Topic 8. Bio-Fluid Mechanics

Topics

- fluid properties
- hydrostatics
- flow – mass balancing
- flow – inviscid flow
- flow – viscous flow
- blood rheology

The second category of biomechanics problems we will consider is bio-fluid mechanics. This refers to the study of how fluids (liquids or gases) flow in response to pressure gradients. Just as electrical current flows in response to a voltage gradient, heat is transferred in response to a temperature gradient, and molecules diffuse in response to a concentration gradient, fluids flow in response to a gradient of pressure. Fluid mechanics is a core subject of chemical engineering, mechanical engineering and civil engineering. The basic principles are the same, but the applications of bio-fluid mechanics often present unique challenges. Understanding how fluids flow through pipes is necessary for selecting appropriate pumps in an industrial setting and so is commonly studied by each of these traditional engineering disciplines. Flow through pipes is important for biomedical engineers as well, only now the pipes may be arteries whose walls are elastic (unlike the rigid pipes encountered in an engine or manufacturing plant), and now the fluid could be blood – a complex mixture of water, proteins, and deformable "semi-solid" cells (unlike the relatively simple fluids usually, but not always, found in the other applications).

We will start with the basics and consider a few applications to fluid flow problems in the body.

Fluid Properties – density

Density, ρ

mass per unit volume

ρ [=] M/L^3

An "incompressible" fluid has a constant density.

Many fluids have negligible density changes as pressure increases, e.g. water, and although $\rho \propto 1/T$, the constant of proportionality is very small.

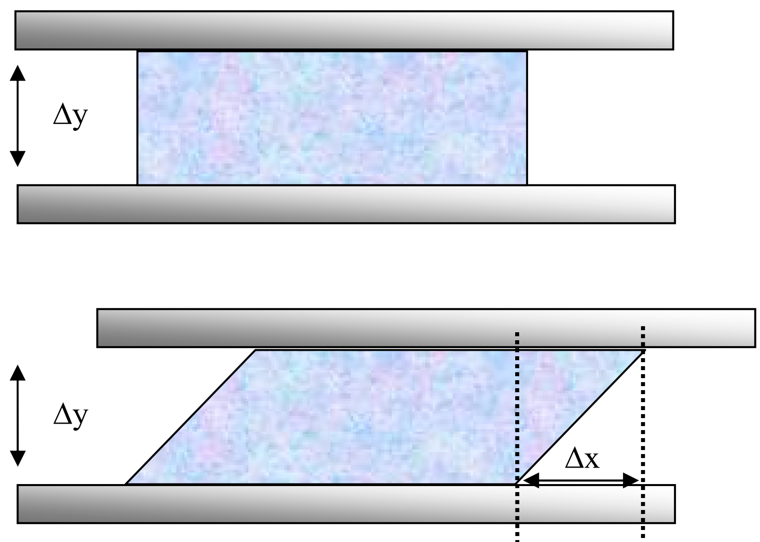
Notable exception: gases, e.g. $PV = nRT \Rightarrow n/V = P/(RT)$

Specific Gravity, S.G. $\equiv \rho(\text{liquid})/\rho(\text{H2O typically @ 15°C, 999 kg/m}^3)$

Fluid Properties - viscosity

Every fluid can be characterized by a *viscosity*. This is a material property that describes a fluid's resistance to flow. Not every fluid will flow at the same rate in response to the same pressure drop. Consider how much less work it requires to drink water through a straw than to drink a milkshake through a straw. The two liquids have much different viscosities.

The viscosity has a fundamental mathematical definition. Consider a volume of fluid held between two parallel plates. The distance between the plates is Δy. The top plate is moved relative to the bottom plate by an amount Δx. One of the key ideas of fluid mechanics is the notion of the *no-slip boundary condition*. This means that the layer of molecules in the fluid that lie directly on a solid surface do not slip along that surface. If the surface is stationary (velocity v = 0), then the layer of molecules on that surface also must be stationary (v = 0). Likewise, the layer of molecules in contact with the top surface must be rigidly coupled to the top surface. So, if the top surface moved by an amount Δx, the topmost layer of fluid must have moved by Δx too. The result is that the fluid is *sheared* between the two surfaces.

