

Suggested Answers to Practice Exam # 1

Please find all graphs at the end of this document.

Exercise # 1. (a) Andy's budget line is

$$x_1 + x_2 = 400.$$

Andy's preferences are Cobb-Douglas. The marginal rate of substitution between good 1 and 2 is

$$MRS(x_1, x_2) = -\frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}} = -\frac{x_2}{x_1}.$$

Since Cobb-Douglas preferences are well-behaved and do not have kinks, the necessary and sufficient condition for an optimal choice is given by

$$MRS(x_1, x_2) = -\frac{p_1}{p_2}.$$

In this case $p_1 = p_2 = 1$. Therefore,

$$-\frac{x_2}{x_1} = -1,$$

or

$$x_1 = x_2.$$

Thus, Andy consumes the same amount of each good. To find his optimal consumption bundle use this condition and the budget constraint:

$$\begin{aligned} x_1 &= x_2 \\ x_1 + x_2 &= 400. \end{aligned}$$

Solving this system yields $x_1^* = x_2^* = 200$. See Figure 1.

(b) The equation for the budget line is

$$\begin{aligned} 0.5x_1 + x_2 &= 400 \text{ if } x_1 \leq 200, \\ x_1 + x_2 &= 500 \text{ if } x_1 > 200. \end{aligned}$$

It is possible to find this line by means of the following argument. If Andy does not buy any food, then he can spend \$400 on other things. The point (0,400) is therefore a point on the budget line. For each dollar that Andy spends on food up to a maximum of \$200, Andy receives \$0.50 back from the government. This means that, in effect, the price of a dollar's worth of food is only \$0.50 until Andy reaches \$200 in food expenditures (at which point his cash rebates from the government

would equal to $0.50(200)=100$ dollars). In other words, the point $(200,300)$ is also a point on the budget line: at this point Andy's net expenditures on food are \$100, leaving him \$300 dollars to spend on other things. In addition, all of the points on the line connecting $(0,400)$ and $(200,300)$ are on the budget line. The slope of this line segment is $-1/2$: for each dollar by which Andy reduces his expenditures on other things, he can buy \$2 worth of food.

Once Andy reaches the point $(200,300)$, the slope of his budget line changes to -1 (i.e. the slope of the budget line in part (a)). This is because Andy does not receive government rebates for food expenditures in excess of \$200. Finally, if Andy spends all of his income on food, he can buy \$500 worth of food: in this case, Andy spends his entire income of \$400, plus government cash rebates worth \$100, on food. All of the points on the line connecting $(200,300)$ and $(500,0)$ are therefore on the budget line. The kink point is $(200,300)$.

See Figure 2 for the graphs.

Exercise # 2. (a) The marginal rate of substitution (MRS) between x and y is

$$\text{MRS}(x,y) = -\frac{\frac{\partial u(x,y)}{\partial x}}{\frac{\partial u(x,y)}{\partial y}} = -\frac{16-4x}{4} = x-4.$$

The MRS measures the rate at which Barb is willing to trade good y for good x . More precisely, given Barb's current consumption bundle, the MRS measures how many units of good y she is willing to give up in order to obtain an additional (marginal) unit of good x .

(b) Barb has smooth convex indifference curves. Therefore, assuming that her choice problem has an interior solution, her optimal consumption bundle is characterized by the tangency condition (i.e. $\text{MRS}=\text{price ratio}$) and the budget equation. The tangency condition is

$$4-x = \frac{p_x}{p_y}$$

and the budget equation is

$$p_x x + p_y y = m.$$

If $p_x = p_y = 2$, and $m = 24$, then the tangency condition becomes

$$4-x = 1,$$

so that $x = 3$. Substituting the solution for x into the budget equation, one obtains $y = 9$.

(c) If $p_x = 6$ and $p_y = 2$, the tangency condition becomes:

$$4-x = 3,$$

so that $x = 1$. From the budget line we then obtain $y = 9$. Barb's optimal bundle in part (b) is $(3,9)$. Barb's optimal bundle in part (c) is $(1,9)$. Barb's utility in part (b) is $u(3,9) = 66$, while her utility in part (c) is $u(1,9) = 50$. Hence Barb is better off in part (b) than in part (c).

