A Simple Model of Pharmaceutical Price Dynamics

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This draft: October 25, 2002

Much of the work on this paper was completed while Vogt was a John M Olin visiting scholar at the Stigler Center for the Study of the Economy and the State, Graduate School of Business, University of Chicago, for which opportunity he is grateful. We thank Sam Peltzman, Gary Becker, Will Manning, the participants at the ELO and CHAS Workshops at the University of Chicago, and the participants at the Health Economics Seminar at RAND for helpful comments. We also wish to thank IMS America for kindly providing the data.

Abstract

Branded pharmaceutical firms use price and promotional strategy to manage public knowledge about their drugs. We propose a dynamic theory of pharmaceutical pricing and conduct an exploratory empirical analysis inspired by the theory. Our theory predicts a pattern of increasing prices and decreasing promotional activities over a drug's life cycle. Prices are kept low and advertising levels high early in the life cycle in order to build public knowledge about the drug. As knowledge grows, prices rise and advertising falls. If the management of this stock is important enough, this tendency of prices to rise can overwhelm the price-decreasing effect of entry by generic competitors late in the drug life cycle. We argue that our theory of price dynamics explains empirical regularities in this industry.

1 Introduction

Most IO models of competition among firms predict that entry of a new competitor will result in a price decrease. In the market for pharmaceutical drugs, however, the price of a branded drug commonly rises after its patent expires and (lower priced) generic drugs enter the market (see Caves et al., 1991; Grabowski and Vernon, 1992; Frank and Salkever, 1995) The behavior of the original patent holder seems counter-intuitive given the usual economic logic of competition.

In this paper, we present a new explanation for this stylized fact. The key to our explanation lies in the pharmaceutical company's dynamic demand management problem. Pricing and advertising strategies set today affect quantity demanded tomorrow. Patent expiry is an important event in the dynamic program shaping the firm's price and advertising path not only at and after expiry but before that point as well.

Because it is costly to learn about drugs, doctors' prescribing habits are sticky. Consequently, they possess an evolving stock of knowledge about each drug. A central problem for the pharmaceutical company is to shepherd this stock to its best advantage. At any point in time, low prices and high advertising increase quantity demanded, but they also shift up demand at future times, since more doctors gain experience with and knowledge of the product.

Much of the relevant information about a drug is molecule rather than manufacturer specific. This means that, after patent expiry, generic entrants enjoy spillover benefits from physicians' stock of knowledge. This reduces the branded manufacturer's incentive to advertise and to keep prices low (both in the post expiry period and in the run-up to patent

expiry).

The solution to the pharmaceutical company's dynamic problem calls for rising prices and falling advertising both before and after patent expiry as it first builds up and then exploits its drug's knowledge stock. There is a discontinuity at the point of patent expiry, due to the increased price competition that generic entry engenders.

In our story, despite the fact that branded prices continue to rise after patent expiry, it is still the case that patent expiry and generic entry has a salutary effect on prices. The price rise over the life cycle of the drug is caused by the pharmaceutical company's response to the inherent dynamics of demand. The price rise after generic entry is *not* caused by generic entry, as it would happen in any event.

2 Background

The literature on the effects of competition on drug pricing has a long history. In an examination of the effect of new products on the existing pharmaceutical prices, Comanor (1964) concludes that the introduction of new branded and patent protected products can increase product differentiation in the marketplace, and potentially increase prices. Temin (1980) agrees with this conclusion, though he uses a different (less theoretically grounded) argument. Peltzman (1973), on the other hand, argues that even though new pharmaceutical products may be introduced at higher prices than than the older drugs that are already on the market, the new drugs may nevertheless induce competitive effects that reduce overall drug prices.

Perloff et al. (1996) develop a theoretical model of new drug introductions that potentially resolves the dispute among these papers. Following Salop (1979), their model considers firm pricing decisions in a setting where drugs are spatially differentiated. When a new firm enters, whether the price of the existing product rises or falls depends upon how closely substitutable the new drug is with the old one. In Bertrand pricing equilibrium, if the new drug is close in product space, then the price of the old and new drugs will fall, relative to the original price of the old drug. On the other hand, if the drugs are moderately differentiated, then drug prices might rise, even if the new and old firms do not collude.

While all of these papers focus on the entry of new and differentiated patented drugs, the recent empirical and theoretical literatures on drug pricing tend to focus on the introduction of generic alternatives to branded drugs. Scherer (1993) provides a concise summary of this literature.

Relevant facts are provided by Grabowski and Vernon (1992), Frank and Salkever (1995), and Caves et al. (1991). Although these papers examine different sets of patented drugs at different times, all of them find that branded drug prices typically rise after patent expiry or fall only weakly. Grabowski and Vernon (1992) estimate that branded prices rise both before and after generic entry, albeit more slowly after entry. Conversely, Caves et al. (1991) estimate that the price of the branded drug falls about 2% after the patent expires. In a fixed effects framework, Frank and Salkever (1995) find that branded drug prices rise when generics enter. In a paper using a policy experiment in Germany, Pavcnik (2000) finds that the presence of generic drugs acts as a check on branded prices: when the German government enacted a policy exposing consumers to more of the financial consequences of

choosing a higher priced drug, branded drug prices fell, and branded drug prices fell more for branded drugs facing more generic competition. The empirical methodology in Grabowski and Vernon (1992) is most similar to ours, and their results are quite similar to ours as well.

Frank and Salkever (1992) present an explanation for the generic competition paradox. Their explanation has by and large become the accepted one for this puzzle–see, for example, Scherer (1993) or Stern (1995). In their story, the market for pharmaceuticals is segmented. There is a highly price elastic hospital segment that responds to price changes in either the branded or generic drugs, and a less elastic insured consumer segment that responds only to price changes in the branded drug. Before the entry of generics, the branded drug serves both segments; thus, the price elasticity it faces is a mixture of the price elasticities of the two segments. Upon generic entry, the branded drug loses the price-elastic segment to the generic drugs, and this lowers the elasticity of the branded drug's demand curve, leading to an increase in price. The law of one price fails here because consumers value "quality" attributes associated with the branded drug such as persuasive advertising or perhaps the greater accumulated experience with the branded version.

In empirical work inspired by Frank and Salkever (1992), Stern (1995) estimates a nested logit model of demand in the market for gout drugs, for sedatives, and for oral diabetic therapies. He finds that the branded drug's demand becomes steeper with generic entry.

There is a related empirical literature concerning the measurement of pharmaceutical price indexes.[1] Berndt et al. (1993) and Berndt and Greenberg (1995) find that the prices of branded pharmaceuticals rise over each drug's life cycle, but that the rate of growth is

[1] We wish to thank an anonymous referee for suggesting the relevance of this literature to us.

slower immediately after introduction. Also, Kelly (1997) finds that the prices of "new" drugs fall immediately after introduction. Because he is interested in updating price indexes for new products, his definition of new drugs includes generics.

There are a number of other papers in the pharmaceutical literature that emphasize parts of the dynamic demand management problem facing firms. However, none of these authors note that the solution to this demand management problem, without recourse to other phenomena, explains the generic competition paradox. Berndt et al. (1995) specify a model of knowledge stock diffusion in the market for anti-ulcer medications. They find evidence that advertising by one firm in a therapeutic category increases demand for other drugs in the same category.

The (theoretical) approach of Hudson (1992) is most similar to ours. In his theory, lowering price reduces present profits, but raises future demand by increasing the set of patients exposed to the drug. However, his framework considers neither advertising, patent expiry, nor the shape of the optimal price path. In another similar paper, Ridley (2002) analyses the effects of price controls in a market with intertemporal stickiness in demand and applies his model to the pharmaceutical industry.

The most recent paper in this vein is Ching (2000). His theoretical model emphasizes learning as an explanation for the slow increase in generic market share after patent expiry. However, like Hudson (1992), he excludes advertising from model. Most of his theoretical effort is aimed at analyzing price dynamics after patent expiry. When he does focus on the generic competition paradox, he relies upon consumer heterogeneity to explain it.

In our paper, stickiness in demand is modeled rather abstractly via a "knowledge stock"

variable. Closely related are the literatures on learning by using and switching costs (see Klemperer, 1995; Schlee, 2001, and references therein), which generate stickiness in demand in a more detailed micro models of consumers. In the context of pharmaceuticals, Crawford and Shum (2000) provide an empirical dynamic learning-by-using model, Coscelli (2000) estimates the effect of physician and patient habit on prescription demand. Finally, in the rational addiction literature (c.f. Becker and Murphy, 1988) intertemporal stickiness in demand arises directly out of individuals' utility functions, as the marginal utility of future consumption rises in current consumption.

