Homework 3

Due Thursday, September 20

1. Do the even numbered cases in problem 1 on page 27 of van Dalen, as well as the following:

$$\models (\varphi \to (\psi \to \vartheta)) \to ((\varphi \to \psi) \to (\varphi \to \vartheta))$$

2. Using algebraic means, determine conjunctive and disjunctive normal forms for the following formulas:

$$\neg(p \leftrightarrow q), \quad ((p \to q) \to p) \to p$$

3. Use the normal forms from the previous problem to determine whether or not each formula is a tautology.

4. (a) Show that all of the truth functions (on $\{0, 1\}$) can be defined in terms of $\{\to, \bot\}$, i.e. that this is a *functionally complete set of connectives.*

   (b) Show that $\{\to, \vee, \wedge\}$ is not a functionally complete set of connectives.

   (c) Conclude that $\{\to, \vee, \wedge, \leftrightarrow, \top\}$ is not a functionally complete set of connectives. (Hint: define the last two in terms of the others.)

⋆ 5. Let X be a set with an equivalence relation, written $x \sim y$. Suppose given a set A, and a function $f : X \to A$ that respects $\sim$, in the sense that $f(x) = f(y)$ whenever $x \sim y$. Show that there is a function

$$\overline{f} : X/\!\sim\ \longrightarrow A$$

such that for all $x \in X$,

$$\overline{f}([x]) = f(x),$$

where the *quotient set* $X/\!\sim\ = \{[x] \mid x \in X\}$ is the set of all *equivalence classes* $[x] = \{y \mid x \sim y\}$.