(d) Let Barb's income be m . If $p_x = 6$ and $p_y = 2$, then $x = 1$ (part (c)). Note that Barb's optimal choice for y does not depend on m . However, Barb's optimal choice for y does depend on m . In particular, Barb's optimal choice for y as a function of m is determined by the budget equation

$$6(1) + 2y = m.$$

Solving for y in terms of m , one obtains:

$$y = \frac{m - 6}{2}.$$

Barb's utility as a function of her income m is therefore

$$u\left(1, \frac{m - 6}{2}\right) = 14 + 4 \left(\frac{m - 6}{2}\right) = 14 + 2(m - 6) = 2m + 2.$$

In order to leave Barb just as well off as she was in part (b), her utility must be 66 (see answer to (b) above). Set

$$2m + 2 = 66$$

to get $m = 32$. The compensating variation is therefore $32 - 24 = 8$.

Exercise #3. Consider the following statements and say whether they are true or false and why. To get credit you should provide a clear justification for your answers.

(a) If two goods are perfect complements and the price of one of them increases, the quantity demanded of both goods decreases.

TRUE. Following the price increase, the demand for the good whose price has increased will fall because perfect complements are ordinary goods. Since the two goods are consumed in fixed proportions, the consumer will also reduce his demand of the other good.

(b) [4 pts.] A non-transitive preference relation \succeq can be represented by some utility function.

FALSE. If a preference relation is non-transitive, it must be the case that for some bundles X , Y , Z

$$\begin{aligned} X &\succeq Y \\ Y &\succeq Z \\ Z &\succ X. \end{aligned}$$

A utility function assigns a number to each bundle with higher numbers assigned to more-preferred bundles. Then, assigning numbers u_x to X , u_y to Y , and u_z to Z , with

$$u_x \geq u_y \geq u_z > u_x$$

we reach the contradiction $u_x > u_x$.

(c) [4 pts.] Consider two goods x and y . If preferences are strictly convex, the absolute value of the marginal rate of substitution between x and y is decreasing along an indifference curve as x increases.

TRUE. This can be seen with the help of a graph. Figure 3 represents an indifference curve corresponding to strictly convex preferences. The tangents to the indifference curve at Y and Z have the same slope as the indifference curve at those points. As you can see the absolute value of the slope of the indifference curve at Z is smaller than the absolute value of the slope of the indifference curve at Y . The same observation can be made for any couple of consumption bundles on that indifference curve. Therefore the absolute value of the MRS decreases with x along the indifference curve.

(d) [4 pts.] The following Cobb-Douglas utility functions represent two different preference relations:

$$\begin{aligned}u_1(x,y) &= 0.3 \log(x) + 0.6 \log(y) \\u_2(x,y) &= 0.6 \log(x) + 1.2 \log(y).\end{aligned}$$

FALSE. They represent the same preference relation because $u_1(x,y)$ is a positive monotonic transformation of $u_2(x,y)$. Specifically,

$$u_1(x,y) = \frac{1}{2}u_2(x,y).$$

(e) [4 pts.] If a consumer is making an optimal choice between two goods x and y , then, independently of his preferences, the following condition must always hold:

$$-\frac{p_x}{p_y} = \text{MRS}(x,y).$$

FALSE. If the optimal bundle is not interior or indifference curves have a kink, then this condition may not hold.

(f) [4 pts.] If the following condition holds

$$-\frac{p_x}{p_y} = \text{MRS}(x,y)$$

then a consumer must be making the optimal choice of x and y , independently of his preferences.

FALSE. For example, if preferences are concave, rather than convex, then the condition above does not guarantee that the consumer is making the optimal choice of x and y .

(g) [4 pts.] A cigar is a luxury good for a consumer that has Cobb-Douglas preferences over cigars and food.

FALSE. If preferences over cigars and food are Cobb-Douglas, then the Engel curve is linear in cigars and income. This means that a consumer's consumption of cigars changes proportionally to his income.

(h) [4 pts.] Consider two goods x and y , with dollar prices p_x and p_y , respectively. The introduction of a 0.07 percent value tax on these two goods does not affect the relative price of x in terms of y .