3 Model

Demand for the branded pharmaceutical product at each instant of time is $D^i(p, X)$. The price of the pharmaceutical is p and the stock of knowledge about the pharmaceutical is X. Demand is indexed by $i \in \{B, A\}$, where B denotes the structure of demand before patent expiry and A denotes demand after patent expiry. We assume that demand is linear:

$$D^i(p, X) = \alpha_0^i - \alpha_p^i p + \alpha_X^i X \tag{1}$$

Over time, the stock of knowledge evolves. We use a modified version of a standard "contagion" model of knowledge diffusion. The simple contagion model would have $\dot{X} = \rho_2 X(1 - X/M)$ — knowledge diffuses quickly and at an accelerating rate early, but then slows down as saturation occurs (X gets close to M). The contagion model essentially assumes that

one comes to know about a drug by coming into contact with someone who already knows about it — whether or not they use the drug themselves. It seems likely that physicians who actually are using the drug are in a better position to impart knowledge. To reflect this, we include a term $\rho_1 D$ to allow knowledge diffusion to be positively affected by current use. Finally, pharmaceutical companies can impart knowledge directly via promotional activities, A. So, the law of motion for knowledge is:

$$\begin{aligned}\dot{X} &= A + \rho_1 D + \rho_2 X(1 - X/M) \\ &= A + \rho_1 D + \rho_2 X - \frac{\rho_2}{M} X^2 \\ &= A + \rho_1 D + \rho_2 X - \rho_3 X^2 \end{aligned} \tag{2}$$

We assume quadratic cost of promotional effort:

$$c(A) = \frac{1}{2\rho_4} A^2 \tag{3}$$

We make a number of additional assumptions on these functions.

A1: All parameters are positive

A2: $r < \rho_2 + \frac{\rho_1 \alpha_X}{2} + \frac{\alpha_X}{2\alpha_p}$

A3: $\alpha_0^i > \alpha_p^i c$

A4: $\alpha_0^B > \alpha_0^A$, $\alpha_p^B < \alpha_p^A$, $\alpha_X^B > \alpha_X^A$

Assumptions 1 and 3 are regularity conditions. Assumption 1 imposes negatively sloping demand, costly promotion, and a variety of similar assumptions, and 3 says that demand is high enough to make it worthwhile to produce the product at all. Assumption 2 restricts the interest rate r so that the firm does not discount the future too steeply.

Assumption 4 says that the effect of the expiry of a patent and subsequent generic entry is to reduce the level of the innovator's demand, increase the slope of the innovator's demand, and to decrease the benefit of knowledge dissemination for the innovator's demand. The first two are quite standard — most oligopoly models would predict this change in a firm's residual demand upon the entry of a competitor. The last part of Assumption 4 captures the knowledge externality discussed by Berndt et al. (1995) — since the generic and branded drugs have the same active ingredient(s), knowledge about the branded drug also provides knowledge about the generic drug.[2]

3.1 The innovator's problem

The instruments available to the firm to influence the path of sales are price and promotional effort. The firm solves the following problem:

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[2] Notice we do not need to assume that the two drugs are identical or that knowledge about the branded drug is the same as knowledge about the generic drug, merely that knowledge about the branded drug spills over to some degree on demand for the generic.

$$max_{\{p,A\}} \int_0^T \exp(-rt) \left\{ (p_t - c) D^B(p_t, X_t) - c(A_t) \right\}$$
$$+ \int_T^\infty \exp(-rt) \left\{ (p_t - c) D^A(p_t, X_t) - c(A_t) \right\}$$

$$\dot{X}_t = A_t + \rho_1 D_t^i + \rho_2 X_t - \rho_3 X_t^2$$

$$D_t^i = \alpha_0^i - \alpha_p^i p_t + \alpha_X^i X_t$$

$$X_t \geq 0$$

$$X_0 = 0$$

There are two epochs in the innovator's optimization problem. From time 0 to time T, there is a patent protecting the innovator's market and the innovator acts as a monopolist. At time T the patent expires, and from then on the innovator faces generic competition. This generic competition shifts the innovator's residual demand curve as described in Assumption 4.

The stock of knowledge about the drug is X. We assume that this starts out at zero when the drug is introduced, $X_0 = 0$. Furthermore, we assume that this stock can not be negative.[3]

[3] In order to impose the constraint, we require that $\chi_{X=0}(A + \rho_1 D) \geq 0$. The indicator variable, $\chi_{X=0}$ is equal to 1 any time X is zero. This constraint is equivalent to $\chi_{X=0}(A + \rho_1(\alpha_0 - \alpha_p p)) \geq 0$ Without any constraint, there is an undesirable potential solution to this problem. The firm could charge prices higher than the choke price and engage in negative promotion early in the program in order to drive X negative. The quadratic form (in X) of $\dot{X}$ would then set up a positive feedback causing X to go to negative infinity

3.2 First order conditions

The Hamiltonian associated with this dynamic program is:

$$H^{CV} = (p-c)\,D(p,X) - c(A) + \lambda\left(A + \rho_1 D + \rho_2 X - \rho_3 X^2\right) + \mu\chi_{X=0}(A + \rho_1(\alpha_0 - \alpha_p p)) \tag{4}$$

Thus, we are led to the following necessary conditions which must hold for all t:

$$H_p^{CV} = 0 = (p - c + \rho_1\lambda + \mu\chi_{X=0})D_p + D \tag{5}$$

$$H_A^{CV} = 0 = c_A(A) - \lambda + \mu\chi_{X=0} \tag{6}$$

$$\dot{\lambda} = r\lambda - (p-c)\,D_X + \lambda\,(\rho_1 D_X + \rho_2 - 2\rho_3 X) \tag{7}$$

$$\dot{X} = A + \rho_1 D + \rho_2 X - \rho_3 X^2 \tag{8}$$

$$X_0 = 0 \tag{9}$$

$$\mu \geq 0,\ \mu\chi_{X=0}(A + \rho_1(\alpha_0 - \alpha_p p)) = 0 \tag{10}$$

Our choice of functional form for demand and cost of promotional effort ensure that H^{CV}

quickly. Later in the program, the firm could charge a price less than c to earn negative per unit profits on a large negative volume of sales, to earn large positive profits. To rule out this case, we need some constraint, and the positivity constraint on X is the easiest one.

is concave in p, A for each λ, so that the first order conditions are sufficient for the problem of maximizing H^{CV} in p, A.

A key element of the solution to the problem is λ, the shadow price or value of knowledge. Its interpretation is the value to the firm of the stream of benefits associated with acquiring one more unit of X. In the appendix, section B, by solving the first order conditions for A and p in terms of λ and X, we establish that, *ceteris paribus*:

- A is increasing in λ.
- p is decreasing in λ.
- p is increasing in X.

Advertising is increasing in λ since the firm will expend more effort to spread knowledge about its drug the more valuable is the knowledge. Since high demand increases the diffusion of knowledge, the firm will want to expand demand when knowledge is most valuable to it. Thus, it will charge a lower price when λ is higher. Finally, a large knowledge stock expands the level of demand, and the firm will tend to take advantage of this by charging a higher price, so that p is increasing in X.

Given the time paths for λ and X, we would then have the optimal time path chosen by the firm for p and A. We solve for the optimal paths of λ and X via a phase diagram. The details of the analysis are confined in the appendix, section B. We summarize the results and their derivation here.

Figure 1 shows the optimal phase path for X and λ both before and after the expiry of the patent, which is labeled T. The path labeled O is the optimal path. At drug introduction,

