Econ 73-250A-F
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Suggested Answers to Practice Exercises for Exam #2

Exercise #1. (a) To show that the technology displays increasing returns to scale, multiply all inputs by a number $t > 1$:

$$\begin{aligned} y_t &= \frac{1}{1{,}000}(tK)(tL)^{\frac{1}{2}} \\ &= \left(t^{1+\frac{1}{2}}\right)\frac{1}{1{,}000}KL^{\frac{1}{2}} \\ &= t^{\frac{3}{2}}y \end{aligned}$$

where

$$y = \frac{1}{1{,}000}KL^{\frac{1}{2}}.$$

Output increases by a factor

$$t^{\frac{3}{2}} > t.$$

Thus, the technology displays increasing returns to scale.

(b) The marginal products are

$$\begin{aligned} MP_L(K,L) &= \frac{1}{2(1{,}000)}KL^{-\frac{1}{2}} \\ MP_K(K,L) &= \frac{1}{1{,}000}L^{\frac{1}{2}}. \end{aligned}$$

(c) The firm solves the problem

$$\begin{aligned} &\min_L (10L + (20)\,1{,}000) \\ &s.t. \\ y &= \frac{1}{1{,}000}(1{,}000)\,L^{\frac{1}{2}}. \end{aligned}$$

From the production function, we get

$$L^* = y^2.$$

The cost function is then

$$c(y) = 10y^2 + 20{,}000.$$

(d) The firm's short run marginal cost function is

$$MC(y) = \frac{\partial c(y)}{\partial y} = 20y.$$

The firm's short run average cost function is

$$AC(y) = \frac{c(y)}{y} = 10y + \frac{20{,}000}{y}.$$

The marginal cost function intersects the average cost function at the point

$$\begin{aligned} 20y &= 10y + \frac{20{,}000}{y} \\ y^* &= \sqrt{2{,}000}. \end{aligned}$$

(e) See Figure 1.

Exercise #2. (a) Equilibrium is obtained by setting quantity supplied equal to quantity demanded:

$$2{,}000w = 12{,}000 - 2{,}000w$$

which gives us

$$w = \frac{12{,}000}{4{,}000} = \$3.$$

The quantity of labor is

$$L_s^* = (2{,}000)\,3 = 6{,}000.$$

(b) With a minimum wage of \$4, now firms will demand only

$$L_d^* = 12{,}000 - (2{,}000)\,4 = 4{,}000$$

person-hours.

(c) See Figure 2.

(d) The workers gain $A = \$(4-3)\,(4{,}000) = \$4{,}000$ and lose $B = \$\,(6{,}000 - 4{,}000)\,(3-2)/2 = \$1{,}000$. Thus, their net gain is \$3,000. Firms lose A and $C = \$(4-3)\,(6{,}000 - 4{,}000)\,/2 = \$1{,}000$. Thus they lose \$5,000. The deadweight loss is $B + C = \$2{,}000$.

Exercise #3. (a) The price elasticity of demand is

$$\begin{aligned} \varepsilon &= \frac{\partial x_d\,(p{,}m)}{\partial p} \frac{p}{x_d\,(p{,}m)} \\ &= -\frac{1}{2}\frac{m}{p^2}\frac{2p^2}{m} \\ &= -1. \end{aligned}$$

The income elasticity of demand is

$$\begin{aligned} \eta &= \frac{\partial x_d\,(p{,}m)}{\partial m} \frac{m}{x_d\,(p{,}m)} \\ &= \frac{1}{2}\frac{1}{p}\frac{2pm}{m} \\ &= 1. \end{aligned}$$

(b) The firm maximizes

$$\Pi = (10)\, 10\sqrt{L} - wL.$$

The first order condition gives the inverse labor demand curve

$$w = 50L^{-\frac{1}{2}}.$$

See Figure 3 for the plot.

