

Rational Expectations.

①

- The Keynesian model of, say, 1965 didn't pay too much attention to expectations. They were assumed to be static, and exogenous.

It was inevitable that expectations be treated in such a cavalier manner. To see why, remember:

$$\frac{M^s}{P} = m(y, i),$$

$$y = c(y-t, i-\pi^e) + I(i-\pi^e) + g,$$

the two components of the IS model.

- we have two equations
- M^s , t and g are policy choices and we take them as exogenous.
- P ~~was~~ is assumed fixed.

That leaves us with 3 variables: y , i and π^e .
And how many equations did we have?

To pin ideas down even further, let us think about price determination in the ~~short run~~ long run. ②

- recall from the classical model that the real rate of interest is determined by the intersection of savings and investment, independently of π^e .
- further, y is determined by the labour market equilibrium and the production function.

So, in the money market:

$$\frac{M}{P} = m(y, i),$$

$$= m(y, r^e + \pi^e) \quad \left\{ \begin{array}{l} \text{using } i = r^e + \pi^e \end{array} \right.$$

we have

policy choice. $\frac{\bar{M}}{\bar{P}} = m(\bar{y}, \bar{r}^e + \bar{\pi}^e).$

$\left\{ \begin{array}{l} \text{labour market} \end{array} \right.$ determined by $S=I$

a) Expected inflation is given: $\pi^e = \bar{\pi}^e$. Then

$$\frac{\bar{M}}{\bar{P}} = m(\bar{y}, \bar{r}^e + \bar{\pi}^e),$$

and rearranging

$$p = \frac{\bar{M}}{m(\bar{y}, \bar{r}e + \bar{\pi}e)},$$

so the price level is uniquely determined.

b) Expected inflation is actually related in some way to realized inflation, $\pi^e = g(\dot{p}/p)$

$$\dot{p} = \frac{dp}{dt}$$

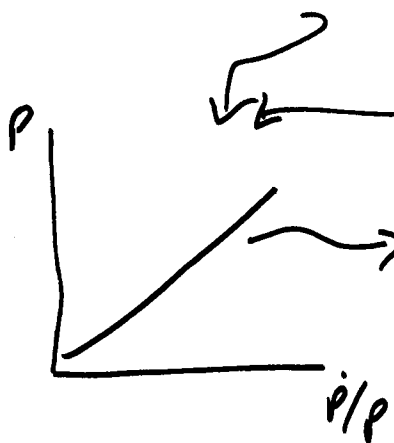
In this case, we have:

$$\dot{p}/p = \pi = \text{actual inflation.}$$

$$p = \frac{\bar{M}}{m(\bar{y}, \bar{r}e + g(\dot{p}/p))}$$

two unknowns, p and $\dot{p}$.
(or, equivalently, p and π)

but only one equation.



so we can only pin down a locus of values of $\{p, \dot{p}/p\}$. unless we are prepared to write down a theory of how π^e is formed, any p is consistent with money

market equilibrium.

So the theory in, say, 1965 was to assume π^e was fixed!

④
• Obviously, as we have seen from our study of the Phillips curve, the assumption of constant π^e can, at times, be dangerously misleading.

• So, in 1968, Friedman and Phelps pointed out that:

- expectations affect the equilibrium
- and expectations change.

In order for this to be a substantive contribution, however, it was necessary to offer an explanation of how expectations change. Put another way - it was necessary to add an extra equation to the model.

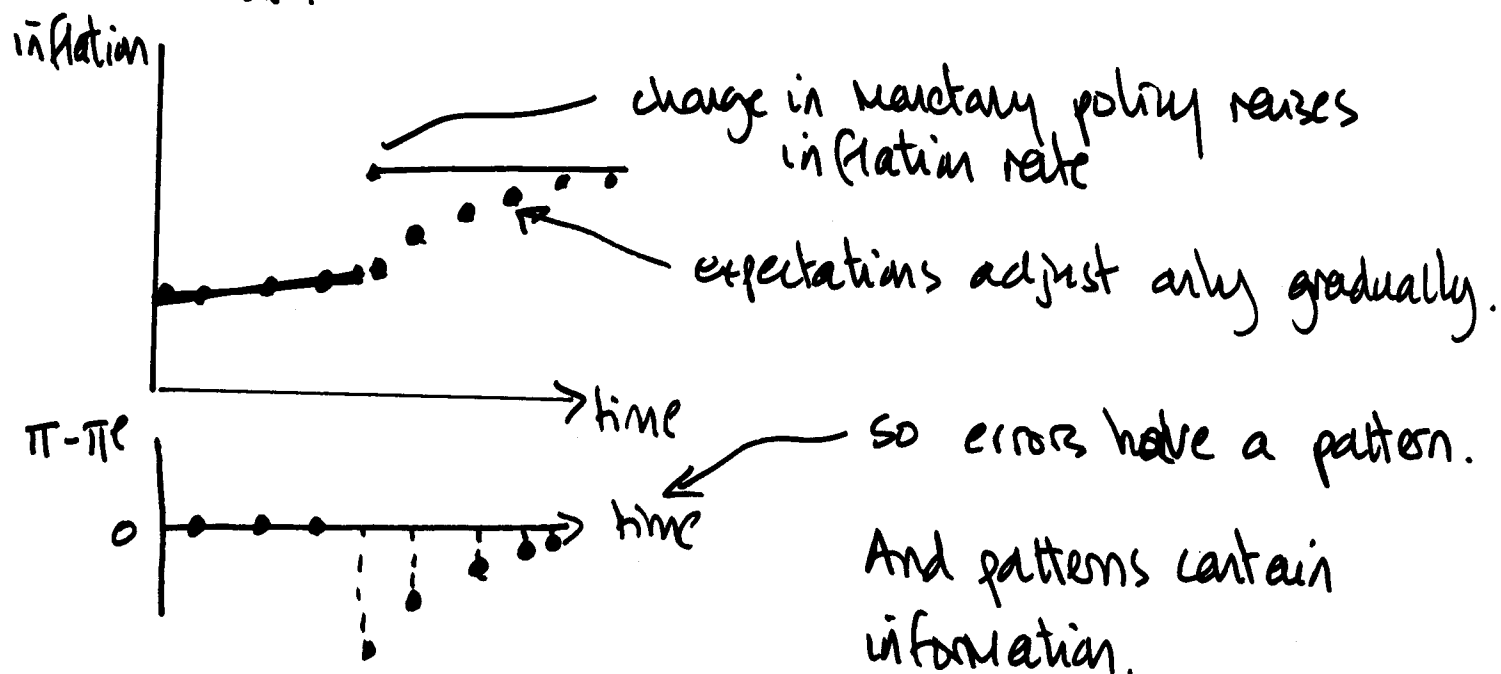
- Friedman & Phelps modeled changes in expectations by the adaptive expectations approach.