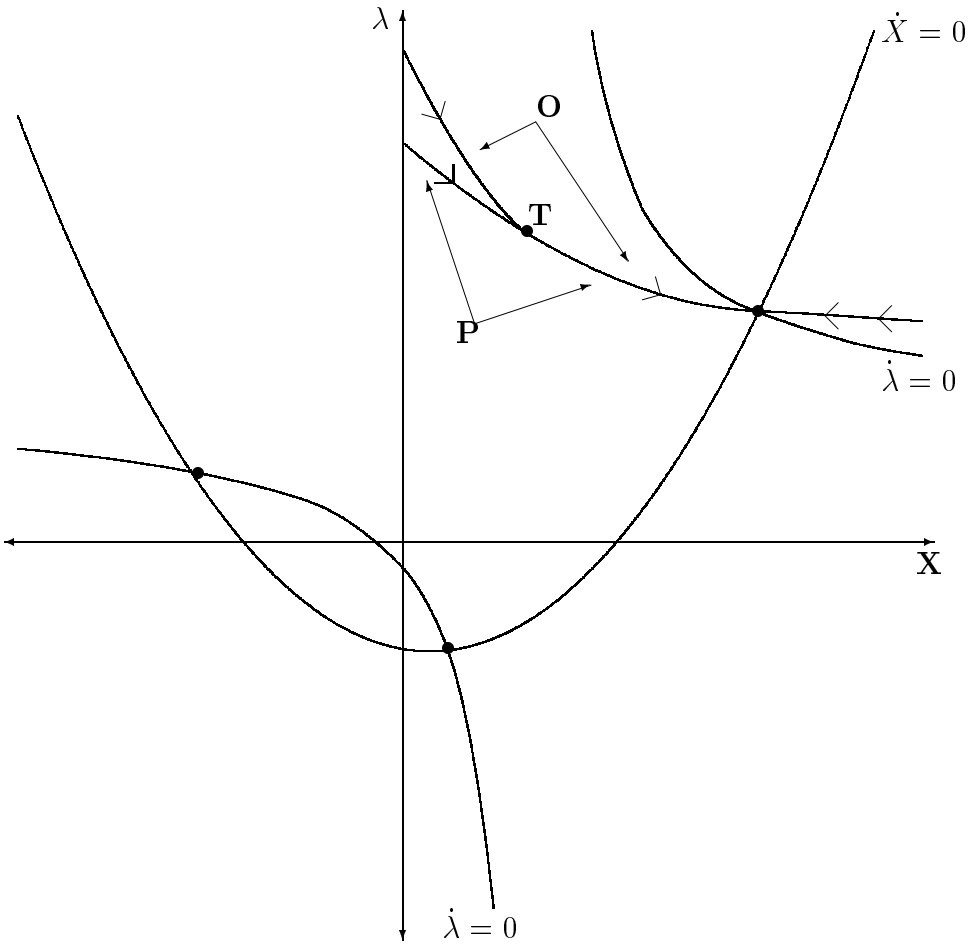


Figure 1: Optimal paths, before and after

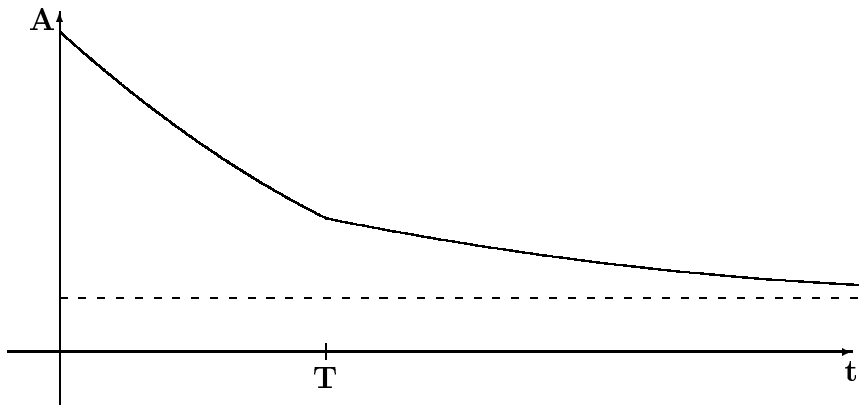


Figure 2: Optimal path for A

knowledge stock is zero and the firm begins at the λ-intercept of path O. As time elapses, the firm moves down path O until coming to rest at the steady state at the intersection of $\dot{X} = 0$ and $\dot{\lambda} = 0$.

The kink at point T is caused by patent expiry. At patent expiry, the rate of change of the knowledge stock drops discretely. Prior to patent expiry, two forces push λ down over time. First, as X accumulates, decreasing returns sets in, making additional units less valuable. This process also continues after the patent expires. Second, before patent expiry, with each passing moment, the firm knows that it will enjoy monopoly profits for a shorter period of time on each extra unit of knowledge. Thus, with each passing moment, each unit of X becomes less valuable. This second effect disappears at the moment of patent expiry.

So, over the course of the drug life cycle, λ starts out high and then falls. While the drug is on patent, λ falls quickly. Then, when the drug loses its patent protection, λ begins to fall more slowly. Knowledge stock is continuously increasing over the course of the drug's life cycle. In the typical case, knowledge stock grows more slowly after patent expiry than before, as the firm makes fewer investments in dissemination.

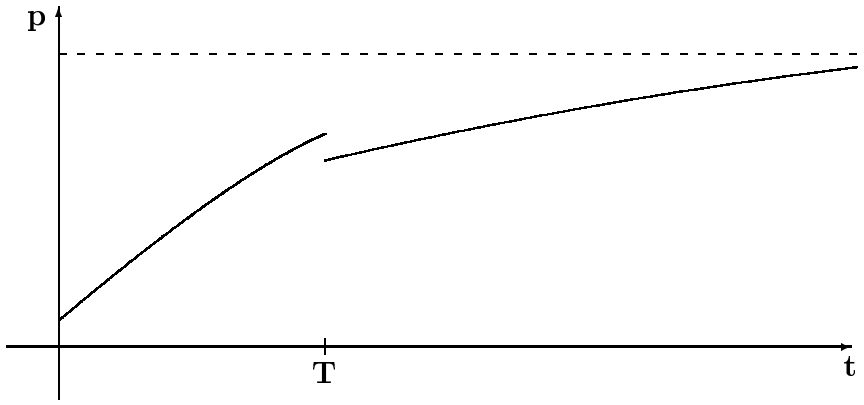


Figure 3: Optimal path for p

Figure 2 graphs the optimal path for advertising over a drug's life cycle. Before patent expiry, the value of additional knowledge diffusion is highest; thus, promotional efforts are highest. As knowledge accumulates and patent expiry approaches, the value of incremental knowledge diffusion decreases, and the firm reduces promotion. The rate of decrease decreases at the date of patent expiry.

Figure 3 graphs the optimal path for price. Early in the program, the firm establishes a low price to stimulate demand and therefore knowledge acquisition and diffusion. As time progresses knowledge expands, which both reduces the long run value of additional knowledge and increases the short term returns to high prices. So, prices rise quickly over time. When the patent expires, the branded firm's demand both shifts in and becomes more elastic. This pushes the firm to reduce prices discretely. In addition, the rate of growth of price decreases for the same reason that the decline in advertising slows — because $\dot{\lambda}$ drops at that point.

3.3 Discussion

In our model, the pharmaceutical firm's central dynamic problem is the management of physicians' information about its products. Knowledge is disseminated via word-of-mouth among physicians and via pharmaceutical company promotional effort. Word-of-mouth dissemination is enhanced when a large number of physicians use the drug. So, the firm wishes to set a low price and high promotional effort early in its program in order to build a stock of knowledge. As time passes, the buildup of the knowledge stock and the impending patent expiry erode the firm's incentive to disseminate knowledge, so its price rises and its promotional effort falls. After the patent has expired, the firm engages in steadily lower promotional effort and steadily higher prices, as the effect of the spillovers in knowledge between branded and generic drugs blunts its incentives to facilitate knowledge dissemination.

While we interpret X as the stock of physician knowledge, one might just as well reinterpret it as the stock of consumer knowledge or the union of the two. In that interpretation A would include direct-to-consumer advertising as well as detailing, journal advertising, and other activities directed at physicians.

3.4 Comparison with Frank and Salkever

Frank and Salkever (1992) is a landmark contribution to the understanding of the generic competition paradox, and any new analysis must be compared against theirs.

First, our model differs from that of Frank and Salkever (1992) in its assumptions. Frank and Salkever assume, in a static model, that the innovator's demand curve becomes *less* elastic with the expiry of the innovating firm's patent. In contrast, in a dynamic model,

Table 1: Comparison of theories

	Frank & Salkever	Bhattacharya & Vogt
Assumptions		
Horizon	static	dynamic
Demand	segmented	homogenous
Generic Entry	decreases |elasticity|	increases |elasticity|
Predictions		
p at entry	increase	decrease
p before entry	N/A	increasing quickly
p after entry	N/A	increasing slowly
Ads before	N/A	decreasing quickly
Ads after	N/A	decreasing slowly

we assume that demand becomes *more* elastic post patent expiry. This assumption is more consonant with economists' typical intuition about the effect of entry on residual demand elasticity. To generate our price and promotional effort dynamics, we instead place the pharmaceutical firm's problem of managing knowledge about its product at the center of the model.

Second, our model differs from that of Frank and Salkever (1992) in its predictions. See table 1 for a comparison of the two theories.

4 Empirics

We offer an exploratory analysis of the price and advertising paths of several drugs. Due to limitations of sample and variables, this is a suggestive rather than a dispositive look at the predictions of our theory.

We sought a therapeutic class which had a number of patent expiries recently and which

were important, both for patients and for pharmaceutical company revenues. The class we chose is β-blockers, which during our sample period were used principally as oral anti-hypertensive agents outside the hospital.

4.1 Data

Our principal data source is IMS America, a company that collects sales and other data for US pharmaceutical markets. IMS kindly provided us with quarterly retail quantity and revenue data for our drugs. The drugs consist of all β-blockers which were on the market at any time during the sample period, from the fourth quarter of 1987 through the first quarter of 1993. These amount to eleven molecules.[4] IMS collects these data through retail pharmacy audits.

Each drug is sold in several different dosage levels; although all of the drugs we study are of an oral form. For our analyses, we choose the most common dosage level for each drug. Unfortunately, in our data we are unable to distinguish different prescription sizes (e.g. 30 tables vs 90 tablets).

Patent expiry data were drawn from the Food and Drug Administration Center for Drug Evaluation and Research's *Approved Drug Products with Therapeutic Equivalence Evaluations*,[5] also known as the FDA Orange Book. This source contains extensive information concerning date of approval, dosages approved, and the expiry dates of the relevant patents.

[4] We have data for both branded and generic drugs of a particular chemical entity; however, the generic data as provided to us are not separated by manufacturer, so that we are unable to separate "branded generics" from other generics.

[5] http://www.fda.gov/cder/ob

Table 2: Variable descriptions

Variable[a]	Description
p	Price of the drug, $ per prescription
q	Quantity of the drug sold, 1000s of prescriptions
ads	Number of journal ads
until	Quarters until patent expiry[b]
onpat	Dummy for drug being on patent
gener	Dummy for drug being a generic
Q	Aggregate quantity of the molecule sold

[a] An observation is one quarter-(branded/generic cell), so that in each quarter before patent expiry there is one obs per molecule and in each quarter after there are two.

[b] This variable is negative after the patent expires and counts down with time.

Table 3: Descriptive statistics

Variable	Mean	Std Dev	25%	75%
p	27.90	7.67	25.60	32.30
q	991.53	1065.56	113.00	1898.50
ads	1.62	3.14	0.00	2.00
until	4.87	15.71	-4.00	16.00
onpat	0.66			
gener	0.17			
Q	1358.92	1437.02	129.00	2938.90
N	265			

Over our sample period, patents expired on four of our eleven drugs. We verified patent expiry dates by looking at the earliest date at which generics were sold for each molecule. In three of the four cases, generics entered within one quarter of patent expiry. Excluding the fourth, Penbutolol, has minimal effect on the results.

