Solutions to Homework #11

3. The first derivation is used in the second one, at the position marked * (there wasn't enough room to write it in).

   a.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[p]_1 \quad [p \leftrightarrow \neg p]_3}{\neg p} \quad [p]_1}{\bot}}{\neg p}\,1 \quad [p \leftrightarrow \neg p]_3}{p} \quad \dfrac{\dfrac{\dfrac{[p]_2 \quad [p \leftrightarrow \neg p]_3}{\neg p} \quad [p]_2}{\bot}}{\neg p}\,2}{\bot}}{\neg(p \leftrightarrow \neg p)}\,3$$

   b.

$$\dfrac{\dfrac{[\exists y\, \forall x\, (S(y,x) \leftrightarrow \neg S(x,x))]_2 \quad \dfrac{\dfrac{[\forall x\, (S(y,x) \leftrightarrow \neg S(x,x))]_1}{S(y,y) \leftrightarrow \neg S(y,y)}}{\bot}\,*}{\bot}\,1}{\neg \exists y\, \forall x\, (S(y,x) \leftrightarrow \neg S(x,x))}\,2$$

4. Forwards direction:

$$\dfrac{\dfrac{[\exists x\, (\varphi(x) \wedge \psi)]_2 \quad \dfrac{\dfrac{\dfrac{[\varphi(x) \wedge \psi]_1}{\varphi(x)}}{\exists x\, \varphi(x)} \quad \dfrac{[\varphi(x) \wedge \psi]_1}{\psi}}{\exists x\, \varphi(x) \wedge \psi}}{\exists x\, \varphi(x) \wedge \psi}\,1}{\exists x\, (\varphi(x) \wedge \psi) \to \exists x\, \varphi(x) \wedge \psi}\,2$$

   Backwards direction:

$$\dfrac{\dfrac{\dfrac{[\exists x\, \varphi(x) \wedge \psi]_2}{\exists x\, \varphi(x)} \quad \dfrac{\dfrac{[\varphi(x)]_1 \quad \dfrac{[\exists x\, \varphi(x) \wedge \psi]_2}{\psi}}{\varphi(x) \wedge \psi}}{\exists x\, (\varphi(x) \wedge \psi)}}{\exists x\, (\varphi(x) \wedge \psi)}\,1}{\exists x\, \varphi(x) \wedge \psi \to \exists x\, (\varphi(x) \wedge \psi)}\,2$$

7. Forwards direction:

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg\exists x\ \varphi(x)]_2 \qquad \dfrac{[\varphi(x)]_1}{\exists x\ \varphi(x)}}{\bot}}{\neg\varphi(x)}\ 1}{\forall x\ \neg\varphi(x)}}{\neg\exists x\ \varphi(x) \to \forall x\ \neg\varphi(x)}\ 2$$

Backwards direction:

$$\dfrac{\dfrac{\dfrac{[\exists x\ \varphi(x)]_2 \qquad \dfrac{[\varphi(x)]_1 \qquad \dfrac{[\forall x\ \neg\varphi(x)]_3}{\neg\varphi(x)}}{\bot}}{\bot}\ 1}{\neg\exists x\ \varphi(x)}\ 2}{\forall x\ \neg\varphi(x) \to \neg\exists x\ \varphi(x)}\ 3$$

9.

$$\dfrac{\dfrac{\forall x\ L(x,b)}{L(b,b)} \qquad \dfrac{\forall x\ (L(b,x) \to x = m)}{L(b,b) \to b = m}}{b = m}$$

10. For I_2:

$$\dfrac{\dfrac{\dfrac{\forall x, y, z\ (x = y \wedge z = y \to x = z)}{u = u \wedge v = u \to u = v} \qquad \dfrac{\dfrac{\forall x\ (x = x)}{u = u} \qquad [v = u]_1}{u = u \wedge v = u}}{\dfrac{u = v}{v = u \to u = v}\ 1}}{\forall v, u\ (v = u \to u = v)}$$

In this derivation, I've combined instances of the $\forall$ rules. On the top left, note that I am substituting u, u, and v for x, y, and z respectively.

For I_3, use I_2:

$$\dfrac{\dfrac{\dfrac{\forall x, y, z\ (x = y \wedge z = y \to x = z \qquad \dfrac{\dfrac{[x = y \wedge y = z]_1}{x = y} \qquad \dfrac{\dfrac{[x = y \wedge y = z]_1}{y = z}}{z = y}\ *}{x = y \wedge z = y}}{x = z}}{x = y \wedge y = z \to x = z}\ 1}{\forall x, y, z\ (x = y \wedge y = z \to x = z)}$$

Here the derivation of I_2 is used at *.

13.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi]_1}{\varphi \vee \neg\varphi} \qquad [\neg(\varphi \vee \neg\varphi)]_2}{\bot}}{\neg\varphi}\,1}{\varphi \vee \neg\varphi} \qquad [\neg(\varphi \vee \neg\varphi)]_2}{\bot}}{\varphi \vee \neg\varphi}\ 2\ \mathrm{RAA}
$$