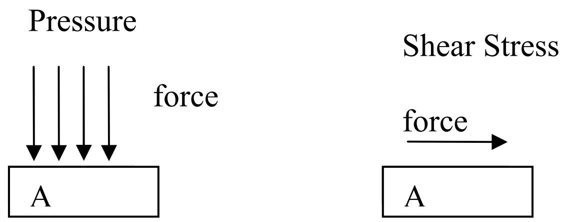


Deformation of a fluid element by moving the top plate by an amount Δx relative to the bottom plate

We define the *strain* (γ) as the ratio of the deformation Δx to the distance Δy.

$$\gamma \equiv \frac{\Delta x}{\Delta y}$$

The act of shearing the fluid requires that the fluid must exert some force on the solid surface (and the bulk fluid must exert a force on the layer of fluid that is "stuck" to the solid surface). We take the magnitude of this force, divided by the area of the surface, and call it the *shear stress* (τ). Shear stress has units of force/area, and thus has the same units as pressure. The difference between pressure (force/area) and shear stress (force/area) is that pressure forces act perpendicular to the surface while shear stress acts parallel to the surface.



The magnitude of the strain is proportional to the magnitude of the shear stress and on the amount of time (Δt) that the stress was applied:

$$\tau \Delta t \propto \frac{\Delta x}{\Delta y}$$

The using μ as a proportionality constant, we can rearrange this to give

$$\tau = \mu \frac{\Delta x / \Delta t}{\Delta y}$$

The ratio $\Delta x / \Delta t$ can be recognized as the difference in velocity of the top plate relative to the bottom plate $\Delta x / \Delta t = \Delta v$. Thus,

$$\tau = \mu \frac{\Delta v}{\Delta y}$$

If the gap between the plates is infinitesimally small (Δy → dy), this is

$$\tau = \mu \frac{dv}{dy}.$$

The derivative dv/dy indicates how the horizontal (*x*-direction) fluid velocity changes with vertical (*y*-direction) distance. This derivative is called the *strain rate*

$$\dot{\gamma} \equiv \frac{dv}{dy}$$

So, we can write

$$\tau = \mu \dot{\gamma}$$

The proportionality constant μ is the viscosity of the fluid. For a given strain rate (*dv/dy*) a more viscous fluid results in a larger shear stress. If a fluid has a constant value for the viscosity, then it is called a *Newtonian fluid*. Most simple fluids are Newtonian, but many complex fluids are non-Newtonian, meaning their viscosity depends on the strain rate. Blood is non-Newtonian.

Units of Viscosity

Examining the dimensions in $\tau = \mu\dot{\gamma}$, we can figure out the units that μ must have.

τ [=] force/area SI unit = Pa
$\dot{\gamma}$ [=] time^{-1} SI unit = s^{-1}

In order for $\tau = \mu\dot{\gamma}$ to be dimensionally consistent, μ must have units of (force/area)xtime. The SI unit for viscosity is Pa·s (Pascal-seconds). A common unit for viscosity in engineering literature is centipoises (cP). The viscosity of water at room temperature is 1 cP. (This is why it remains a somewhat popular unit – it is easy to remember.) 1 Pa·s = 1000 cP.

Some examples of non-Newtonian behavior are *shear-thinning* fluids (viscosity decreases with increasing shear rate – paints are shear-thinning), *shear-thickening* fluids (viscosity increases with increasing shear rate – some particle dispersions in liquids are shear-thickening), and *yield stress* (solid-like behavior with no shear until a threshold stress is reached, at which point the material flows as a fluid – ketchup has a yield stress.) These non-Newtonian behaviors have their origins in complex structures that macromolecular solutes can form in the liquid.

Note, sometimes the "kinematic viscosity" is used in equations. This is the viscosity divided by the density: "nu", ν = μ/ρ

Examining the dimensions of ν…

$$\nu = \frac{\mu}{\rho} \; [=] \; \frac{M/(L \bullet t)}{M/L^3} \; [=] \; \frac{L^2}{t}$$