Descriptions of the variables used in our analyses appear in Table 2 and descriptive statistics for these variables appear in Table 3. We capture the time remaining until patent expiry in the variable "until." One could argue for defining until in terms of the date of generic entry instead. The two definitions only differ for one drug and our results are minimally affected by moving from one definition to the other.

Although such data exist, IMS did not share with us information on pharmaceutical promotional expenditures. Consequently, we developed a (rough) proxy for promotional effort by manually counting the number of journal advertisements for each drug in each quarter of our sample period. We counted ads in all issues of *JAMA*, the *New England Journal of Medicine*, and *Circulation*. *JAMA* and *NEJM* are the two most widely read medical journals by American doctors, and *Circulation* is a widely read cardiology specialty journal.

4.2 Specification

4.2.1 Price and Advertising Dynamics

In order to examine the match between our model's predictions, shown in Table 1, and the evolution in the market for β-blockers, we regress prices and advertising effort on quarterly time trend variables. Time is measured as quarters until patent expiry, with negative values

denoting post-expiry quarters. In the regressions, the effect of the time trend is permitted to differ for the branded drug before and after expiry and is also permitted to differ between the branded and generic drugs. In addition, we include a dummy variable that indicates whether the drug is on patent. This last term permits the regression to pick up any discrete effect that patent expiry may have over and above a change in the trends. Finally, we include dummies for each branded drug and for each generic drug. Given that we include individual dummies and time until expiry, it is not possible to include a time trend, since it would be perfectly collinear with these.[6]

We estimate two different versions of the price trend regressions: one using price and one using log price as the dependent variable. We run each price regression twice using two different subsets of the β-blocker data. The first subset includes all 11 branded drugs and their generics, over the period between the fourth quarter of 1987 and the first quarter of 1993. This yields 265 separate drug-quarter observations, since some drugs are not present at the beginning of our observation period. The second subset includes just the four drugs that expire during the observation period and their generics. There are 104 separate drug-quarter observations in this subset. These are the only drugs for which we observe prices and advertising effort both before and after patent expiry.

Since there are several quarters with no advertising for some of the drugs, we are not able to run a log version of our advertising model. Like the price regressions, however, we run the advertising regression on two data subsets: all branded drugs and expiring branded drugs

[6] This problem is a familiar one in the analysis of panel data — it is impossible separately to identify cohort, age, and time effects in an unrestricted regression.

only. We run our advertising regressions using only observations from branded drugs, since generics never advertise in our sample. This leaves 221 separate drug-quarter observations in the first subset, and 82 separate observations in the second.

All the price and advertising dynamics regressions are estimated using the GEE procedure Liang and Zeger (1986). In the reported results, we use a linear link function and normal, AR(1) errors. The estimated results are asymptotically equivalent to GLS with AR(1) errors. We have tried a variety of other error structures (both autoregressive and moving average) with lag depth of up to four quarters, and the results from those were not materially different from those reported here.

4.2.2 Demand Regressions

In addition to the price and advertising dynamics regressions, we estimate demand regressions. The purpose of these regressions is to characterize the relative importance of price, advertising, and knowledge stock on demand. We measure knowledge stock as total quantity of the molecule sold in the previous quarter. Molecule sales include both the branded and generic versions of the drugs. All the demand regressions include separate drug dummies for both branded and generic drugs.

We estimate each of these two demand regression versions using ordinary least squares (OLS) and two-stage least squares (IV). The motivation for the latter econometric technique is that our theory tells us firms manipulate the key right-hand side variables in our demand regression; that is, they choose price and advertising to manipulate the evolution of the knowledge stock. If this is correct, then price and ads will be endogenous.

Our instruments (detailed in Table 6), are largely related to the timing of drug introduction and patent expiry. As our theory demonstrates, these variables are integral to determining the firm's optimal pricing, advertising, and knowledge stock management strategy, so they are certainly correlated with the endogenous right-hand- side variables. We assume that these variables do not affect demand.[7] Also, because these variables depend on the vagaries of both the research and development process and the regulatory process, they are plausible instruments. Of course, in a model with endogenous research and development and control of the regulatory process by firms, these variables would be correlated with the unobserved determinants of demand.

4.3 Results

4.3.1 Pricing and Advertising Dynamics

Table 4 shows the results from the pricing and advertising dynamics regressions. Columns one through three report results on the full sample, and columns four through six report results from the sample of drugs which went off patent in our data. Looking at the fourth column, we see that price rises before patent expiry for the branded drug at a rate of about \$0.74 (=0.24+0.50) per quarter. At patent expiry, the price drops by \$0.29 and thereafter rises by about \$0.24 per quarter. Generic prices, by contrast, are roughly flat over time. The log specification and the specifications using all drugs confirm the basic results: branded

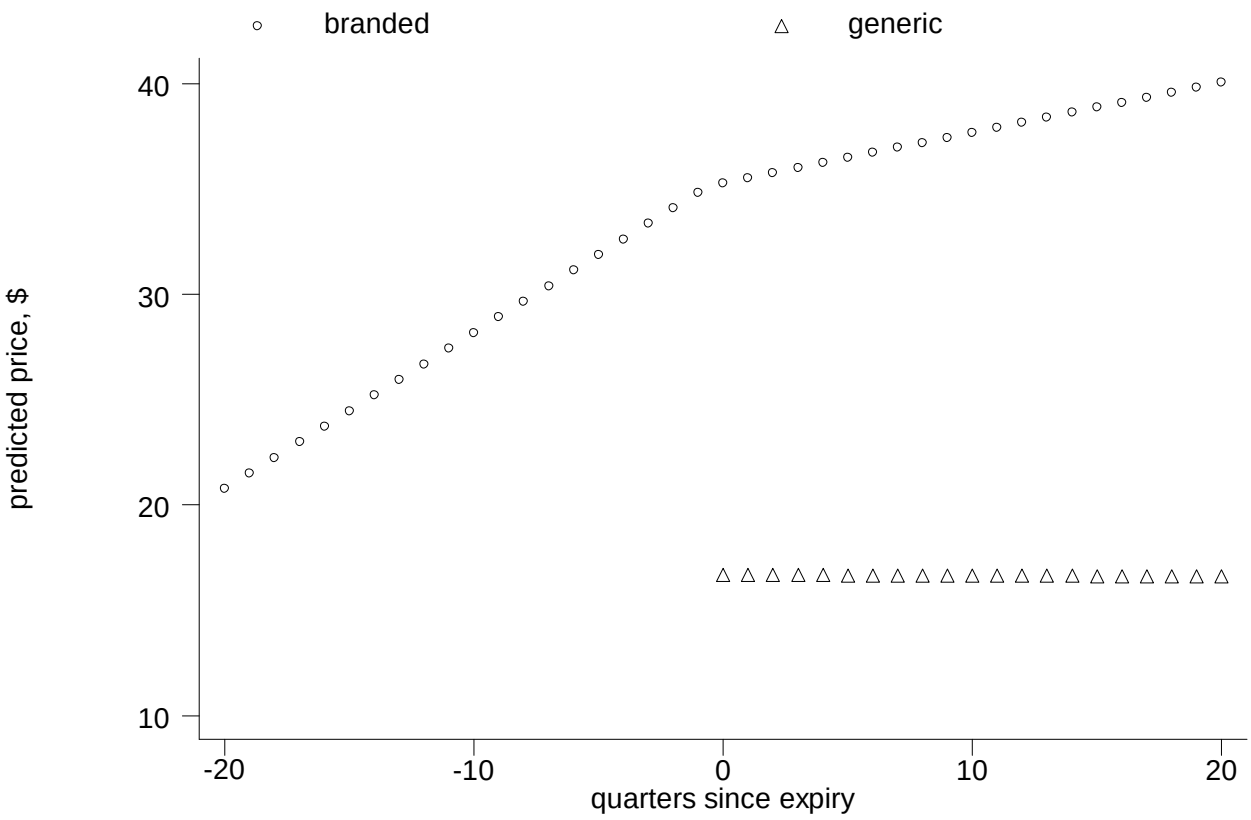
[7] In a model with forward-looking, informed consumers, this assumption would be untenable. What we have in mind is that the knowledge stock is resident in physicians and that they are constrained, by ethics or fear of malpractice suits, to provide the drug which is myopically best for each patient.

Table 4: Price and advertising dynamics regressions

	All drugs			Expiring drugs		
Variable	p	$\ln p$	ads	p	$\ln p$	ads
until	-0.458[a]	-0.018	0.057	-0.239	-0.009	0.214
	(0.093)	(0.003)	(0.084)	(0.148)	(0.005)	(0.154)
until*onpat	-0.104	0.001	-0.011	-0.503	-0.014	-0.225
	(0.105)	(0.004)	(0.094)	(0.175)	(0.006)	(0.181)
until*gener	0.349	0.006		0.243	0.001	
	(0.135)	(0.005)		(0.225)	(0.008)	
onpat	-0.140	0.010	0.427	0.292	0.020	0.190
	(0.740)	(0.029)	(0.802)	(0.831)	(0.033)	(0.884)
drug dumys	Yes	Yes	Yes	Yes	Yes	Yes
drug*gener	Yes	Yes	No	Yes	Yes	No
N	265	265	221[b]	104	104	82[b]

[a] Standard errors are in parentheses.
[b] Generic drugs are omitted from this regression.

