\vec{V}_1^2}{2} + \cancel{\rho_{blood}\, g z_1} = P_2 + \rho_{blood}\frac{\vec{V}_2^2}{2} + \cancel{\rho_{blood}\, g z_2}$$

assume $z_1 \approx z_2$ **+6 pts**

know: P_1, ρ_{blood}, $\vec{V}_1$, d_1, d_2

what about $\vec{V}_2$? : continuity eqn ...

$$\vec{V}_1 A_1 = \vec{V}_2 A_2$$ **+6 pts**

$$\vec{V}_2 = \frac{\vec{V}_1 A_1}{A_2} = \vec{V}_1 \frac{\pi d_1^2/4}{\pi d_2^2/4} = \vec{V}_1 \frac{d_1^2}{d_2^2}$$

$$= \left(50.0\,\frac{cm}{s}\right)\frac{(1.0\,cm)^2}{(0.5\,cm)^2} = 200\,\frac{cm}{s}$$

Now: $$P_1 - P_2 = \frac{\rho_{blood}}{2}\left(\vec{V}_2^2 - \vec{V}_1^2\right)$$

$$(100\,mmHg) - P_2 = \frac{(1.056\,g/cm^3)}{2}\left(\left(200\,\frac{cm}{s}\right)^2 - \left(50\,\frac{cm}{s}\right)^2\right)\left\langle\frac{7.5\times10^{-4}\,mmHg}{g/(cm\cdot s^2)}\right\rangle$$

+2 pts

$$-P_2 = -85.15\,mmHg$$

$$P_2 \approx 85\,mmHg$$ **+1 pt #, +1 pt sig fig**

b. $$Re = \frac{\rho_{blood}\vec{V}_2 d_2}{\mu}$$ **+6 pts**

$$= \frac{(1.056\,g/cm^3)(200\,cm/s)(0.5\,cm)}{(3.0\,cP)\left\langle\frac{0.01\,g/(cm\cdot s)}{1\,cP}\right\rangle} = 3520$$ **+2 pts**

Since $Re > 2100$, the flow in the stenosis is turbulent

+2 pts

3. (26 pts) The basal metabolic rate, BMR, of newborns is given by the correlation:

$$BMR = 0.054W^{1.5}H$$

where BMR is in kcal/hr, W is weight in kg and H is height in cm.

a. (6 pts) What are the units of the constant 0.054 in the equation above?

b. (6 pts) Rewrite the equation above so that BMR may be calculated by plugging in values for W in lbm and H in inches.

c. (6 pts) A large population of infants from a given nursery is found to have $W = 4.5 \pm 0.7$ kg and $H = 40.2 \pm 9.9$ cm; here population data is given as the (mean ± one standard deviation). Determine the value (mean ± one standard error) for the BMR in kcal/hr for this population of infants.

d. (8 pts) If 20 such infants were present in a well-insulated nursery and the air conditioning failed, at what rate in °C/hr (mean ± one standard error), would the temperature rise in the nursery? You may assume a room volume of 499 m^3, $\rho_{air} = 1.1769$ kg/m^3, and $Cp_{air} = 0.2405$ kcal/(kg•°C).

3 a. $BMR = 0.054 W^{1.5} H \Rightarrow \frac{kcal}{hr} [=] (?)\ kg^{1.5} \cdot cm$

$\therefore 0.054 [=] \frac{kcal}{hr \cdot kg^{1.5} \cdot cm}$ +6 pts

b. $0.054 \frac{kcal}{hr \cdot kg^{1.5} \cdot cm} \left\langle \frac{1\ kg}{2.2\ lbm} \right\rangle^{1.5} \left\langle \frac{2.54\ cm}{1\ in} \right\rangle = 0.040933 \frac{kcal}{hr \cdot lbm^{1.5} \cdot in}$