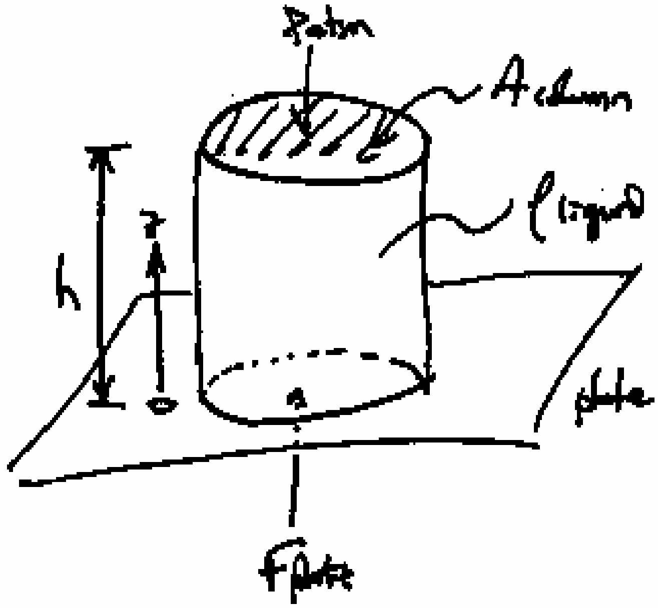
1 centistoke = 0.01 cm^2/s

Hydrostatics

Flow / No Flow considerations (for Newtonian Fluid)

No Flow: "Hydrostatics" ≡ fluid at rest in a gravitational field, $\tau = 0$, and pressure variations are solely to weight of fluid

Consider a fluid at rest in a column, its behavior is described by a force balance...



force balance @ $z = 0$

$$F_{plate} = \underbrace{F_{liquid\ column}}_{m_{liquid}\, g \,=\, \rho_{liquid}\, g\, V_{liquid}} + P_{atm} \cdot A_{column}$$

So $\dfrac{F_{plate}}{A_{column}} = \dfrac{\rho_{liquid}\, g\, V_{liquid} + P_{atm} A_{column}}{A_{column}}$

$\Rightarrow$ $\underset{\text{or } P_{abs}}{P_{plate}} = \rho_{liquid}\, g\, h + P_{atm}$, an absolute pressure

note P_{plate} doesn't depend on A_{column}, only on the height of the column ...

$P_{abs} - P_{atm} = P_{gage} = \rho_{liquid}\, g h$

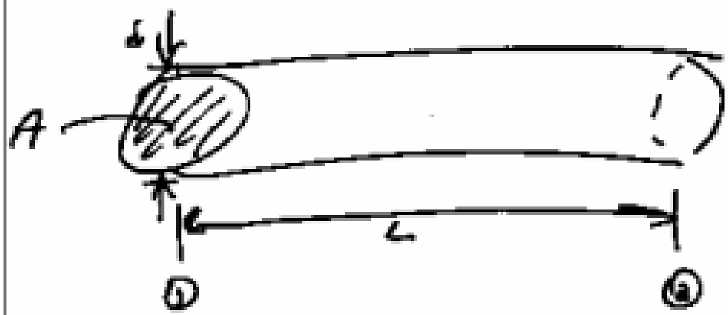
relative or gage pressure: gages measure pressures relative to atmospheric pressure...

a sphygmomanometer – device used to measure blood pressure – measures pressures in terms of the height of a column of mercury

Hydrostatics examples: sphygmomanometer, blood pressure standing up versus lying down

Flow – mass balancing considerations

Flow: in a circular tube



① Consider a mass balance:

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 \dot{V}_1 = \rho_1 \dot{V}_2$$

now, volumetric flow rate = average velocity × area

So, $\dot{V} = \vec{V} \cdot A$

and $\rho_1 \vec{V}_1 A_1 = \rho_2 \vec{V}_2 A_2$

If our fluid is incompressible, $\rho_1 = \rho_2 = \rho$, a constant

$\therefore \quad \vec{V}_1 A_1 = \vec{V}_2 A_2$ • form of the "Continuity Equation"

Two Types of Flow

For modeling purposes, there are two primary types of flow – *turbulent flow* and *laminar flow*. Turbulent flow is associate with wild eddies and recirculations. The velocity fluctuates throughout the fluid, but on average, there is a constant velocity across the entire flowing cross-section. If you put a drop of dye in a liquid in turbulent flow, it rapidly mixes with the fluid. Turbulent flow is associated with high kinetic energy flows.

In laminar flow, fluid follows smooth streamlines. The velocity varies smoothly across the flowing cross-section. If you put a drop of dye in a liquid in laminar flow, the drop tends to move smoothly downstream with very little mixing.

The transition from laminar to turbulent flow occurs at a characteristic value of a dimensionless number called the *Reynolds number*, Re.

$\mathrm{Re} = \dfrac{Dv\rho}{\mu}$ where D is the diameter of the vessel, v is the velocity, ρ is the density, and μ is the viscosity. The Reynolds number is the ratio of "inertial forces" that tend to keep a fluid moving to "viscous forces" that tend to slow a fluid down. For flow in a circular cross-section pipe, the transition from laminar to turbulent flow is observed experimentally to occur at Re = 2000 for all Newtonian fluids.

