

HOMEWORK 7  
Due Thursday, November 1

1. Verify that the following syllogism is correct by giving a deduction.

No Greeks are slaves.  
Some slaves are women.  
Therefore, some women are not Greek.

2. Formalize the following argument and prove that it is correct.

Romeo loves Juliet.  
Romeo is a Montague.  
Juliet is a Capulet.  
Any Montague who loves a Capulet will come to a bad end.  
Therefore, Romeo will come to a bad end.

3. The language of *additive monoids* has a constant symbol  $0$  and a binary function symbol, written  $x + y$ . The axioms are associativity  $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$ ; commutativity  $\forall x \forall y (x + y = y + x)$ ; and  $0$  is a (two-sided) unit  $\forall x (0 + x = x)$  and  $\forall x (x + 0 = x)$ .

Define the *even* elements by

$$E(x) =_{\text{df}} \exists z (x = z + z).$$

Use deduction to show that the sum of two even elements is even.

- ★ 4. Taking only the introduction and elimination rules for the existential quantifier as given, derive the corresponding rules for the universal quantifier, defined by  $\forall = \neg \exists \neg$  (in the way done in Lemma 2.9.1 of van Dalen). Be sure to take account of the side conditions on the rules.