(c) The Laffer curve relates a government tax revenue to the tax rate. It has an inverted U-shape. That is, if the tax rate t were 0, there would not be any revenue for the government. Similarly, if $t = 1$ (100% tax rate) nobody would want to work, and tax revenue would also be zero. For values of $0 < t < 1$ government revenue is positive. A reduction in t has two effects on revenue: 1) for given hours of work it decreases revenue; 2) it induces people to work more hours and thus increases revenue.

At relatively high values of t (t close to 1), a decrease in t might generate an increase in tax revenue, as effect number 2) dominates effect number 1). For relatively low values of t, instead, a decrease in tax rates is likely to generate a decrease in government revenue, i.e., effect 1) dominates.

FIGURE 1

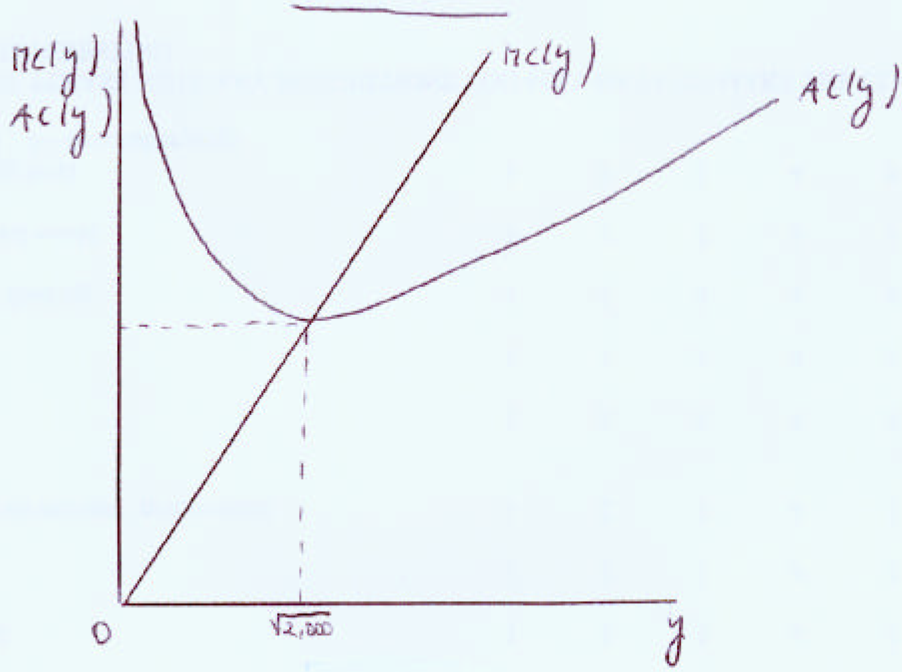

FIGURE 2

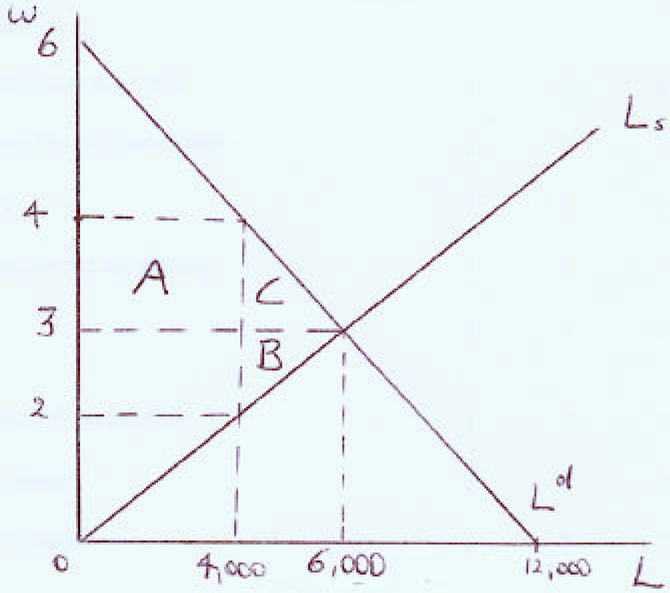

FIGURE 3