Is turbulence an issue in blood flow in humans?

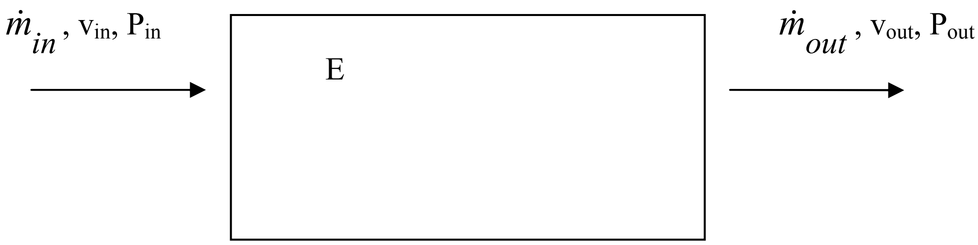
Vessel	Radius, cm	Re
Proximal aorta	1.5	1500
Femoral artery	0.27	180
Left main coronary artery	0.425	270
Left anterior descending coronary artery	0.17	80
Right coronary artery	0.097	233
Terminal arteries	0.05	17

From Truskey, Yuan and Katz *Transport Phenomena in Biological Systems*, Pearson Prentice Hall Bioengineering, 2004.

The way we describe flows mathematically depends on the value of Re. Flows where Re is large can be modeled as inviscid flows (viscosity effects are negligible in determining how the fluid flows); flows where Re is not-so-large to small can be modeled as viscous flows (viscosity effects determine how the fluid flows).

Inviscid Flows – The Bernoulli Equation

Flows that are characterized by large shear rates can produce large *viscous losses*, meaning that energy is lost from the flowing fluid as heat. The origin of this heat is the friction between flowing molecules. Just as friction produces the heat you feel when you rub your hands together, friction between molecules flowing past one another produces heat. In many flows, we can neglect these viscous losses, and use the energy balance equation to derive a useful engineering equation that helps us keep track of pressure variations in non-uniform conduits.



Defining our system as a fluid element containing an amount of energy E, we write

$$\frac{dE}{dt} = \dot{m}_{in}\left(u + \tfrac{1}{2}v^2 + gh\right)_{in} - \dot{m}_{out}\left(u + \tfrac{1}{2}v^2 + gh\right)_{out} + \dot{Q} + \dot{W}$$

Because we assumed no viscous losses, we neglect heat transfer ($\dot{Q} = 0$). We assume steady-state flow, so dE/dt = 0. Conservation of mass tells us that $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. We also assume isothermal flow conditions (isothermal = constant temperature, so u does not change). Thus

$$0 = \dot{m}\left(\frac{v_{in}^2}{2} - \frac{v_{out}^2}{2} + gh_{in} - gh_{out}\right) + \dot{W}$$

Recalling again our heart analysis, the flow work done on the fluid element by fluid entering the system is

$$\dot{W}_{in} = P_{in}\dot{V} = \frac{\dot{m}P_{in}}{\rho}$$

and the work done by the fluid element on fluid in the outlet is

$$\dot{W}_{out} = -P_{out}\dot{V} = -\frac{\dot{m}P_{out}}{\rho}$$

Thus, the net flow work is

$$\dot{W} = \frac{\dot{m}\left(P_{in} - P_{out}\right)}{\rho}$$

Substituting this into the energy balance gives

$$0 = \dot{m}\left(\frac{v_{in}^2}{2} - \frac{v_{out}^2}{2} + gh_{in} - gh_{out} + \frac{P_{in}}{\rho} - \frac{P_{out}}{\rho} \right)$$

Or,

$$\frac{v_{in}^2}{2} + gh_{in} + \frac{P_{in}}{\rho} = \frac{v_{out}^2}{2} + gh_{out} + \frac{P_{out}}{\rho}$$

We wrote this in terms of an inlet and an outlet, but in more general terms, we can use this to relate v, h and P at any two points "1" and "2" in a flowing, isothermal fluid.

$$\frac{v_1^2}{2} + gh_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + gh_2 + \frac{P_2}{\rho}$$

This is the *Bernoulli Equation*.

the "steady state "Bernoulli Eqn" or "mechanical energy balance"

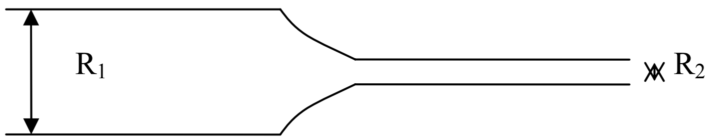
valid for – inviscid flow (neglecting viscosity effects)
- steady state
- adiabatic
- no non-flow work
- no thermal effects
- incompressible fluid.