- The modeling strategy was convenient
 - by repeated substitution, π^e could be expressed as a function of past, observed inflation rates.
 - it appeared to work empirically.

However, by the early 1970's, adaptive expectations was on its way out.

The problem with adaptive expectations.

- It is backward looking, so whenever policy changes / or the environment changes, systematic errors are made:



And information is valuable.

So why don't people learn from these systematic and predictable mistakes.

- Another way of thinking about the dissatisfaction with adaptive expectations is to realize that if I can predict the forecasting mistakes people make, I can make profits off of them (e.g. lend money to them at artificially high nominal interest rates when I know that they will overestimate inflation, and borrow from them at low nominal rates when I know they will underestimate inflation).

A Lane Economics Joke.

- Two economists in the street. One says "there's a \$100 bill on the sidewalk." The other doesn't even look. "No there isn't," he says, "because if there were someone would have already picked it up."

- Economists often tell this joke to motivate rational expectations: any other form of expectations leave unexploited profit opportunities, and economists don't like that

- Personally, I don't buy the motivation

TWO ATLANTIC HIGHLAND BOYS REWARDED FOR THEIR HONESTY

by Allan Dean

ATLANTIC HIGHLANDS, NJ - Most anyone who found a hundred dollar bill on the street would pocket the money and count themselves lucky. For two boys in Atlantic Highlands the "find of their lives" presented an opportunity to for a lesson in morality and patience.

Christopher Dean, 11 and Christopher Clark, 12, found a hundred dollar bill on the sidewalk and turned it over to police. They were awarded the money yesterday when no one stepped forward to claim it.

Atlantic Highlands Journal
24 January 2002.



So what happens after adaptive expectations?

There are really several questions:

Q1. What was proposed instead of adaptive expectations?

Q2. How was it incorporated into macroeconomics?

Q3. Who was responsible for its development?

Q4. What were its implications for macroeconomic theory and policy.

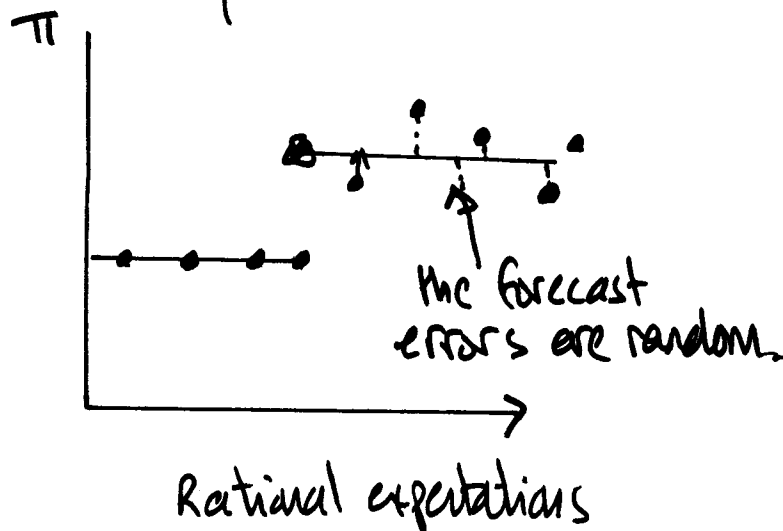
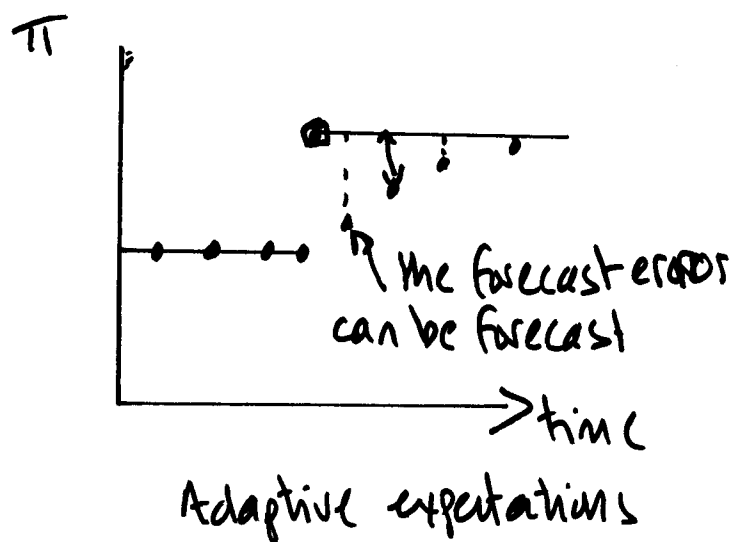
We will answer each of these in turn.

Q4 really has a long answer that will take up the entire rest of the course. The successor to adaptive expectations was rational expectations, and it had such a profound impact on the field that the 1970s is often referred to as the period of the rational expectations revolution.

We will approach the question in this section by looking at several particular applications in which rational expectations was a central part.

The concept of rational expectations.

- Rational expectations assumes simply that people make optimal use of information. We assume firms optimize about choosing inputs, consumers optimize about consumption choices. The Methodology of economics then insists that we should assume optimizing about information.
- RE does not assume that people make better forecasts than, say, adaptive expectations. It does say that people don't make systematic errors that can be corrected easily.
- Put another way, there should be no information contained in people's errors - they should be random.



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That's a nice concept, but how do you put it into practice?

- Imagine you have the following simple model of how the economy behaves over time:

$$y_t = \alpha E[y_{t+1} | I_t] + c x_t$$

output in period t depends on...

 ... the expected value of output in $t+1$ given information that is available in period t .

 + the value of some exogenous variable x .

constant coefficient constant coefficient.

We would like to know the value of y_t in terms only of observable variables. To do that, we have to write down a model of how expectations $E[y_{t+1} | I_t]$ are formed.

- Just as a reminder, adaptive expectations would write down

$$\begin{aligned}
 E[y_{t+1} | I_t] &= E[y_t | I_{t-1}] + \alpha [y_t - E[y_t | I_{t-1}]] \\
 &= (1-\alpha) E[y_t | I_{t-1}] + \alpha y_t
 \end{aligned}$$

and by repeated substitution:

$$E[y_{t+1} | I_t] = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} - \dots$$

So the solution would be

(10)

$$y_t = a E[y_{t+1} | I_t] + c x_t$$

$$= a y_t + a\alpha(1-\alpha)y_{t-1} + a\alpha(1-\alpha)^2 y_{t-2} + \dots + c x_t$$

$\therefore$ ~~From~~

$$(1-a\alpha)y_t = a\alpha \sum_{i=1}^{\infty} (1-\alpha)^i y_{t-i} + c x_t$$

$$\therefore y_t = \left(\frac{a\alpha}{1-a\alpha} \right) \sum_{i=1}^{\infty} (1-\alpha)^i y_{t-i} + \frac{c}{1-a\alpha} x_t$$

But, of course, we're not happy with adaptive expectations. In modeling rational expectations we assume:

- Individuals we are trying to model understand their environment. i.e. they know the model.
- so the best way to model their expectation of y_{t+1} is to find the mathematical expectation from the model itself.