Figure 4: Predicted price: expiring drugs



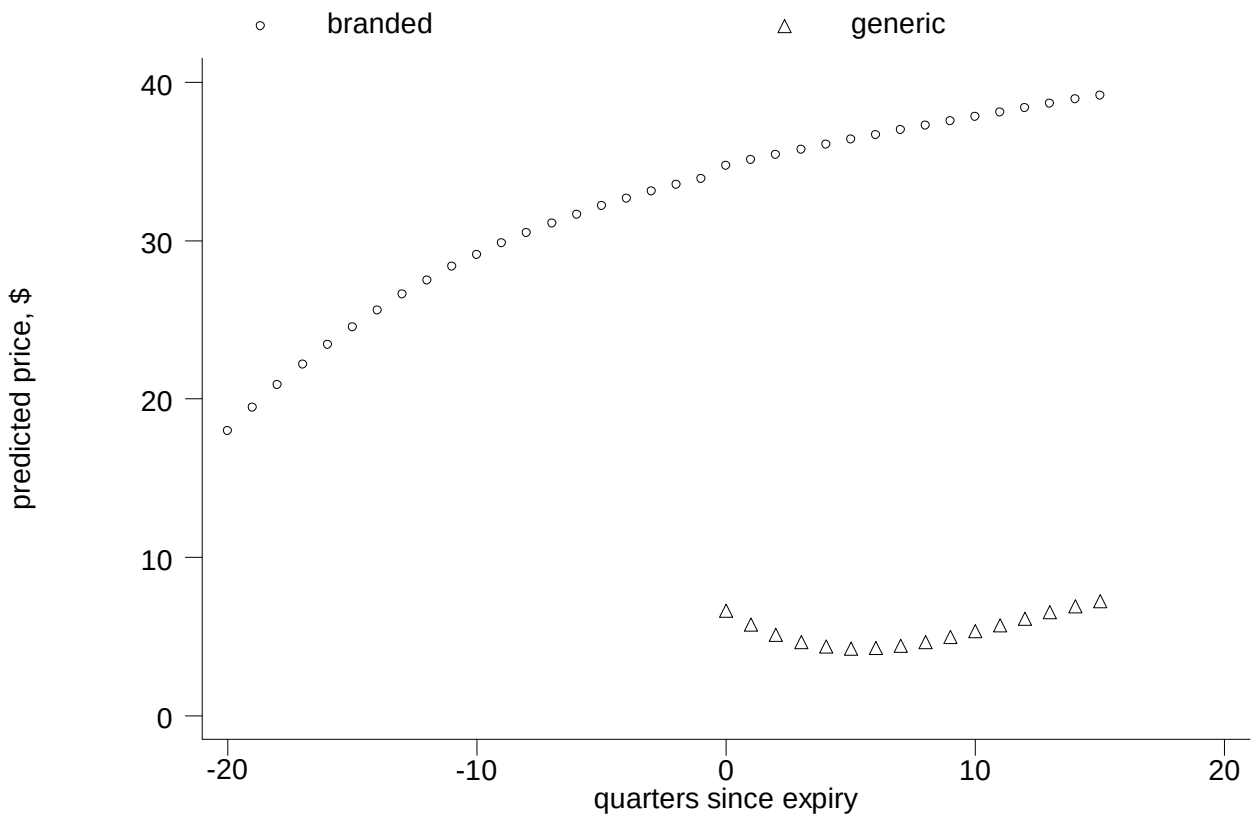
prices are rising both before and after patent expiry, the rise is faster before expiry and branded prices change little at the date of expiry. Figure 4 plots predicted values from the model of column three for the average expiring drug.

To guard against our results being driven by the linear functional form assumption on the price dynamics, we estimated a smoothed version of this model. We smooth using the overlap polynomial method of MaCurdy et al. (1990). This method is also used in Bhattacharya et al. (1996). Overlap polynomials are smooth spline functions which use the cumulative normal distribution rather than the step function to "turn on" and "turn off" the sections of the spline. This leads to an estimated function with derivatives of all orders. Details of the method can be found in the above-referenced papers.

For this application, we estimated an overlap polynomial in time since expiry. We estimate on the sample of expiring drugs and we include a complete set of drug and generic-drug dummies as well as a dummy for being on patent. In essence, we estimate the model of column four of Table 4 but with the time trend flexibly modeled. Knots are placed at time since expiry equal to 15, 7, 3, 0, -3, -10, -20 and a smoothing parameter of 25 is used. The spline functions in the reported results are levels (i.e. zeroth order polynomials); although, we experimented with higher order polynomials with similar results.

Results from this analysis appear in Figure 5. The basic results from the previous analysis continue to hold here. Price rises both before and after expiry, but faster before. Price changes very little at the time of patent expiry. Generic prices are lower than are branded and the generic price trends up over time. The biggest difference between these results and the previous ones is in the behavior of generic prices shortly after expiry. Here, they trend

Figure 5: Predicted price, smoothed: expiring drugs



downward for about a year after expiry — evidently, generic entry happens gradually over about a year rather than all at once as our theory and our simpler empirical model assume.[8] This is consistent with prior work, (see Grabowski and Vernon, 1992; Reiffen and Ward, 2002).

Some previous work has found that some branded drug prices fall in the first few years after introduction (Lu and Comanor, 1998). Although prices in our sample rise over the first few years, this prior result is inconsistent with our theory, which predicts rising prices over the whole life cycle. A pattern of declining prices early in a drug's life cycle may be explained by signalling (Bagwell and Riordan, 1991) or by consumers overestimating the imperfectly observable quality of the new drug (Shapiro, 1983).

There is little of statistical significance in the advertising dynamics regression. In neither sample are there significant differences between the before and after expiry periods in levels, trends, or jointly in levels and trends. A graph of the fit time path of ads appears in Figure 6.

4.3.2 Demand regressions

Table 5 show the results of the demand regressions using OLS. Equation 5.1 contains a simple demand specification in which quantity demanded depends on price, ads, knowledge stock (proxied by lagged aggregate molecule quantity), and drug dummies. Estimates from this equation have demand downward-sloping but inelastic and show a strong effect of lagged on

[8] Under the Hatch-Waxman Act of 1984 decisions, the first generic entrant may enjoy a six month exclusivity period on the generic market, if it successfully challenges the patent of the innovator. However, prior to 1997, only three generic entrants were granted such exclusivity. See, for example, http://www.fda.gov/cder/ogd/Exclusivity/sld011.htm