$\therefore BMR = 0.040 W^{1.5} H$ for $W [=] lbm$ and $H [=] in$ +6 pts

c. $\overline{BMR} = (0.054)\overline{W}^{1.5}\overline{H}$

$= \left(0.054 \frac{kcal}{hr \cdot kg^{1.5} \cdot cm}\right)(4.5\ kg)^{1.5}(40.2\ cm)$

$= 20.72\ kcal/hr$

$Var(BMR) = \left(\frac{\partial BMR}{\partial W}\right)^2\Big|_{\overline{W},\overline{H}} Var(W) + \left(\frac{\partial BMR}{\partial H}\right)^2\Big|_{\overline{W},\overline{H}} Var(H)$ +4 pts

$\left(\frac{\partial BMR}{\partial W}\right) = 0.054(1.5) W^{0.5} H$

$= \left(0.054 \frac{kcal}{hr \cdot kg^{1.5} \cdot cm}\right)(1.5)(4.5\ kg)^{0.5}(40.2\ cm)$

$= 6.907 \frac{kcal}{hr \cdot kg}$

$\left(\frac{\partial BMR}{\partial H}\right) = 0.054 W^{1.5}$

$= \left(0.054 \frac{kcal}{hr \cdot kg^{1.5} \cdot cm}\right)(4.5\ kg)^{1.5}$

$= 0.5155 \frac{kcal}{hr \cdot cm}$

$$Var(BMR) = \left(6.907 \frac{kcal}{hr \cdot kg}\right)^2 (0.7\,kg)^2 + \left(0.5155 \frac{kcal}{hr \cdot cm}\right)^2 (9.9)^2$$
$$= 35.94 \frac{kcal^2}{hr^2}$$
$$S.E.(BMR) = \sqrt{Var(BMR)} = \sqrt{35.94 \frac{kcal^2}{hr^2}} = 5.995 \frac{kcal}{hr}$$

$\therefore BMR = 21 \pm 6 \frac{kcal}{hr}$ +1 pt #, +1 pt sig fig

d. Energy balance on system (air in nursery)
$$\frac{dE_{sys}}{dt} = \dot{Q} - \cancel{\dot{W}}^{\,0}$$
$$\frac{d[m_{sys}(u_{sys} + \cancel{v^2_{sys}}^{\,0} + \cancel{gz_{sys}}^{\,0})]}{dt} = \dot{Q} = n\,BMR$$

$$\frac{d(m_{sys} u_{sys})}{dt} = m_{sys} \frac{du_{sys}}{dt} = m_{sys} C_{p_{sys}} \frac{dT_{sys}}{dt} = n\,BMR$$

$$\Rightarrow \frac{dT_{air}}{dt} = \frac{n\,BMR}{m_{air} C_{p_{air}}} = \frac{n\,BMR}{\rho_{air} V_{air} C_{p_{air}}}$$ +4 pts

$$\frac{dT_{air}}{dt} = \frac{20\,(21\,kcal/hr)}{(1.1769 \frac{kg}{m^3})(499\,m^3)(0.2405 \frac{kcal}{kg \cdot °C})} = 2.974 \frac{°C}{hr}$$

$$Var(dT/dt) = \left(\frac{\partial(dT/dt)}{\partial BMR}\bigg|_{\overline{BMR}}\right)^2 Var(BMR)$$

$$\Rightarrow S.E.(dT/dt) = \left(\frac{\partial(dT/dt)}{\partial BMR}\right)\bigg|_{\overline{BMR}} S.E.(BMR)$$ +2 pts

$$\frac{\partial(dT/dt)}{\partial BMR} = \frac{n}{\rho_{air} V_{air} C_{p_{air}}} = \frac{20}{(1.1769 \frac{kg}{m^3})(499\,m^3)(0.2405 \frac{kcal}{kg \cdot °C})}$$
$$= 0.1416\ °C/kcal$$

$$S.E.(dT/dt) = (0.1416\ °C/kcal)(5.995\ kcal/hr) = 0.8489\ °C/hr$$

$\therefore dT/dt = +3.0 \pm 0.8\ °C/hr$ +1 pt 3, +1 pt sig fig