There are several ways this can be done. We will look at one way, the method of undetermined coefficients.

Solving by the method of undetermined coefficients.

- The method involves guessing a functional form and then trying to find the coefficients in the functional form we have assumed. We can then verify whether or not the guess was correct.

Our problem is to solve:

$$y_t = a E[y_{t+1} | I_t] + c x_t. \quad (1)$$

R.E. is forward-looking (who ~~guesses~~^{cares} about past values if they don't influence the future). So we guess y_t depends on the current value of x and its future values (which are assumed to be known).

$$y_t = \sum_{i=0}^{\infty} \lambda_i x_{t+i} \quad (2)$$

$\lambda_0, \lambda_1, \lambda_2 \dots$ are the unknown coefficients.

Note: the original model was linear, so we guess the solution is also a linear equation.

If our guess is correct, then R.E. gives:

$$E[y_{t+1} | I_t] = \sum_{i=0}^{\infty} \lambda_i x_{t+i+1} \quad (3)$$

} make sure we discuss this in class.

So now we have a guess for y_t (eqn 2) and a guess for $E[y_{t+1} | I_t]$ (eqn 3). Lets put them in our model to get

$$\sum_{i=0}^{\infty} \lambda_i x_{t+i} = a \sum_{i=0}^{\infty} \lambda_i x_{t+i+1} + c x_t$$

To find out what these λ_i 's are, lets match up the coefficients on each side of the equation:

variable	left-side	right side
x_t	λ_0	c
x_{t+1}	λ_1	$a \lambda_0$
x_{t+2}	λ_2	$a \lambda_1$
$\vdots$		

} these must be equal on the left and right sides.

So: $\lambda_0 = c$
 $\lambda_1 = a \lambda_0 = ac$
 $\lambda_2 = a \lambda_1 = a^2 c$ and so on.

And we can write the solution as (using eqn 2):

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$$y_t = c x_t + a c x_{t+1} + a^2 c x_{t+2} + \dots$$

$$= c \sum_{i=0}^{\infty} a^i x_{t+i}.$$

so current y depends on the entire future path of x , and not at all on past values of y (contrast with the adaptive expectations model).

Some notes about what we have done

- Guessing the correct functional form is difficult, + takes practice (I will talk about verifying the guess in class). I do not expect you to master this on your own in the time we have left. I do expect you to understand what we are doing when we look at particular models.
- Guessing the solution is nearly impossible unless the model is simple and linear. So in looking at R.E. applications, we will write down simple linear models!

The people.

- The idea of rational expectations was first published by John Muth in 1961. He was at CMU and rumor has it, he did it to annoy Herb Simon.
- The ideas were absorbed into macroeconomics by Robert Lucas (at CMU for much of the '60s, now Chicago, '95 Nobel) and Tom Sargent (Stanford). The main work to introduce the ideas appeared in 1972-75.

"It took us longer than we like to recall to understand how thoroughly the idea of rational expectations would cause us to change the way we did macroeconomics."

Sargent: "Expectations and the Non-neutrality of Lucas;" p. 4.

- Very quickly, the implications of R.E. for macro were found to be profound:
 - introduced new ideas
 - changed the methodology of macro research (look how different the simple model is from the earlier stuff we have looked at).

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- macroeconomics became more mathematical:
simple, but precise + rigorous models; none of
the handwaving of the early Keynesian material.

We are going to look at some very specific and
profound early implications of rational expectations
models.

- 1972, 75 • Money neutrality (Lucas; Sargent & Neil Wallace)
- 1977 • time inconsistency (Finn Kydland & Ed Prescott)
- 1974 • Ricardian equivalence (Robert Barro)
- 1976 • Exchange rate overshooting (Rudiger Dornbusch)

When we have done this, we will come back to look
at modern theories of business cycles.

Money Neutrality.

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- Concept of combining RE. into a basic model of monetary policy was first carried out by Robert Lucas in 1972 (Journal of Economic Theory). It was subsequently expanded upon by Sargent & Wallace (see online reading).
- Basic idea: firms cannot immediately distinguish between a change in the demand for their product and a change in the general price level.
 - In the short-run, firms may confuse a nominal shock (a change in the general price level) with a real shock (a change in relative prices).
 - This confusion allows monetary shocks to have real effects in the short-run.
- Lucas is trying to deal here with the apparent implications of RE that monetary policy has no impact on output, while at the same time there is evidence that money shocks lead (cause?) output fluctuation.
~ Christmas!

- This confusion about relative price changes and general price changes leads us to a familiar equation:

$$y_t = \beta [P_t - E(P_t | I_{t-1})] \quad (1)$$

so, what will matter is not the equation itself, which we have seen before, but how expectations are formed.

} given an expected price, if the realized price is higher firms confuse this with increased relative price of their own good, and they respond by producing more.

- NOTES:
1. y_t is modeled as deviations from $\bar{y}$.
 2. y_t, P_t are the logarithms of output and price. (This will help us later in keeping all equations linear).

- The demand side of the economy is very simple, and is based on the money identity $MV = PY$. When we use logarithms, we can write this as:

$$y_t = m_t - p_t + v_t. \quad (2)$$

- Velocity is random, but assumed to be correlated with its own past values: $v_t = \rho v_{t-1} + u_t, E[u_t] = 0 \quad (3)$

- Finally, we need to write down some theory about how monetary policy behaves.

why? Because R.E. assumes forward-looking behavior in expectations formation, and this requires some idea about how key variables, such as the money supply will behave in the future:

$$m_t = -\phi v_{t-1} + e_t \quad (4)$$

contract the money supply if the previous period's velocity shock was positive.

random noise (error) because the Fed cannot control the money supply precisely. $E(e_t) = 0$.

Solving the Model.

We have:

$$y_t = \beta [p_t - E(p_t) | I_{t-1}] \quad (1)$$

$$\begin{aligned} y_t &= m_t - p_t + v_t \\ &= m_t - p_t + \phi v_{t-1} + u_t \end{aligned} \quad (2)$$

$$m_t = -\phi v_{t-1} + e_t \quad (3)$$

Combine these 3 equations to get a single equation involving only one of the three unknown endogenous variables — m , p and y .

- combine (1) + (2) to remove y
- use (3) to remove m :

$$-\phi V_{t-1} + e_t - p_t + \rho V_{t-1} + u_t = \beta (p_t - E[p_t | I_{t-1}]) \quad (5)$$

Solving by the method of undetermined coefficients.

- Assume a solution of the form

$$p_t = \alpha_0 u_t + \alpha_1 V_{t-1} + \alpha_2 e_t \quad (6)$$

$\alpha_0, \alpha_1, \alpha_2$ are unknown.