TRUE. Without the tax, the relative price of x in terms of y is

$$\frac{p_x}{p_y}.$$

After the tax, the relative price of x in terms of y is

$$\frac{(1 + 0.07) p_x}{(1 + 0.07) p_y} = \frac{p_x}{p_y}.$$

(i) [4 pts.] The marginal rate of substitution measures the rate at which the market is willing to substitute one good for the other.

FALSE. The marginal rate of substitution measures the rate at which the consumer is willing to substitute one good for the other.

(j) [4 pts.] An indifference curve represents the collection of all the consumption bundles that a consumer can buy.

FALSE. An indifference curve represents the collection of all the consumption bundles among which a consumer is indifferent.

Figure 1

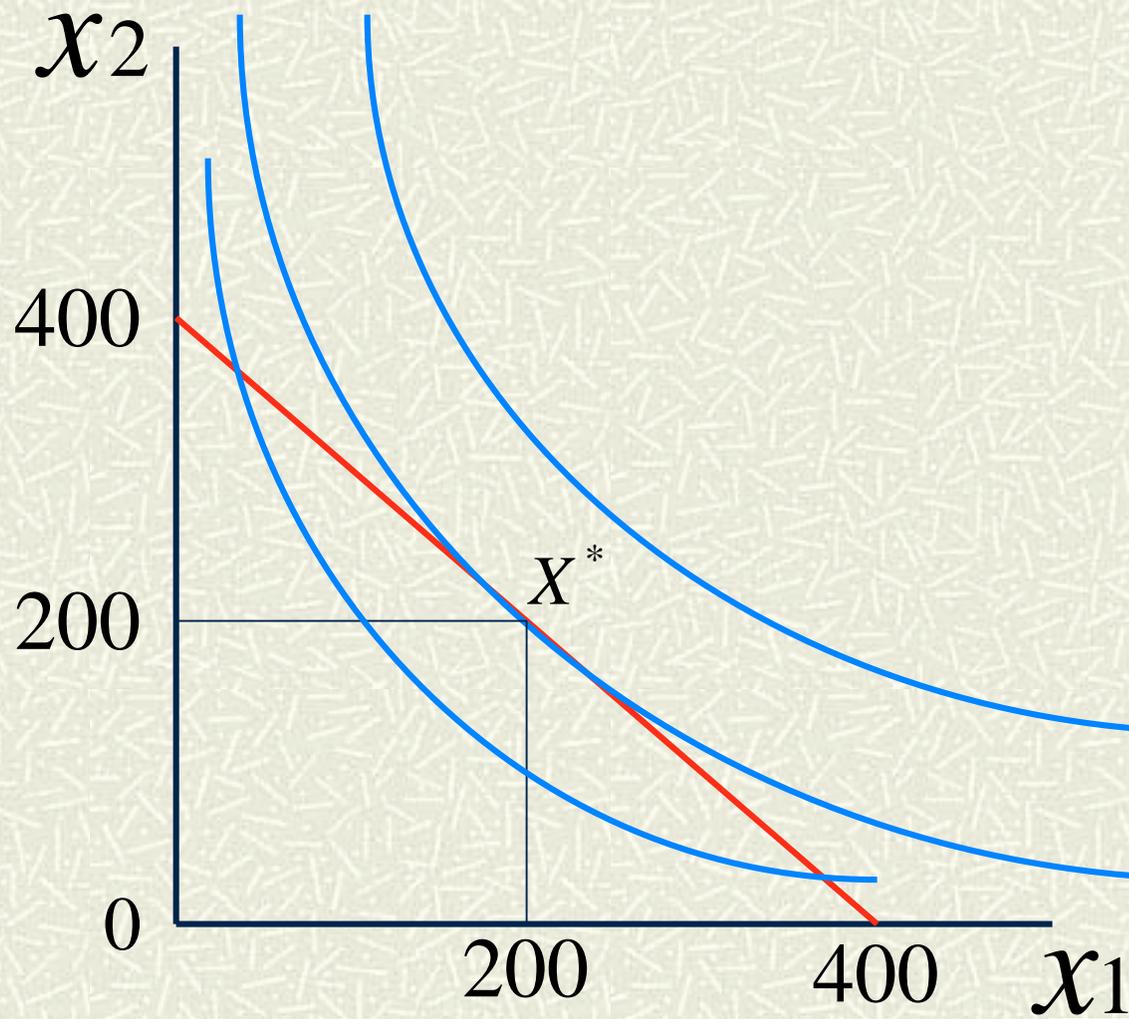


Figure 2

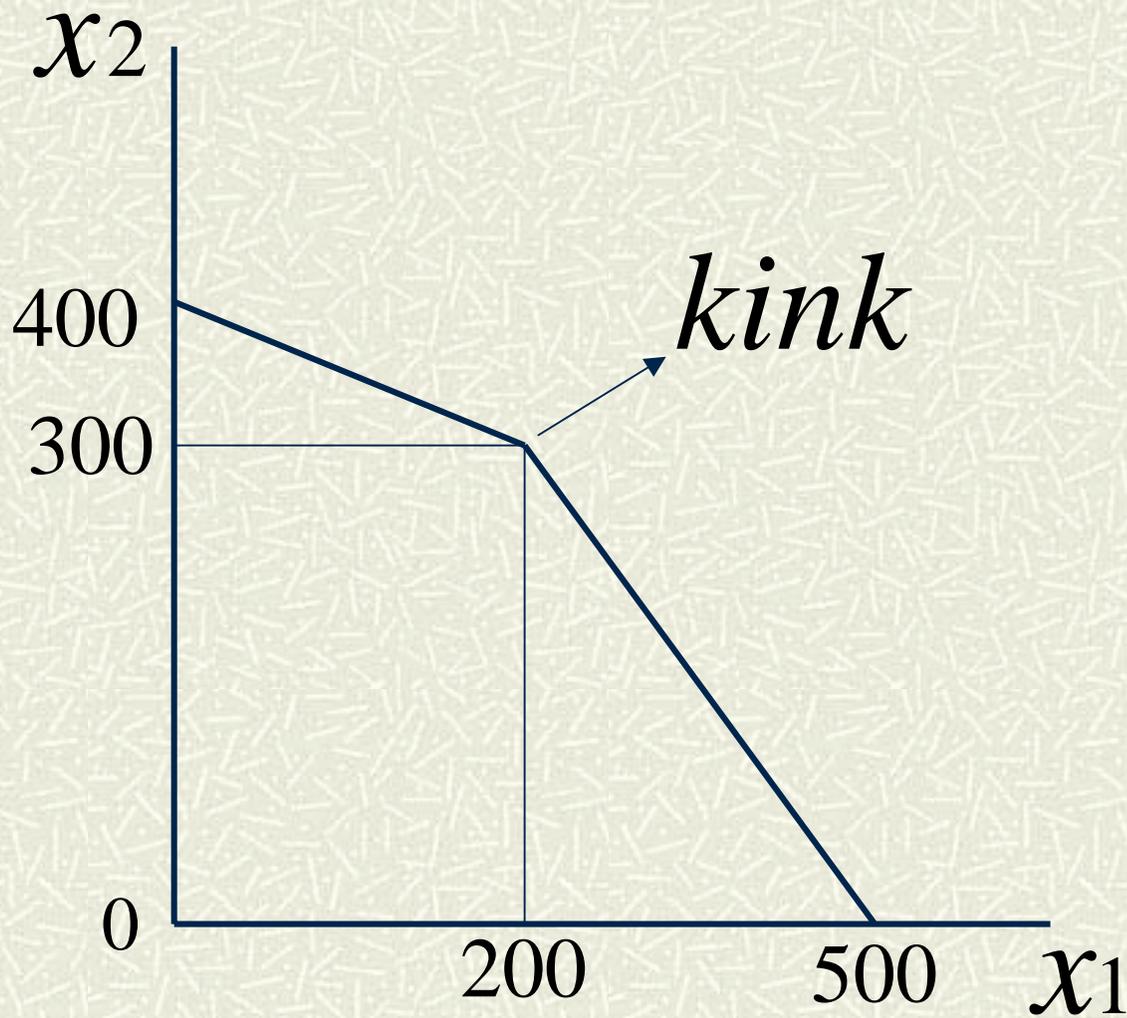


Figure 3