This is a special case that may be used to estimate behavior of flowing system, e.g. pressure drops ($\Delta P = P_2 - P_1$), assuming viscous effects can be neglected – low viscosity fluids, high Re., e.g. ΔP across a heart valve

The Bernoulli Equation is also useful when the flow geometry irregular…

Inviscid Flow Examples:

Consider blood flowing into a narrow constriction. Find the difference in pressure at points 1 and 2, if the cylindrical vessel has a radius R_1 at point 1 and a radius R_2 at point 2.



Assuming a horizontal arrangement, $h_1 = h_2$, so we neglect those terms. By conservation of mass, the flowrate must be constant. Thus the average velocity at any location must be

$$\langle v \rangle = \frac{\dot{m}}{\pi R^2 \rho}$$

Substituting the following for v_1 and v_2:

$$\langle v \rangle_1 = \frac{\dot{m}}{\pi R_1^2 \rho}$$

$$\langle v \rangle_2 = \frac{\dot{m}}{\pi R_2^2 \rho}$$

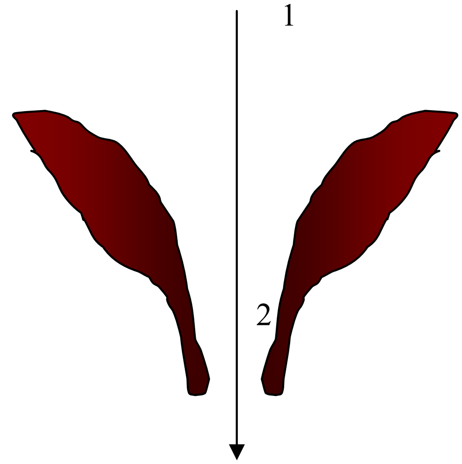
into the Bernoulli Equation, we can rearrange to get

$$P_2 - P_1 = \frac{\dot{m}^2}{2\pi^2 \rho}\left(\frac{1}{R_1^4} - \frac{1}{R_2^4}\right)$$

Since $R_2 < R_1$, we find that $P_2 - P_1 < 0$. There is a drop in pressure at the constriction. This is due to the acceleration of the fluid as it passes through the constriction.

Extension to a stenotic heart valve

Schematic of heart valve



Stenosis is a narrowing within a flow channel, so a stenotic heart valve is one with a narrowed cross-section for blood flow. This can result from degeneration of the valves, congenital defects, or diseases such as bacterial endocarditis or rheumatic heart fever. Sometimes the valve is normal but the aorta downstream is constricted. Stenosis causes the heart to work harder to pump the blood. The heart compensates by becoming more muscular (hypertrophy), but this causes other problems with heart function that may be serious.

Mild stenosis can be treated with anti-clotting drugs or drugs that regulate heart function. About 90,000 surgeries are performed annually for stenosis. This is typically valve replacement surgery. About 19,600 fatalities occur in the US annually from valve defects.

The Bernoulli equation is used to non-invasively measure pressure drops across stenotic valves. The technology is Doppler Ultrasound Imaging (Doppler Flow Mapping). This provides a local measure of blood velocity, from which the pressure is calculated.

Referring to the drawing above, determine the pressure in the valve (position 2) relative to the pressure in the heart chamber just before the valve inlet (position 1).

Neglecting small differences in h, the Bernoulli equation tells us that

$$P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right)$$

Doctors like to report pressure in mm Hg (1 mm Hg = 133.32 Pa). If the Doppler device reports velocities in meters/second, and we use the density of blood $\rho = 1070$ kg/m^3, this becomes

$$P_1 - P_2 = 4\left(v_2^2 - v_1^2\right)$$

with the pressure difference reported in mm Hg.

Usually, the velocity in the chamber is much less than that in the valve. Using the maximum velocity measured in the valve, and invoking the viscous flow result that $v_{max} = 2v_{avg}$, we can then write

$$P_1 - P_2 = 4v_{max}^2$$

Here the Bernoulli Equation made it possible to non-invasively measure the local pressure in the valve, rather than the alternative, which is having to invasively insert a pressure probe via a catheter.