} equation contains all the terms in (5) above

If (6) is correct, then

$$E[p_t | I_{t-1}] = \alpha_1 V_{t-1} \quad (7)$$

← because $E[u_t | I_{t-1}] = 0$
 $E[e_t | I_{t-1}] = 0$

Substitute (6) and (7) into (5):

$$[\rho - \phi - \alpha_1] V_{t-1} + [1 - \alpha_2 - \beta \alpha_2] e_t + [1 - \alpha_0 - \beta \alpha_0] u_t = 0.$$

This equation must be true whatever the realized values of v_{t-1} , u_t and e_t are. And the only way to make that so is if each coefficient is zero:

$$[p - \phi - \alpha_1] = 0$$

$$1 - \alpha_1 - \beta \alpha_2 = 0$$

$$1 - \alpha_0 - \beta \alpha_0 = 0$$

Hence:

$$\alpha_1 = p - \phi$$

$$\alpha_2 = \alpha_0 = \frac{1}{1+\beta}$$

These coefficients imply

$$\begin{aligned} p_t &= \frac{1}{1+\beta} u_t + (p - \phi) v_{t-1} + \frac{1}{1+\beta} e_t \\ &= (p - \phi) v_{t-1} + \frac{1}{1+\beta} (u_t + e_t) \end{aligned} \quad (8)$$

of course, what we really care about is output. From (8), we have

$$E(p_t | \mathcal{I}_{t-1}) = (p - \phi) v_{t-1}$$

so

$$p_t - E(p_t | \mathcal{I}_{t-1}) = \frac{1}{1+\beta} (u_t + e_t)$$

and

$$y_t = \beta (p_t - E(p_t | \mathcal{I}_{t-1})) = \frac{\beta}{1+\beta} (u_t + e_t)$$

Let's put up these equations again:

(21)

$$p_t = (\rho - \phi) v_{t-1} + \frac{1}{1+\beta} (u_t + e_t)$$
$$y_t = \frac{\beta}{1+\beta} (u_t + e_t)$$

ϕ captures the Fed's systematic, predictable monetary policy. At time $t-1$, we observe v_{t-1} , so we know that $m_t = -\phi v_{t-1} + e$ will change by an amount ϕv_{t-1} .

- this is the anticipated component of monetary policy.

- it has an impact on price, but it does not appear in the equation for output!

Theorem: The Fed's choice of ϕ , its systematic monetary policy, has no influence on output.
(money neutrality)

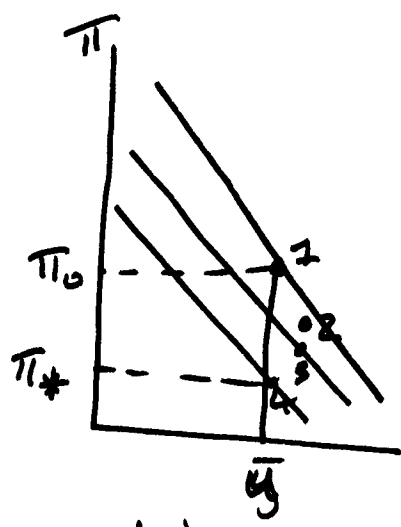
Corollary: The only component of monetary policy that can influence output is the unanticipated component, e_t . This is not helpful to the Fed, because e_t consists of its mistakes, that it cannot control.

Good News + Bad News.

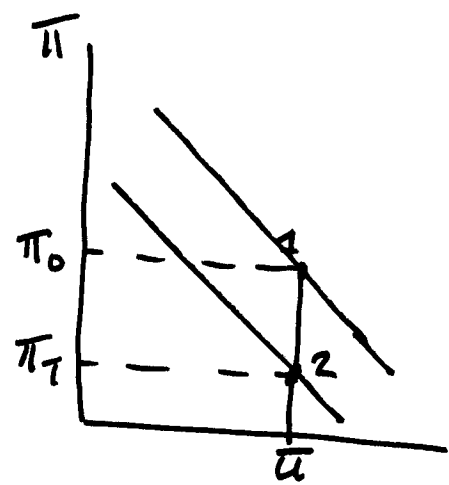
• Good news:

Announced policies affect the price level but not output. Hence, one can get inflation out of the system by simply announcing that money supply increases from now on will be smaller.

in the model, monetary policy changes can be announced by announcing a change in the parameter ϕ .



adaptive expectations implies a sacrifice because it takes time for π^e to decline...



.. under RE, if target inflation, π_T , is announced, π^e falls to π_T . The effect? A zero sacrifice ratio.

Did this work in the early 1980s?

(Volcker + Thatcher deflations)

Not only were they monetarist experiments, they were also R.E. experiments

- Bad news:

Guidance for policymakers such as "increase the money supply if the economy is about to slide into a recession" is self-defeating. Such a policy would raise inflation, but it would not save jobs. This would obviously be disappointing to Keynesians!

Is this money neutrality result robust?

No!

- One way in which systematic monetary policy ~~to~~ could have an impact is if the Fed bases its systematic monetary policy choices on the outcome of variables it has observed, but that the public has not yet observed.

Problem set #5
asks you to prove
this formally.

Intuitively this means that even though the Fed is behaving in a systematic way, from the perspective of the public it is unanticipated.

- Another way is to introduce price-stickiness. This we look at now

The Fischer Model

(24) (2)

Stan Fischer (1977), Journal of Political Economy.

- First Keynesian response to the Lucas & Sargent-Wallace papers. The model introduces wage rigidities resulting from long-term labor contracts.
 1. Nominal wages set for two periods
 2. Labor contracts do not restrict employment.
 3. Full price indexation of contracts not possible.
- Contract setting is staggered. In each period, half the economy is setting a two-period nominal contract.
 - w_t^1 = wage specified in the first period of contracts set in period t .
 - w_t^2 = wage specified in the second period of contracts set in period $t-1$.
- Contracts are set at beginning of period t , so there is no current information available.

That is,

w_t^1 is set based on I_{t-1} (info. available in period $t-1$)

w_t^2 is set based on I_{t-2} .

For simplicity, we assumed that wages are set on the basis of expectations about the price level.

- Imagine that the equilibrium real wage, w/p , equals 1.

- Then, firms and workers would attempt to set $w = p$, based on their expectations about what p will be:

$$w_t^1 = E[p_t | I_{t-1}]$$

$$w_t^2 = E[p_t | I_{t-2}]$$

- Output is assumed to be a simple two-period extension of the aggregate supply curve we saw before:

$$y_t = \frac{\beta}{2} (p_t - E[p_t | I_{t-1}]) + \frac{\beta}{2} (p_t - E[p_t | I_{t-2}])$$

- Demand is the same as before:

$$y_t = m_t - p_t + v_t$$

$$v_t = \rho v_{t-1} + u_t$$

and the money supply rule is the same as before:

(26)

$$M_t = -\phi V_{t-1} + e_t.$$

So - recap:

- the only change is that every contract lasts for two periods; half of all contracts adjust to the new economy / monetary policy in each period.
- what we will now show is that, unlike the previous model, the systematic component of monetary policy, ϕ , will affect output.

