Homework 6

Due Thursday, October 25

1. Consider a first-order language, with relation symbols $<$ and $=$, and constant symbol 0. Consider the interpretation: the natural numbers, with "less-than" and "equality", and the distinguished element 0. Formalize the following statements:
   (a) "x is less than or equal to y"
   (b) "0 is the smallest number"
   (c) "there is a smallest number"
   (d) "there is no largest number"
   (e) "every number has an immediate successor" (in other words, for every number, there is another one that is the "next largest")
   (f) "every number is greater than some (other) number"
   (g) "there is some number that every (other) number is greater than"
   (h) Add the operations $+$ and $\cdot$, and the constant 1, and formalize the following:
      i. "all square numbers are positive"
      ii. "there's just one even prime number"
      iii. "between every two squares there is always a prime"

2. Consider a first-order language with predicate symbols L, P, M and F. The intended interpretation is all people, living or dead, with $L(x, y)$ meaning "x loves y", $P(x, y)$ meaning "x is a parent of y", $M(x)$ meaning "x is male", $F(x)$ meaning "x is female". Formalize the following statements:
   (a) "everyone loves their grandmother"
   (b) "some fathers love women who are not the mothers of their (the fathers') children"
   (c) "not all aunts are loved"
   (d) "to love and to be loved, that is to be a parent!"

3. Give natural deduction proofs of the following formulas (using the 4 quantifier rules, and not defining $\exists\varphi$ as $\neg\forall\neg\varphi$!).

   (a) $\neg\exists x\varphi \to \forall x\neg\varphi$

   (b) $\exists x\neg\varphi \to \neg\forall x\varphi$

   (c) $(\exists x\varphi \to \psi) \to \forall x(\varphi \to \psi)$, where x is not free in ψ.

   (d) $(\exists x\varphi \wedge \psi) \to \exists x(\varphi \wedge \psi)$, where x is not free in ψ.

   (e) $(\forall x\varphi \vee \psi) \to \forall x(\varphi \vee \psi)$, where x is not free in ψ.

   In each case, the converse also holds, but you need not show this (unless you want some additional practice!).