Viscous Flows – The Hagen-Poiseuille Law (Laminar Flow in a Cylindrical Vessel)

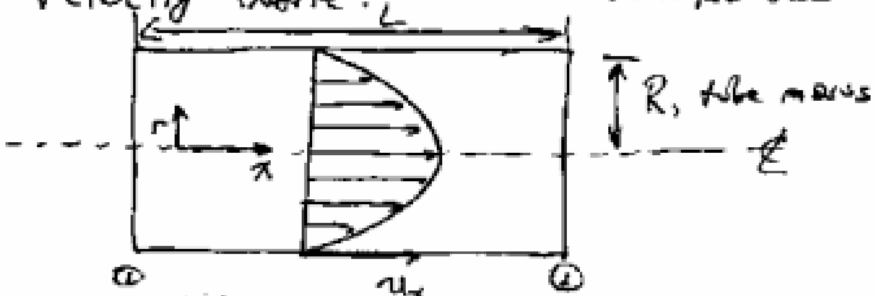
Small Re flows (<~100) where viscosity effects determine how the fluid flows – where energy is dissipated by viscosity (e.g. Volvo 4WD example…).

Because of the relevance to blood flow, we here present the *Hagen-Poiseuille Law* that relates the pressure drop between the entrance and exit of a pipe of length L and radius R to the volumetric flowrate, Q [=] volume/time, of a Newtonian fluid in laminar flow. The analysis is based on a force balance on differential fluid shell element.

This analysis requires some calculus as we must integrate the force balance on this diff. element across the tube – makes sense as we know that due to no-slip condition, there will be velocity differences between the walls and center of tube

Summary of results, $\left\{ \begin{array}{l} Re < 2100 \text{ (laminar flow)} \\ \mu = \text{constant (Newtonian)} \\ \text{well-developed flow} \end{array} \right.$

Velocity Profile:



note, detailed picture of velocity field results

$$u_x = -\frac{1}{4\mu}\underbrace{\frac{dP}{dx}}\left(R^2 - r^2\right) \quad \text{, a parabolic profile}$$

$$\text{pressure drop} = \frac{\Delta P}{L} = \frac{P_2 - P_1}{L} < 0$$

note $u_x(r = R) = 0$, no slip, as expected

$\frac{\partial u_x}{\partial r} = 0$ @ $r = 0$, max velocity at center of tube so $u_x(r=0) = -\frac{1}{4\mu}\frac{\partial P}{\partial x} R^2 = u_{x,max}$

$\therefore u_x = u_{x,max}\left(1 - r^2/R^2\right)$

some derived quantities for this flow field:

average velocity, $\vec{V}$

$$\vec{V} = \frac{\int_0^{2\pi}\int_0^{R} u_x \, r\,\partial r\,\partial\theta}{\int_0^{2\pi}\int_0^{R} r\,\partial r\,\partial\theta} = \frac{1}{\pi R^2}\int_0^R u_x 2\pi r\,\partial r$$

$$= -\frac{R^2}{8\mu}\frac{\partial p}{\partial x} = \frac{1}{2} u_{x\,max}$$

volumetric flow rate $\dot{V} (= Q)$

$$\dot{V} (= Q) = \vec{V} \cdot A = -\frac{R^2}{8\mu}\frac{dp}{dx} \cdot \pi R^2 = -\frac{\pi R^4}{8\mu}\frac{\Delta P}{L}$$

rearranging for pressure drop,

$$\Delta P = \frac{8\mu \dot{V} L}{\pi R^4}$$

makes intuitive sense as $\Delta P \uparrow$ when $\mu\uparrow, \dot{V}\uparrow, L\uparrow, R\downarrow$

shear stress

$\tau = \mu \frac{\partial u_x}{\partial r}$, $\frac{\partial u_x}{\partial r} = \frac{1}{2\mu}\frac{\partial P}{\partial x} r$

$\tau = \frac{1}{2}\frac{\Delta P}{L} r$

$\tau = 0$ at center

$\tau = \frac{1}{2}\frac{\Delta P}{L} R$, max at wall of tube

Can use these results to calculate details of the flow field, e.g. stress on the wall of the tube, pressure drop, etc.

The centerline velocity v_{max} is equal to twice the average velocity $\langle v \rangle$, where $\langle v \rangle = Q/\pi R^2$.