Step 1. Get an equation involving prices only.

- Square demand and supply:

$$\begin{aligned}\frac{\beta}{2}(p_t - E[p_t | I_{t-1}]) + \frac{\beta}{2}(p_t - E[p_t | I_{t-2}]) &= m_t - p_t + v_t \\ &= -\phi V_{t-1} + e_t - p_t + v_t \\ &= -\phi V_{t-1} + e_t - p_t + \rho V_{t-1} + u_t \\ &= (\rho - \phi) V_{t-1} + e_t + u_t - p_t\end{aligned}$$

Rearrange by bringing p_t over to the left-hand side:

$$(1+\beta)p_t - \frac{\beta}{2} E[p_t | I_{t-1}] - \frac{\beta}{2} E[p_t | I_{t-2}] = (p-\phi)V_{t-1} + e_t + u_t$$

$$= (p-\phi)\rho V_{t-2} + (p-\phi)u_{t-1} + e_t + u_t$$

I want to take
my exogenous variables
back to period $t-2$, using
 $V_{t-1} = \rho V_{t-2} + u_{t-1}$.

Step 2. Guess a functional form for the solution based on all the exogenous variables:

$$p_t = \alpha_0 u_t + \alpha_1 u_{t-1} + \alpha_2 V_{t-2} + \alpha_3 e_t$$

Step 3: Impose rational expectations:

$$E[p_t | I_{t-1}] = \alpha_1 u_{t-1} + \alpha_2 V_{t-2}$$

— stuff known in $t-1$

$$E[p_t | I_{t-2}] = \alpha_2 V_{t-2}$$

— stuff known in $t-2$.

Step 4: Substitute back into the original solution equation:

$$(1+\beta)[\alpha_0 u_t + \alpha_1 u_{t-1} + \alpha_2 V_{t-2} + \alpha_3 e_t]$$

$$- \beta/2 [\alpha_1 u_{t-1} + \alpha_2 V_{t-2}] - \frac{\beta}{2} [\alpha_2 V_{t-2}] = (p-\phi)\rho V_{t-2} + (p-\phi)u_{t-1} + e_t + u_t$$

Step 5 collect terms on each variable, and solve for the unknowns. (28)

variable	Left hand side	right hand side	
u_t	$(1+\beta)\alpha_0$	1	(a)
u_{t-1}	$(1+\beta)\alpha_1 - \frac{\beta}{2}\alpha_1$	$p-\phi$	(b)
c_t	$(1+\beta)\alpha_3$	1	(c)
v_{t-2}	$(1+\beta)\alpha_2 - \frac{\beta}{2}\alpha_2 - \frac{\beta}{2}\alpha_2$	$(p-\phi)p$	(d)

from (a): $\alpha_0 = \frac{1}{1+\beta}$

(c) $\alpha_3 = \frac{1}{1+\beta}$

(b): $(1+\frac{\beta}{2})\alpha_1 = p-\phi \Rightarrow \alpha_1 = \frac{2(p-\phi)}{2+\beta}$

(d): $\alpha_2 = (p-\phi)p$

Step 6 . solve for substitute the solutions for p into the supply curve.

This gives us

$$y_t = \frac{\beta}{1+\beta} (u_t + e_t) + \frac{\beta(p-\phi)}{2+\beta} u_{t-1}$$

Compare this with what we had before:

$$y_t = \frac{\beta}{1+\beta} (u_t + e_t)$$

Unlike Lucas' result, output has a predictable component that depends on systematic monetary policy, ϕ .

Conclusion

- Using monetary policy to fine-tune the economy is possible even under rational expectations, as long as there is some rigidity in wage contracts.
- Put another way, Stan Fischer's 1977 paper resurrected the role for activist monetary policy that appeared to have been undermined by Lucas only 5 years earlier.

The optimal rule.

Here is an interesting aside to the issue of money neutrality, that came out of Fisher's paper.

If one is prepared to write down what we think the objectives of the Fed are, we can then also think about the optimal choice of ϕ .

The best ϕ will depend on the particular objective. One possibility is that it wants to stabilize y (and u_t) as best as it can.

Now as

$$y_t = \underbrace{\left(\frac{\beta}{1+\beta}\right)(u_t + e_t)}_{\text{unpredictable component of output.}} + \frac{\beta(p-\phi)}{2+\beta} u_{t-1}$$

predictable component of output, because y_t depends on a variable observed in $t-1$.

we have

$$\text{var } y_t = \left(\frac{\beta}{1+\beta}\right)^2 (\text{var } u + \text{var } e) + \left(\frac{\beta(p-\phi)}{2+\beta}\right)^2 (\text{var } u).$$

which is minimized by setting $\phi = p$, and with this rule

$$y_t = \frac{\beta}{1+\beta}(u_t + e_t) \sim \text{if Fed adopts this optimal rule, output has no predictable component.}$$

Rules vs. Discretion.

- Despite the early conclusions of RE models, most economists agree that monetary policy has short-run impacts on output. Nonetheless, there has been considerable debate about what implications this has for the conduct of monetary policy.
- Keynesians have had the view that central banks can usefully engage in discretionary policy - evaluate where the economy is heading and then decide how to react to stabilize the economy.
- But as early as the 1940s, Milton Friedman had been arguing in favor of tying the central bank down by means of rules. Central banks, he argued, did such a lousy job, they should be stopped.
- Lucas, as we have seen, argued that (i) monetary policy has no desirable impacts on output when expectations are formed rationally, (ii) only mistakes in policy have an impact.

• This Lucas version of RE now appears to be exaggerated

— central banks may have superior information
(the problem set)

— there may be wage stickiness
(Stan Fischer) .

But, in 1977, R.E. theorists came up with another danger of monetary policy — time inconsistency.

Finn Kydland + Ed Prescott (1977): "The Inconsistency of optimal plans,"
Journal of Political Economy, 85(3):

- Time inconsistency is a problem in which a policy that seems optimal to announce now is no longer optimal when the time comes to enact it.

"Time inconsistency [is] a familiar problem faced by all decision makers — from parents to prime minister — who are trying to affect the behavior of others. Mr + Mrs. Smith want their daughter to go to college, but they are also keen that she get a summer job to learn how to act responsibly. So they offer to make up the rest of her university fees if she will get a job and earn some money; if she does not get a job and save, though, they warn her that she will get nothing. The snag is that Mr. + Mrs. Smith's plan is

time inconsistent, and their daughter knows it. Even if she does not get a summer job, she knows her parents will relent. They will pay her fees because their long-term interest is for her to go to college. She decides to take a holiday.