Figure 6: Predicted ads: expiring drugs

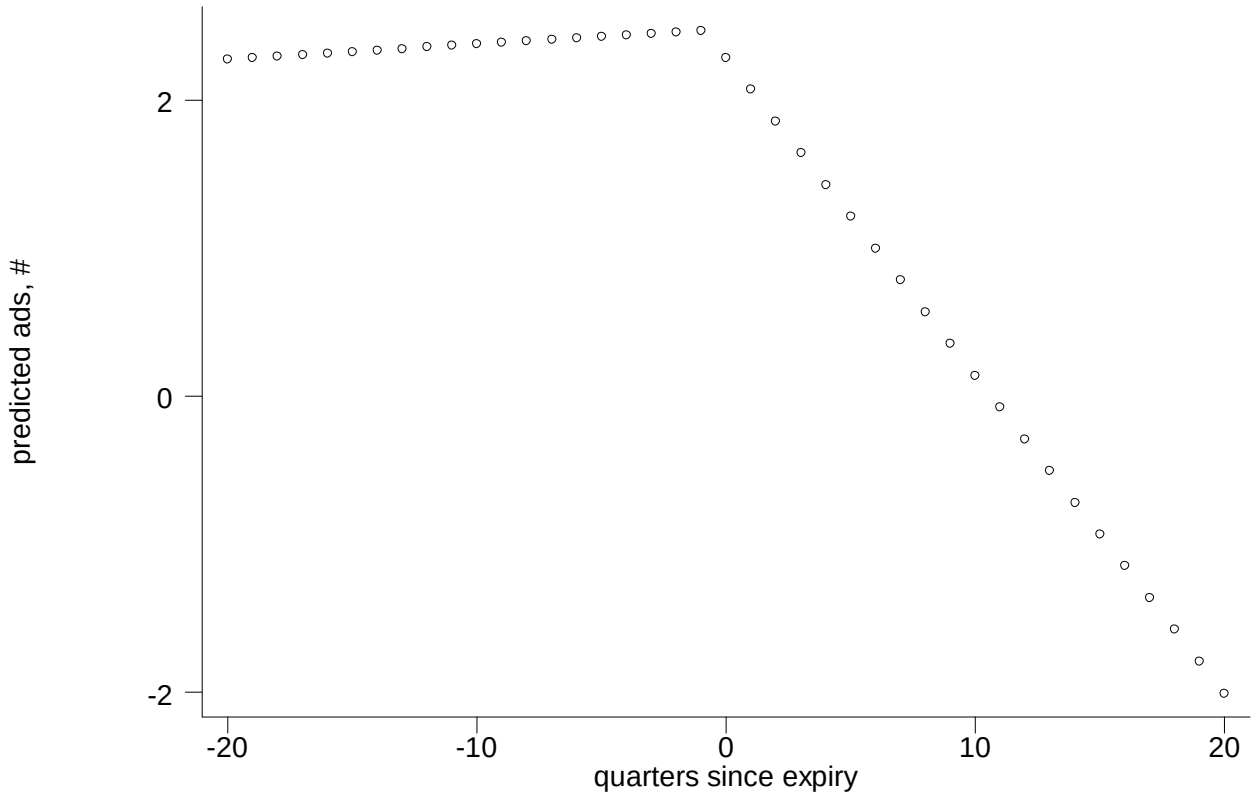


Table 5: Demand regressions: OLS

Variable[a]	eq 5.1	eq 5.2	eq 5.3
$\ln p$	-0.704	-1.433	-1.431
	(0.113)	(0.192)	(0.175)
$\ln p$*onpat		1.011	1.014
		(0.212)	(0.193)
$\ln p$*gener		0.300	-1.138
		(0.341)	(0.374)
ads	0.010	0.009	0.010
	(0.007)	(0.006)	(0.005)
$\ln Q(-1)$	0.437	0.481	0.482
	(0.030)	(0.036)	(0.033)
$\ln Q(-1)$*onpat		-0.029	-0.031
		(0.027)	(0.025)
$\ln Q(-1)$*gener		-2.216	0.372
		(0.262)	(0.444)
onpat		-3.180	-3.182
		(0.741)	(0.675)
until*gener			0.077
			(0.011)
drug dumys	Yes	Yes	Yes
drug*gener	Yes	Yes	Yes
N	250	250	250
R^2	0.990	0.993	0.994

[a] The dependent variable is $\ln q$.

current demand. Ads have no significant measured effect (although the point estimate is quite large), but this is not surprising given how noisy our measure for it is. All this is as our theory would predict.

To get at the more nuanced predictions of our theory: that branded demand should become more elastic after patent expiry and that branded demand should become less sensitive to the knowledge stock after expiry, we estimate the specification Equation 5.2. Here, the interaction between onpat and price is positive, showing that demand elasticity is estimated to be lower in absolute value prior to patent expiry. Furthermore, the beneficial effect of knowledge stock on branded demand is estimated to decline after expiry, though that result is not significant. Surprisingly, the demand for generics is estimated to be less elastic than is the demand for branded drugs. Even more surprisingly, generic demand is estimated to have *negative* dependence on knowledge stock.

The surprising result in Equation 5.2 are driven, we think, by a variable omitted both from the empirical work and from our theory. Evidently, it takes time for knowledge about generics to diffuse. By our theory, generic demand should jump up immediately upon patent expiry. This does not happen in our data. Generic quantity increases gradually, presumably reflecting the diffusion of knowledge about the existence of generics, or lags in pharmacies stocking generics. To allow for this effect, we construct Equation 5.3. This specification contains a generic-specific time trend. Including the generic time trend provides more intuitive estimates. Generic demand is more elastic than is branded. Generic drugs have positive spillover effects from the knowledge stock, and generic demand rises over time.

The two-stage least squares results appear in Table 6, and the contrast between the OLS

Table 6: Demand regressions: IV

Variable[a,b]	eq 6.1	eq 6.2	eq 6.3
$\ln p$	-0.793	-2.151	-2.146
	(0.128)	(0.249)	(0.236)
$\ln p$*onpat		1.885	1.916
		(0.303)	(0.287)
$\ln p$*gener		0.986	-1.211
		(0.410)	(0.494)
ads	0.020	0.024	0.032
	(0.012)	(0.012)	(0.012)
$\ln Q(-1)$	0.433	0.500	0.507
	(0.031)	(0.039)	(0.037)
$\ln Q(-1)$*onpat		-0.057	-0.066
		(0.031)	(0.030)
$\ln Q(-1)$*gener		-2.236	0.867
		(0.281)	(0.507)
onpat		-6.097	-6.162
		(1.039)	(0.983)
until*gener			0.097
			(0.013)
drug dumys	Yes	Yes	Yes
drug*gener	Yes	Yes	Yes
N	250	250	250
R^2	0.990	0.992	0.993

[a] The dependent variable is $\ln q$.

[b] The instrument set consists of onpat, until, $\ln Q(-1)$, until*onpat, squares and interactions of these, drug dummies, and generic interacted with all of these.

and these results is modest. As one would expect, price elasticities of demand are estimated to be higher once the endogeneity of price is accounted for. Otherwise the results are quite similar. Looking at Equation 6.3, price elasticity is higher for branded drugs after patent expiry. Also, knowledge stock has a larger beneficial effect on branded demand and the difference between this effect before and after patent expiry is small, though in the "wrong" direction. Generic demand is more elastic than is branded, knowledge stock is beneficial to generics, and demand for generics rises over time.

5 Conclusion

We have exhibited a theoretical model which both explains and adds nuance to the "stylized fact" that branded pharmaceutical prices rise in the face of generic entry. In our story, a pattern of price rising over time is generated by the firm's dynamic demand management problem — rising prices reflect the firm taking payoffs from earlier investments in knowledge stock. Generic entry causes an instantaneous price decline from the usual competitive effects; however, the same forces which push up branded prices before expiry also do so after expiry, and the (slower) upward trend reasserts itself.

Our empirical results are exploratory in nature and are imprecisely estimated; however, they provide suggestive confirmation of several aspects of our theory. First, prices and to a lesser extent advertising move as our theory predicts. The upward price trend is evident from the moment of introduction of the drug. Generic entry does not cause a measurable instantaneous effect on prices, but the trend growth in prices is muted after generics enter. Finally, ads decline more quickly after patent expiry than they do before.

Our demand regressions also provide some confirmation for our theory. First, lagged demand, a proxy for knowledge stock, robustly influences contemporaneous demand. Second, branded demand becomes more elastic when patent protection lapses and generics enter.

References

Bagwell, K. and Riordan, M. (1991). High and declining prices signal product quality. *American Economic Review*, 81(1):224–239.

Becker, G. S. and Murphy, K. M. (1988). A theory of rational addiction. *Journal of Political Economy*, 96(4):675–700.

Berndt, E., Bui, L., Reiley, D., and Urban, G. (1995). Information, marketing, and pricing in the u.s. antiulcer drug market. *American Economic Review*, 85(2):100–105.

Berndt, E. and Greenberg, P. (1995). An updated and extended study of the price growth of prescription pharmaceutical preparations. In Helms, R., editor, *Competitive Strategies in the Pharmaceutical Industry*, pages 35–48. AEI Press.

Berndt, E., Griliches, Z., and Rossett, J. (1993). Auditing the producer price index: micro evidence from prescription pharmaceutical preparations. *Journal of Business and Economic Statistics*, 11(3):251–67.

Bhattacharya, J., Garber, A., and MaCurdy, T. (1996). Cause-specific mortality amoung medicare enrollees, nber working paper #5306.

Caves, R., Whinston, M., and Hurwitz, M. (1991). Patent expiration, entry, and competition in the us pharmaceutical industry. *Brookings Papers on Economic Activity*, pages 1–48.

Ching, A. (2000). Dynamic equilibrium in the u.s. prescription drug market after patent expiration, mimeo, university of minnesota.

Comanor, W. (1964). Research and competitive product differentiation in the pharmaceutical industry in the united states. *Economica*.

Coscelli, A. (2000). The importance of doctors' and patients' preferences in the prescription decision. *Journal of Industrial Economics*, 48(3).

Crawford, G. and Shum, M. (2000). Uncertainty and learning in pharmaceutical demand.

Frank, R. and Salkever, D. (1992). Pricing, patent loss and the market for pharmaceuticals. *Southern Economic Journal*, 59(2):165–79.

Frank, R. and Salkever, D. (1995). Generic entry and the pricing of pharmaceuticals, nber working paper #5306.

Grabowski, H. and Vernon, J. (1992). Brand loyalty, entry, and price competition in pharmaceuticals after the 1984 drug act. *Journal of Law and Economics*, 35(2):331–350.

Hudson, J. (1992). Pricing dynamics in the pharmaceutical industry. *Applied Economics*, 24:103–112.

Kelly, G. G. (1997). Improving the ppi sample for prescription pharmaceuticals. *Monthly Labor Review*, 120(10):10–17.

Klemperer, P. (1995). Competition when consumers have switching costs: an overview with applications ot industrial organization, macroeconomics, and trade. *Review of Economic Studies*, 62:515–39.

Liang, K. Y. and Zeger, S. L. (1986). Longitudinal data analysis using generalized linear models. *Biometrika*, 73:13–22.

Lu, Z. J. and Comanor, W. S. (1998). Strategic pricing of new pharmaceuticals. *Review of Economics and Statistics*, 80(1):108–118.

MaCurdy, T., Green, D., and Paarsch, H. (1990). Assessing empirical approaches for analyzing taxes and labor supply. *Journal of Human Resources*, 25(3):415–90.

Pavcnik, N. (2000). Do pharmaceutical prices respond to insurance? nber working paper #w7865.

Peltzman, S. (1973). An evaluation of consumer protection legislation: The 1962 drug amendments. *Journal of Political Economy*, 81:1049–91.

Perloff, J., Suslow, V., and Seguin, P. (1996). Higher prices from entry: Pricing of brand-name drugs, mimeo uc berkeley.