As we discussed earlier in the application of the energy balance to the power to pump the human heart, the power required to pump a fluid is

$$\dot{W} = Q\Delta P$$

Thus, if the flow is laminar, we can relate DP to the viscosity, tube length, tube radius and volumetric flow rate via the Hagen-Poiseuille Law to arrive at

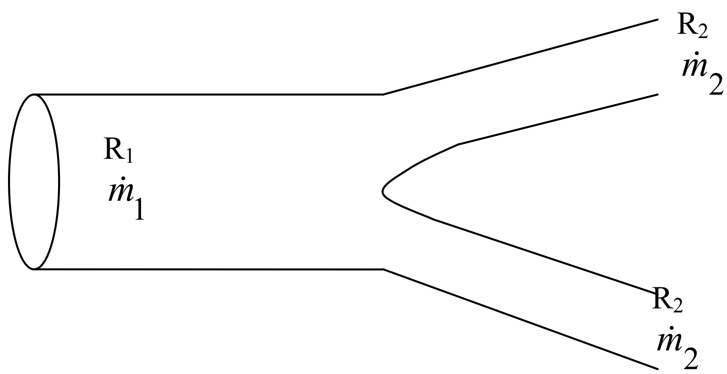
$$\dot{W} = \frac{8\mu L Q^2}{\pi R^4}$$

Viscous Flow Examples: Estimate Shear Stress and Shear Rate at wall of Aorta; Pressure Drop in Capillaries

Alternative Invisicid flow treatment of pressure drop in the capillaries.

In the text, the example sketched in Figure 11.8 illustrates how the Hagen-Poiseuille Law can be used to calculate how the pressure varies at a branching point in a capillary network.

We can also use the Bernoulli Equation to estimate how the degree of size reduction in a branching blood vessel may lead to pressure buildup or pressure decreases.



Consider a single artery of radius R_1 that splits into two branches, both of which have radius R_2. At steady-state, there is no buildup of blood, so the flowrate $\dot{m}_1$ entering the branchpoint, must equal the total flowrate out of the branchpoint, which is equal to $2\dot{m}_2$. (We assume that since the branches have equal sizes, they split the blood flow equally.)

Again neglecting height differences, the Bernoulli Equation is

$$P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right)$$

Calculate the relationship between $\langle v_2 \rangle$ and $\langle v_1 \rangle$, where we use the < > to indicate an average velocity

$$\langle v_1 \rangle = \frac{\dot{m}_1}{\rho \pi R_1^2}$$

$$\langle v_2 \rangle = \frac{\dot{m}_2}{\rho \pi R_2^2}$$

Now, setting $\dot{m} = \dot{m}_1 = 2\dot{m}_2$, this becomes

$$\langle v_1 \rangle = \frac{\dot{m}}{\rho \pi R_1^2}$$

$$\langle v_2 \rangle = \frac{\dot{m}}{2\rho \pi R_2^2}$$

and the Bernoulli Equation becomes

$$P_1 - P_2 = \frac{\rho}{2}\left(\left(\frac{\dot{m}}{2\rho\pi R_2^2}\right)^2 - \left(\frac{\dot{m}}{\rho\pi R_1^2}\right)^2\right)$$

$$P_1 - P_2 = \frac{\dot{m}^2}{2\rho\pi^2}\left(\frac{1}{4R_2^4} - \frac{1}{R_1^4}\right)$$

What if $R_2 = R_1/2$? What is the local pressure change at that branchpoint?

$$P_1 - P_2 = \frac{\dot{m}^2}{2\rho\pi^2}\left(\frac{1}{4\left[\frac{R_1}{2}\right]^4} - \frac{1}{R_1^4}\right)$$

$$P_1 - P_2 = \frac{\dot{m}^2}{2\rho\pi^2 R_1^4}(4-1) = \frac{3\dot{m}^2}{2\rho\pi^2 R_1^4}$$

Is there a relationship between R_2 and R_1 that gives no pressure difference at the branchpoint ($P_1 - P_2 = 0$)?

$$0 = \frac{\dot{m}^2}{2\rho\pi^2}\left(\frac{1}{4R_2^4} - \frac{1}{R_1^4}\right)$$

$$4R_2^4 = R_1^4$$

$$R_2 = \frac{R_1}{4^{1/4}} = \frac{R_1}{1.414}$$

This would be the optimal branching geometry for one vessel splitting into two vessels. What if one vessel split into 3, or 4, or more vessels?

Blood Rheology

<u>Blood Properties</u>

we're familiar with air ($\mu \approx 0$), water ($\mu \sim 1$ cP small, but finite), oil ($\mu \sim 100$'s cP) behavior, what about blood?