The Economist (see reading)

These sorts of problems are common in monetary policy:

- Central bank would like to see low inflation. It therefore announces that it will attain low inflation next year by allowing only a small increase in the money supply.
- If workers & firms believe the Bank, they will negotiate only a moderate increase in the nominal wage.
- But then, once contracts have been set, the central bank's incentives will have changed: it is no longer optimal to limit money supply growth. An increase in the money supply, with wages held fixed, will stimulate demand, causing a short-term increase in output.
- The problem is that neither workers nor firms will believe the bank's announcement. They will anticipate the large money supply increase and negotiate large wage increases. So the money supply increase is anticipated - all you get is a rise in prices, with no stimulus to output.

- Time inconsistency with monetary policy is likely to end up with higher inflation than would be the case if the central bank was really forced to make a commitment to the announced policy. So, in order to make a policy of low money supply growth credible, the central bank needs to find a commitment mechanism.

A Formal Model

Assume that aggregate supply is given by the now familiar equation:

$$y = \bar{y} + b(\pi - \pi^e)$$

Kydland + Prescott assume that the central bank would like to see an output level above the natural rate. But they also acknowledge that inflation is costly. A simple way to capture this is to assume that the central bank faces a loss function that it wants to minimize:

$$L = \frac{1}{2} (y - y^*)^2 + \frac{1}{2} a (\pi - \pi^*)^2,$$

π^* target long-run inflation.
 y^* is the optimal output level

We will imagine that the central bank gets to choose inflation directly.

Situation 1

- The bank makes a binding commitment about what inflation will be. Since the commitment is binding, expected inflation equals actual inflation. Thus,

$$\pi^e = \pi \Rightarrow y = \bar{y}.$$

- So, the bank simply chooses π to minimize its loss function. The solution is to choose

$$\pi = \pi^* \Rightarrow L = \frac{1}{2} (\bar{y} - y^*)^2$$

Situation 2

- The bank cannot make a binding commitment. Instead, it make a choice given the prevailing value of π^e , which it cannot influence by its ~~choice~~ announcement.

- To deal with this situation, substitute AS into the loss function:

$$L = \frac{1}{2} [\bar{y} + b(\pi - \pi^e) - y^*]^2 + \frac{1}{2} a (\pi - \pi^e)^2.$$

and now choose π to minimize the loss function:

$$\frac{\partial L}{\partial \pi} = [\bar{y} + b(\pi - \pi^e) - y^*] b + a(\pi - \pi^e) = 0.$$

so,

$$\pi = \frac{b^2 \pi^e + a \pi^* + b(y^* - \bar{y})}{a + b^2}.$$

or

$$\pi = \pi^* + \frac{b}{a+b^2} (y^* - \bar{y}) + \frac{b^2}{a+b^2} (\pi^e - \pi^*)$$

Now we impose RE. Since there is no uncertainty, $\pi^e = \pi$.

Thus,

$$\pi = \pi^e,$$

and

$$\pi - \frac{b^2}{a+b^2} \pi = \pi^* + \frac{b}{a+b^2} (y^* - \bar{y}) - \frac{b^2}{a+b^2} \pi^*$$

$$\pi \left(\frac{a}{a+b^2} \right) = \frac{a}{a+b^2} \pi^* + \frac{b}{a+b^2} (y^* - \bar{y})$$

$$\text{or } \pi = \pi^* + \frac{b}{a} (y^* - \bar{y})$$

• so without commitment, the bank ends up setting an inflation rate greater than π^* .

• but as $\pi^e = \pi$, it follows that $\bar{y} = y$.

So, the lack of a commitment technology causes the central bank to set a higher inflation rate than it would have done if it were able just to announce π^* and stick to it.

Note also the cost of doing this:

- with commitment, the loss function was

$$L^C = \frac{1}{2} (y - y^*)^2 + \frac{1}{2} a (\pi - \pi^*)^2$$

$$= \frac{1}{2} (\bar{y} - y^*)^2$$

— because $\pi = \pi^*$
+ $y = \bar{y}$.

- without commitment, the loss function is:

$$L^N = \frac{1}{2} (y - y^*)^2 + \frac{1}{2} a (\pi - \pi^*)^2$$

$$= \frac{1}{2} (\bar{y} - y^*)^2 + \frac{1}{2} a \left(\pi^* + \frac{b}{a} (y^* - \bar{y}) - \pi^* \right)^2$$

$$= \frac{1}{2} (\bar{y} - y^*)^2 + \frac{1}{2} \frac{b^2}{a} (y^* - \bar{y})^2$$

$$= \frac{1}{2} \left(\frac{a + b^2}{a} \right) (y^* - \bar{y})^2$$

$$= \frac{1}{2} (y^* - \bar{y})^2 + \frac{1}{2} \frac{b^2}{a} (y^* - \bar{y})^2$$

$$= L^C + \frac{1}{2} \frac{b^2}{a} (y^* - \bar{y})^2$$

} cost of time inconsistency.

Let us just summarize the modeling strategy:

- with commitment. whatever the bank announces, is believed, so we set $\pi^e = \pi$ and then ask what the optimal level of π is.
- without commitment. whatever the bank announces is not necessarily believed, so we take π^e as given, choose the optimal π and then set $\pi^e = \pi$.

The strategy formally told us what we had arrived at intuitively - that if you can't make commitments, you will end up being worse off. What you need is a way to make your initial announcement credible.

What is this credibility and how can I get me some?

"A central bank is credible if businesses and consumers have come to believe that the bank will act systematically to attain a reasonably small set of ultimate objectives. In a limited sense, a central bank with credibility may be interpreted as one that in a large sense behaves as though it were following a "rule."

Jeffrey Fuhrer
V.P. Federal Reserve Bank of Boston

Three ways to earn credibility

- Impose rules on the central bank
- Reputation effects.
- Delegation.

Imposing rules on the central bank.

- The government can tell the central bank what to do.
Problem is that governments often have even more political incentive to renege on announcements and produce surprise inflation than do central banks.
- Economists have made a distinction between
 - goal independence
 - instrument independence.