Reiffen, D. and Ward, M. R. (2002). Generic drug industry dynamics. ftc be working paper.

Ridley, D. (2002). The wrong prescription: price regulation in a market with consumer start-up costs.

Salop, S. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics and Management Science*, 10:141–156.

Scherer, F. (1993). Pricing, profits, and technological progress in the pharmaceutical industry. *Journal of Economic Perspectives*, 7(3):97–115.

Schlee, E. E. (2001). Buyer experimentation and introductory pricing. *Journal of Economic Behavior and Organization*, 44:347–62.

Shapiro, C. (1983). Optimal pricing of experience goods. *Bell Journal of Economics*, 14:497–507.

Stern, S. (1995). When entry causes prices to rise: An exploration of product demand in pharmaceutical markets, mimeo stanford university.

Temin, P. (1980). *Taking Your Medicine: Drug Refulation in the United States.* Harvard University Press.

Appendix

A Drugs studied

Table 7: Drugs studied

Branded Name	Generic Name	Patent Expiry
Blocadrin	Timolol	04/1989
Brevibloc	Esmolol	06/2000
Cartrol	Carteolol	10/1994
Corgard	Nadolol	01/1993
Inderal LA	Propranolol	02/1996
Inderal	Propranolol	12/1984
Kerlone	Betaxolol	01/1999
Levatol	Penbutolol	12/1992
Lopressor & Lopressor IV	Metoprolol	12/1993
Satral	Acebutolol	12/1993
Tenormin	Atenolol	09/1991
Toprol	Metoprolol Tartarate	01/1995
Visken	Pindolol	09/1992

B Solution to dynamic program

B.1 The solution from T on

We begin by analyzing the solution starting from time T. From T on, the problem is time-invariant, and the initial stock of X at time T we will denote K — this is the amount of knowledge accumulated over the interval $[0, T]$.

We draw the phase diagram for the unconstrained problem. Then we turn to the phase diagram in the presence of the constraint (see equation 10).

B.1.1 phase diagram, unconstrained

Ignoring the constraint, Equation 6 can be solved immediately to give A as a function of λ as:

$$A(\lambda) = (c_A)^{-1}(\lambda) = \rho_4 \lambda \tag{11}$$

Similarly, Equation 5 gives $p = p(\lambda, X)$:

$$p(\lambda, X) = \frac{1}{2}\left(c + \frac{\alpha_0 + \alpha_X X}{\alpha_p} - \rho_1 \lambda\right) \tag{12}$$

These solutions then may be substituted back into Equation 7 and Equation 8 to form a system of two autonomous differential equations. Then, the initial condition on X, $X_0 = 0$

along with a condition on λ would be enough, often, to provide a solution.

Since there are neither initial nor final conditions for λ in this infinite horizon problem, there will be a large family of time paths consistent with these first order conditions, and we will need to rule out all but one of them later.

Now, let us substitute back into the laws of motion for X and λ. First, let's consider $\dot{X}$ and substitute Equation 12 and Equation 11 into Equation 8.

$$\begin{aligned}
\dot{X} &= A + \rho_1 D + \rho_2 X - \rho_3 X^2 \\
&= \rho_4 \lambda + \frac{\rho_1}{2}\left[\alpha_0 + \alpha_X X - \alpha_p c + \alpha_p \rho_1 \lambda\right] + \rho_2 X - \rho_3 X^2 \\
&= \left(\rho_4 + \rho_1^2 \frac{\alpha_p}{2}\right)\lambda + \frac{\rho_1}{2}\left(\alpha_0 - \alpha_p c\right) + \left(\rho_2 + \alpha_X \frac{\rho_1}{2}\right) X - \rho_3 X^2
\end{aligned} \tag{13}$$

Similarly, substituting into Equation 7 yields:

$$\begin{aligned}
\dot{\lambda} &= r\lambda - (p - c) D_X - \lambda\left(\rho_1 D_X + \rho_2 - 2\rho_3 X\right) \\
&= \lambda\left(r - \frac{\rho_1 \alpha_X}{2} - \rho_2 + 2\rho_3 X\right) - \frac{\alpha_X}{2}\left(\frac{\alpha_0}{\alpha_p} - c\right) - \frac{\alpha_X^2}{2\alpha_p} X
\end{aligned} \tag{14}$$

From the time of patent expiry on, this problem is an infinite horizon, autonomous, discounted optimal control problem. We employ a phase diagram to solve the problem. We will look for a path leading to a steady state, so we solve Equation 13 for $\dot{X} = 0$ and Equation 14 for $\dot{\lambda} = 0$.

$$
\begin{aligned}
0 &= \dot{X} \\
&= \left(\rho_4 + \rho_1^2 \frac{\alpha_p}{2}\right) \lambda + \frac{\rho_1}{2} (\alpha_0 - \alpha_p c) + \left(\rho_2 + \alpha_X \frac{\rho_1}{2}\right) X - \rho_3 X^2 \\
\lambda &= \frac{1}{\rho_4 + \rho_1^2 \alpha_p / 2} \left(\rho_3 X^2 - \left(\rho_2 + \alpha_X \frac{\rho_1}{2}\right) X - \frac{\rho_1}{2} (\alpha_0 - \alpha_p c)\right)
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
0 &= \dot{\lambda} \\
&= \lambda \left(r - \frac{\rho_2 \alpha_X}{2} - \rho_2 + 2\rho_3 X\right) - \frac{\alpha_X}{2} \left(\frac{\alpha_0}{\alpha_p} - c\right) - \frac{\alpha_X^2}{2\alpha_p} X \\
\lambda &= \frac{\frac{\alpha_X}{2} \left(\frac{\alpha_0}{\alpha_p} - c\right) + \frac{\alpha_X^2}{2\alpha_p} X}{r - \frac{\rho_1 \alpha_X}{2} - \rho_2 + 2\rho_3 X}
\end{aligned} \tag{16}
$$

The graph of $\dot{X} = 0$ has λ quadratic in X with a positive coefficient on the squared term, a negative coefficient on the linear term, and a negative λ intercept. The graph of $\dot{\lambda} = 0$ is a negatively sloping hyperbola with a negative λ intercept, a positive vertical asymptote, and a positive horizontal asymptote. In addition, it is easy to check that $\dot{X}_\lambda > 0$, $\dot{X}_X < 0$ near $\dot{X} = 0$ when the graph of $\dot{X} = 0$ is sloping up, $\dot{\lambda}_\lambda < 0$, and $\dot{\lambda}_X < 0$.

Figure 7 contains a phase diagram for this problem. First, let us find all of the steady states. Recall that $\dot{X} = 0$ is a convex parabola with a negative λ intercept and that $\dot{\lambda} = 0$ is a negatively sloping hyperbola with a negative λ intercept. Intersections of these two graphs are the steady state levels of λ and X.

If the λ intercept of $\dot{X} = 0$ is less than the λ intercept of $\dot{\lambda} = 0$ then there are three

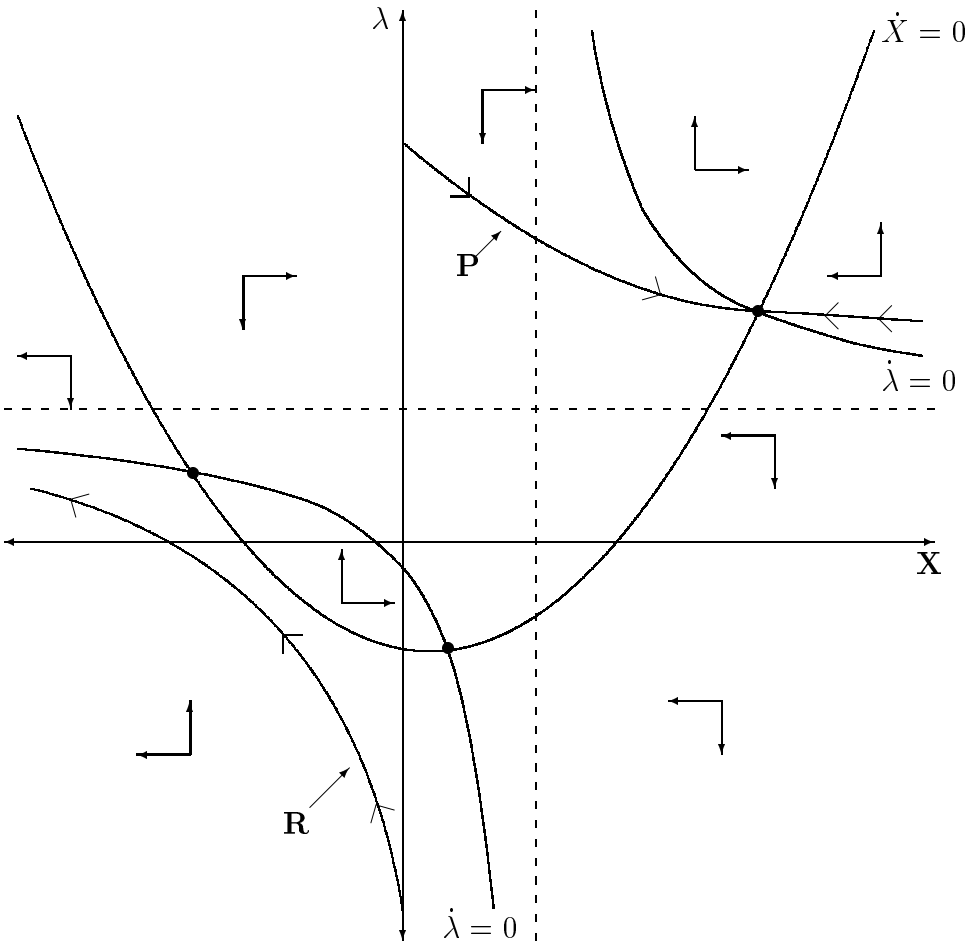


Figure 7: Phase diagram for knowledge stock

intersections between the two graphs (this is the case pictured in Figure 7). One intersection must lie in the first quadrant. One intersection must lie in the second quadrant. The last intersection may lie either in the third or fourth quadrant, and we have drawn it in the fourth.