Blood is a solution of cells and proteins and other small molecules

<u>Whole Blood</u> (serum)

~45 vol % "hematocrit" → <u>cells</u> (erythrocytes)
95% red blood cells (RBCs)
4.9% platelets
~0.13% white blood cells (leukocytes)

~55 vol % → <u>plasma</u>
dilute, 0.15 N electrolyte soln w/ ~8 wt% proteins
+ carbohydrates,
organic acids,
nitrogenous compounds (urea)
lipids,
vitamins,
hormones,
enzymes,

RBCs 8.4 μm × 2.4 μm biconcave disks
platelets 2-5 μm
leukocytes 7-20 μm

s.g. ~ 1.056

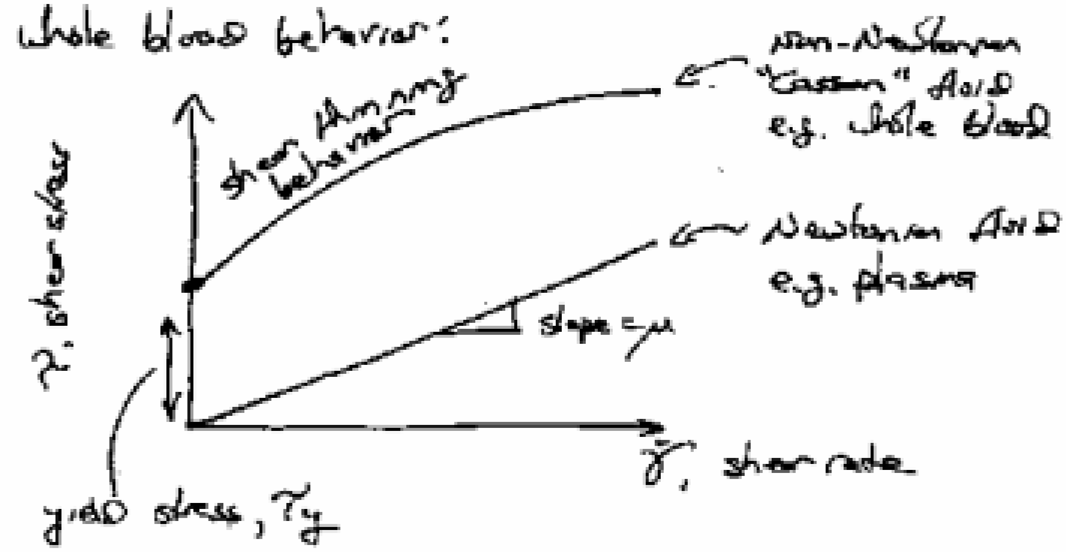
$\mu \sim 3.0$ cP @ high $\dot{\gamma}$ ...

Blood Rheology

Whole blood is a concentrated solution of particles (cells) and dilute solution of polymers (proteins). During flow, the cells interact (RBCs stack into "rouleaux") causing the flow behavior to be very different than water ⇒ non-Newtonian (e.g. μ is NOT constant).

Plasma, since it is a dilute solution, does behave like a Newtonian fluid with $\mu \approx 1.2$ cP, a little more viscous than H_2O

Whole blood behavior:



yield stress: – must apply some force (shear stress) before fluid will start to move. If $\tau < \tau_y$, fluid behaves as rigid solid
– cells stack up, pack together; need to apply some stress to disrupt packing and allow fluid to move

These RBC stacks are known as "rouleaux"

Shear thinning – harder fluid is sheared, the less viscous it becomes
– harder blood is sheared, the less opportunity there is for cells to interact, as interactions decrease, solution becomes less viscous

Casson equation:

$$\tau^{1/2} = \tau_y^{1/2} + \left[\frac{\mu_{plasma}}{(1-H)^{\alpha K - 1}}\right]^{1/2} \dot{\gamma}^{1/2}$$

where $\tau_y^{1/2} = (H - 0.10)(C_F + 0.5)$

H ↑ hematocrit (expressed as fraction)

C_F conc. Fibrinogen, g/100 ml plasma

note if $H < 10\%$, yield stress disappears

$\alpha K = f(\text{protein content})$

if $H \to 0$ (plasma) recover $\tau = \mu_{plasma}\dot{\gamma}$ as expected

At high shear rates ($> 100\ s^{-1}$), whole blood behaves as if it were Newtonian, with $\mu \sim 3.0$ ($\tau_y^{1/2}$ term becomes negligible ...)