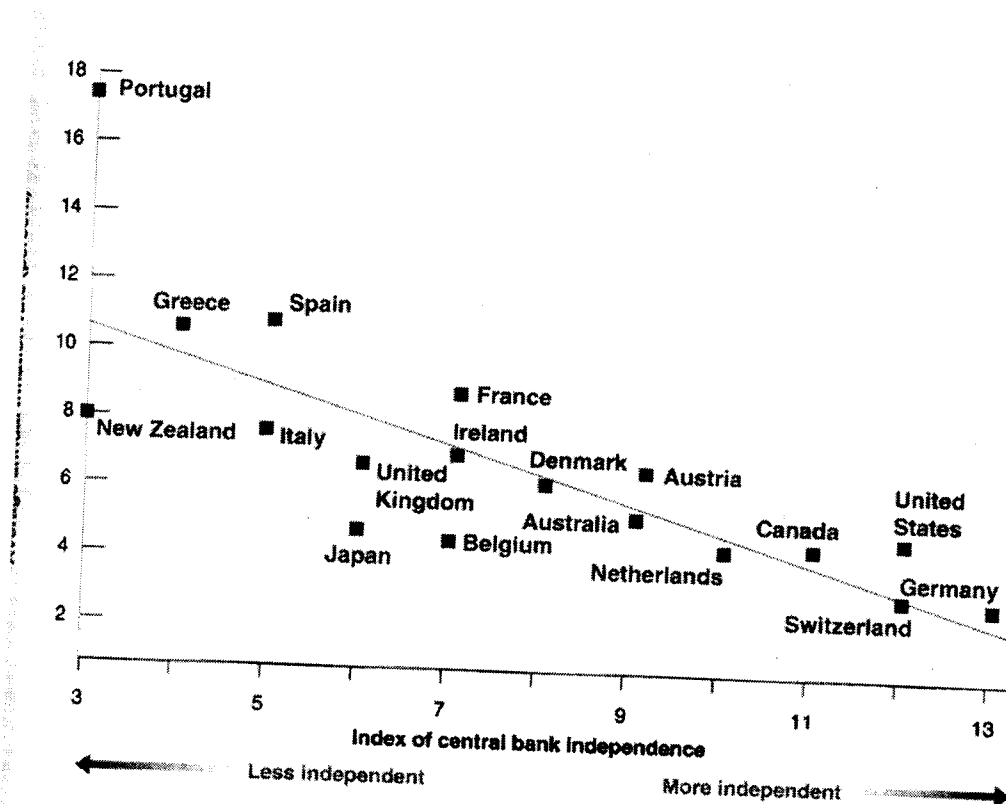
exists when a bank is free to choose its own targets or objectives

exists when a bank is free to choose its own policies that it thinks will meet the targets.

e.g. 1989 Reserve Bank of New Zealand Act gave the bank the singular mandate of price stability and granted it immunity from political pressure. A target of 0-2% inflation was mandated by law. But the bank was left to choose its own instruments.

• Economists have been circumspect about the nature of the restraints that should be imposed on central banks.

- imposing outside control by government can be counter productive; - because evidence shows that the central bank will perform better if it is relatively independent.



- so economists favor setting targets for the bank, but then leaving them alone.

Reputation effects.

Recall the following 'losses' calculated from the Kydland-Prescott model:

with commitment
and no cheating : $L^c = \frac{1}{2} (\bar{y} - y^*)^2$

without commitment : $L^{nc} = \frac{1}{2} (\bar{y} - y^*)^2 + \frac{1}{2} \frac{b^2}{a} (y^* - \bar{y})$

And now consider the pay-off from a central bank that everyone believes will commit to its announced policy of $\pi = \pi^*$, but then - when everyone has set their expectations $\pi^e = \pi^*$ - the bank reneges.

- This sneaky bank's loss function is given by

$$\begin{aligned} L^R &\stackrel{\sim \text{for 'reneege'}}{=} \frac{1}{2} [\bar{y} + b(\pi - \pi^e) - y^*]^2 + \frac{1}{2} a (\pi - \pi^e)^2 \\ &= \frac{1}{2} [\bar{y} + b(\pi - \pi^*) - y^*]^2 + \frac{1}{2} a (\pi - \pi^*)^2 \end{aligned}$$

Having set $\pi^e = \pi^*$, the optimal way to renege is found by:

$$\frac{dL^R}{d\pi} = [\bar{y} + b(\pi - \pi^*) - y^*]b + a(\pi - \pi^*) = 0$$

solving : $\Rightarrow \pi = \pi^* + \left(\frac{b}{a+b^2}\right)(y^* - \bar{y}) > \pi^*$

And this cheating gives a payoff of

(42)

$$L^R = \frac{1}{2} \left[\bar{y} + b \left(\frac{b}{a+b^2} \right) (y^* - \bar{y}) - y^* \right]^2 + \frac{1}{2} a \left(\frac{b}{a+b^2} (y^* - \bar{y}) \right)^2$$

$$= \frac{1}{2} \left(\frac{a}{a+b^2} \right) (\bar{y} - y^*)^2$$

$$= \left(\frac{a}{a+b^2} \right) L^C < L^C$$

So, to recap: $L^R = \left(\frac{a}{a+b^2} \right) L^C$

and $L^{NE} = \left(\frac{a+b^2}{a} \right) L^C.$

Now consider the following set of beliefs:

- If the central bank has never reneged in the past, people believe it will keep its word today.
- If the central bank has ever reneged in the past, people believe it will not be able to keep its word today.

So, a central bank that reneges today will be viewed as a no-commitment central bank in the future.

Given an interest rate of r , the ~~cost~~^{cost} of renegeing in (43)
the current time period is therefore

$$\begin{aligned}
 & L^R + \frac{1}{1+r} L^{NC} + \frac{1}{(1+r)^2} L^{NC} + \frac{1}{(1+r)^3} L^{NC} + \dots \\
 &= L^R + \frac{1}{r} L^{NC} \\
 &= \left(\frac{a}{a+b^2} \right) L^C + \frac{1}{r} \left(\frac{a+b^2}{a} \right) L^C \\
 &= \left[\left(\frac{a}{a+b^2} \right) + \left(\frac{a+b^2}{ar} \right) \right] L^C \quad \sim \text{lifetime losses to bank from renegeing.}
 \end{aligned}$$

If the central bank does not renege, its costs are

$$\begin{aligned}
 & L^C + \frac{L^C}{1+r} + \frac{L^C}{(1+r)^2} + \dots \\
 &= \frac{1+r}{r} \cdot L^C \quad \sim \text{lifetime losses from not renegeing.}
 \end{aligned}$$

A central bank wants to minimize its losses, so it will not renege - and thereby preserve its reputation - as long as

$$\begin{aligned}
 & \frac{1+r}{r} < \frac{a}{a+b^2} + \frac{a+b^2}{ar} \\
 \Rightarrow & \underline{r < \frac{a+b^2}{a} = 1 + \frac{b^2}{a}}
 \end{aligned}$$

As $r \approx 0.05$, say,
it is much lower than
the amount necessary
to prevent renegeing.

- As long as the central bank cares enough about future punishments from cheating, its own self-interest will stop it from cheating. That is, reputation effects provide a mechanism to persuade the central bank to commit.
- This view of reputation effects has led some observers to conclude that even time consistency is not a serious problem.

Delegating the problem away.

Recall that, faced with time inconsistency and an inability to commit, a central banker sets too high an inflation rate for his/her preferences.

- One way to deal with this is for society to delegate monetary policy making to a central banker whose preferences are different from those of society as a whole.
- Specifically, society should appoint a central banker who cares less about having $y > \bar{y}$, and who dislikes inflation more.

Characterizing Central Bank Rules

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Taylor Rules.

We have seen that time consistency demands that the Fed follow some sort of a rule in setting monetary policy. The question is what should the rule look like?