On the other hand, if the λ intercept of $\dot{X} = 0$ is greater than the λ intercept of $\dot{\lambda} = 0$, there is a single intersection between the two graphs: the one in the first quadrant.

In the next section, we will argue that the path labeled "P" is the solution. For now, consider path "R." This is the type of potential solution we discussed in footnote 3. It eventually has the firm charging a negative price and earning negative per-unit profits, but making money by selling a large negative number of units.

B.1.2 phase diagram, with constraint

Imposing the constraint that $X \geq 0$ only affects the parts of the phase plane in which $X = 0$ and $\dot{X} < 0$. Thus, the affected portion of the phase plane is the λ-axis below the λ intercept of $\dot{X} = 0$. The effects of the constraint are to make $\dot{X} = 0$ along that portion of the axis and to change the magnitude but not the sign of $\dot{\lambda}$. The half of the phase plane in which $X < 0$ is no longer relevant since no path may reach it.

The paths in the phase plane fall into the following categories:

1. Path(s) like P leading to the steady state in the first quadrant

2. Paths along which $\lambda, X \longrightarrow +\infty$ monotonically eventually.

3. Paths along which $\lambda < 0$ eventually forever (like S and all paths which dip below $\lambda = 0$)

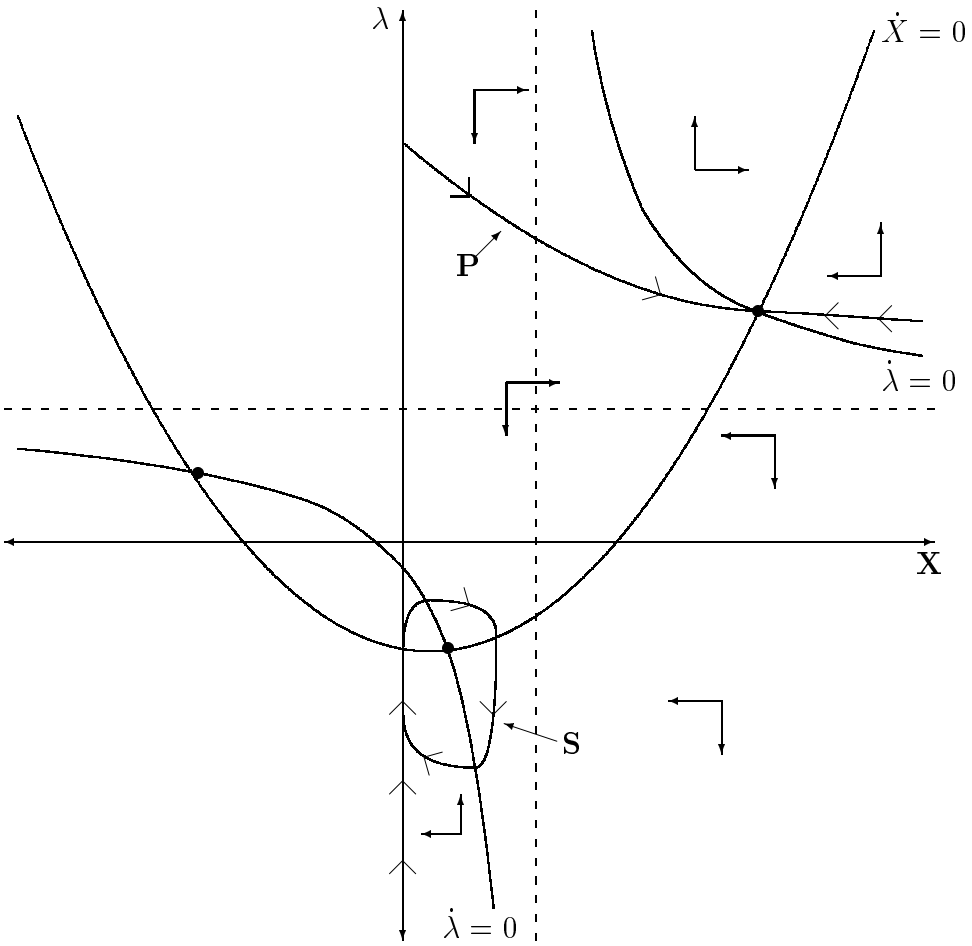


Figure 8: Phase diagram for knowledge stock, constrained

The paths in category 3 are not optimal. Along these paths, λ eventually is always negative. On the negative-λ portion, the firm is charging higher than the profit maximizing price and engaging in negative promotion in order to reduce X. The firm could do better by charging the myopic profit maximizing price and engaging in no promotion. This would both earn higher profits for any value of X and would lead to higher values of X.

The paths in category 2 are not optimal. As $X \longrightarrow +\infty$, it eventually becomes necessary for $p < c$ in order to maintain $\dot{X} > 0$, because $\rho_3 X^2$ grows more rapidly than $\rho_2 X + \rho_1 \alpha_X X$, so that $\alpha_0 - \alpha_p p$ must grow without bound in order to keep $\dot{X} > 0$. A path like this cannot be optimal, since it earns negative profits forever after some point. The firm could do better by engaging in myopic maximization eventually (since positive profits forever must be better than negative profits forever).

Since only the paths in category 1 are left, if there is an optimum, then it must correspond to a steady-state path like P. We can check that this path eventually leads to positive profits. Consider the steady state point itself. At this point, $\dot{\lambda} = 0$ and $\dot{X} = 0$ by definition of steady state. Because $\dot{X}$ is quadratic, we know that $\dot{X}_X < 0$ at the larger (in X) root of $\dot{X} = 0$. It is also true that:

$$\dot{\lambda} = r\lambda - (p - c)\alpha_X - \lambda(\dot{X}_X)$$

At the steady state in the first quadrant, $\lambda > 0$. As we argue above, at the steady state, $\dot{X}_X < 0$. Thus, both $r\lambda$ and $-\lambda\dot{X}_X$ are greater than zero. This means that, for $\dot{\lambda}$ to be equal to zero, it must be the case that $(p - c) > 0$. So, near the steady-state, the firm is

earning positive per period profits. So, on its face, a steady state path, P could reasonably be optimal.

Along a path like P, λ is monotonically decreasing in time, ie $\dot{\lambda} < 0$. Similarly, X is monotonically increasing, ie $\dot{X} > 0$.

Also, we may characterize the behavior of price and promotional effort from T to ∞. Since A is an increasing function of λ and since it does not depend upon X, we can conclude that A decreases along the optimal path. Similarly, since p is a decreasing function of λ and an increasing function of X, since $\dot{\lambda} < 0$, and since $\dot{X} > 0$ we may conclude that p is increasing along the optimal path.

B.2 The solution from 0 to T

The first order conditions described in Equations 5 through 10 must hold from 0 to T as well, of course. The difference is that, from 0 to T, α_0 is higher, α_p is higher, and α_X is lower. It is straightforward to verify that these conditions imply that $\dot{\lambda}$ is higher for each value of X. Also, since $\dot{\lambda}$ is decreasing in X and X is increasing over time, we can conclude that $\dot{\lambda}$ is higher before patent expiry than it is after.

Figure 1 shows a typical phase path for X and λ both before and after the expiry of the patent at the point labeled T. The path labeled O is the optimal path, and the path labeled P is (as in Figure 8) the optimal path with demand as it is after T. So the whole path P is the path that would be optimal were the patent to expire immediately, at time 0.

At the time that the patent expires, T, the paths for λ, X are continuous; however, they are not typically differentiable, since $\dot{\lambda}$ and $\dot{X}$ change discontinuously at those points. As

we discussed above, $\dot{\lambda}$ rises (falls in absolute value at T).

These changes have implications for the time paths of A and p. Since A depends positively on λ and only on λ, $\dot{A}$ rises (falls in absolute value) at time T. A typical optimal time path for A appears in Figure 2.

The case for p is not quite so simple. Since p depends negatively on λ and positively on X, $\dot{p}$ also depends negatively on $\dot{\lambda}$ and positively on $\dot{X}$. At time T, $\dot{\lambda}$ increases discontinuously (decreases in absolute value) and this tends to cause $\dot{p}$ to decrease.

$\dot{X}$ will decrease at time T. Consider the law of motion for X:

$$\dot{X} = A + \rho_1 D + \rho_2 X - \rho_3 X^2$$

Since X and A are continuous, no discontinuous change in X may come from these. D does change discontinuously at T; however, it is easy to show that, under our assumptions, D may not rise at T. Thus, since $\dot{\lambda}$ rises and $\dot{X}$ falls at time T, $\dot{p}$ must fall at time T. It is equally easy to show that p falls at time T discontinuously. Figure 3 depicts the path of prices.