John Taylor (1993)* suggested a simple and specific rule for the US.

$$i_t = \pi_t + r^* + 0.5(\pi_t - \pi^*) + 0.5(\tilde{y}_t)$$

interpreted as the
Federal funds rate

where $\tilde{y}_t = 100 \times \left(\frac{\text{GDP} - \text{Potential GDP}}{\text{Potential GDP}} \right)$
"output gap"

long run
equilibrium real
rate of interest.

we can also write this as

$$r_t = r^* + 0.5(\pi_t - \pi^*) + 0.5(\tilde{y}_t)$$

~~fig.~~ ~~over~~

* "Discretion versus policy rules in practice" Carnegie-Rochester Conference Series on Public Policy, vol. 39

which can be read as:

$$r_t = r^* + 0.5 (\pi_t - \pi^*) + 0.5 (\tilde{y}_t)$$

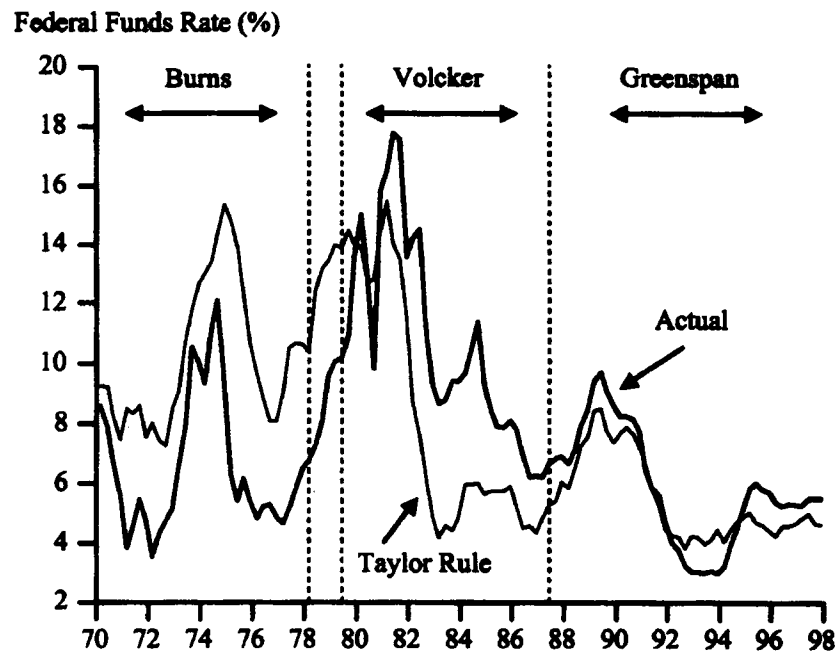
(1) raise the real rate of interest
 (2) ... above its long run equilibrium rate ...
 (3) ... by 0.5 percent ...
 (4) ... for each 1% excess of inflation over the target inflation rate ...
 (5) ... and by 0.5% ...
 (6) ... for each 1% that output is above potential.

NOTE that the rule simply says that interest rates should rise (= money supply should contract) if inflation or output is too high.

Taylor noted

"The policy rule... has the general properties of the rules that have emerged from recent research.... [and] although there is not a consensus about the size of the coefficients..... [they] are round numbers that make for easy discussion."

Now, the so-called Taylor rule was intended as a prescription for what the Fed should do. Surprisingly, it turned out to also be a good way to characterize what the Fed does do.



Source: Judd, J. + G. Rudebusch (1998):
FREBSF Economic Review, No. 3

plots the interest rate that would have been set if the Taylor rule had been ~~set~~^{followed} exactly, along with the actual interest rate set.

- 1970-80 monetary policy was too 'loose'
- 1981-88 monetary policy was too 'tight'
- 1988-98 monetary policy was about right.

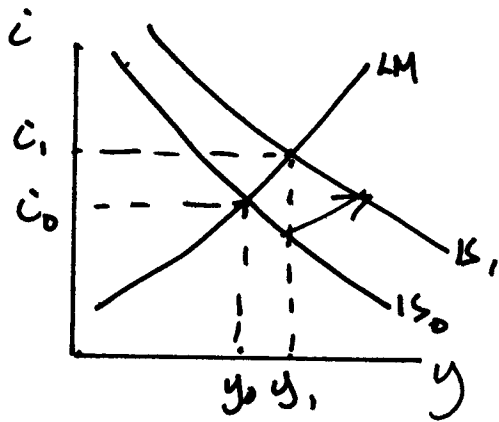
So now we have a reasonable quantitative way to characterize monetary policy.

Ricardian Equivalence

Barro, R. (1974): "Are Government Bonds Net wealth?" Journal of Political Economy.

This is the fiscal policy counterpart to the money neutrality result.

Recall from the IS-LM model:



A cut in taxes raises people's disposable income, increasing their consumption and thus stimulating the economy: the IS curve shifts right.

The implication of this is that if a government decides to finance its expenditure by borrowing instead of by taxing, it can stimulate output.

Barro's paper argued that the Keynesian story is wrong.

Barro's paper relies in an essential way on the forward looking nature of individuals assumed by rational expectations.

The basic theory is thus:

- Suppose govt. cut taxes by \$100 and issued a \$100 bond. Consumers, who bought the bond, would be left with exactly the same money available for purchases as before.
- But, if they felt wealthier because they owned this bond, they would spend more, so bond-financing stimulates the economy.
- Barro's paper explains that this argument ignores the fact that in the future the same consumers will have to pay higher taxes to cover the interest and principal on the bond.
- So consumers increase their saving now to prepare for the tax increase. And under the right conditions of rational expectations, competitive markets, etc., the amount they save is exactly equal to the amount that taxes were cut so that consumption spending is unaltered by the tax cut.

This proposition became known as Ricardian Equivalence, because it subsequently turned out that David Ricardo had raised the idea in 1817.

Ricardian equivalence is a proposition about the effect of fiscal policy that can annoy conservatives and liberals alike.

- liberals (often Keynesians) believe deficit-financed government expenditure stimulates the economy. Ricardian equivalence says it won't.
- fiscal conservatives have been fretting over the size of the budget deficit. Ricardian equivalence says it doesn't matter.

Note: Ricardian equivalence can be viewed as the public finance equivalent of Modigliani-Miller theorem (that it doesn't matter with firms finance their investments with debt or equity).

Objections to Ricardian Equivalence.

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- government debt 'lives' larger than people do. As I won't be around to suffer the higher taxes, I don't need to save to pay them.
 - Response: but we care about our children (we leave bequests!), and this is equivalent to living forever.
 - Counter: but 20% of US taxpayers don't have children.
- taxes are distortionary, so they matter.
 - Response: True. so we need to use non-distortionary taxes
- people don't have all the information - the majority don't have a clue about what the deficit is.
 - Response: er, OK!

The nature of these objections has led the majority of economists to conclude that

- Ricardian equivalence at best is an approximation
- It is more relevant if tax cuts are clearly temporary
- But in most circumstances, tax policy matters.