Econ 73-250A-F Spring 2001 Prof. Daniele Coen-Pirani

## Suggested Answers to Problem Set #3

Please refer to the end of the document for all diagrams.

**Exercise** #1. (a) We first need to determine the demand function of an individual EFF. Given the standard Cobb-Douglas preferences, we know that the demand function for football games is given by:

$$x_g = \frac{1}{2}\frac{m}{p_g} = \frac{50}{p_g}$$

Since there are 50 EFF, to obtain their aggregate demand for football games (denoted by  $X_g^{EFF}$ ), we need to sum their individual demands. Since EFF's demands are equal we get

$$X_g^{EFF} = 50\left(\frac{50}{p_g}\right) = \frac{2500}{p_g}.$$

See Figure 1 for the plot.

(b) Also LFF have Cobb-Douglas preferences, but with different exponents. Therefore, an LFF's demand function looks like this:

$$x_g = \frac{1}{4}\frac{m}{p_g} = \frac{25}{p_g}$$

Following the same steps as above, we get that the aggregate demand by LFF for football games (denoted by  $X_q^{LFF}$ ) is

$$X_g^{LFF} = 50\left(\frac{25}{p_g}\right) = \frac{1250}{p_g}.$$

See Figure 1 for the plot.

(c) To find the aggregate demand function for football games in Pittsford (denoted by  $X_g$ ), just sum the aggregate demand by EFF and the aggregate demand by LFF, to get

$$X_g = X_g^{EFF} + X_g^{LFF} = \frac{3750}{p_g}.$$

See Figure 1 for the plot.

(d) The price elasticity of this demand function is given by

$$\varepsilon = \frac{\partial X_g}{\partial p_g} \frac{p_g}{X_g} = -\frac{3750}{(p_g)^2} \frac{(p_g)^2}{3750} = -1.$$

(e) When EFF have an income equal to \$50, their individual demand is:

$$x_g = \frac{1}{2} \frac{50}{p_g} = \frac{25}{p_g}$$

and their aggregate demand is

$$X_g^{EFF} = 50\left(\frac{25}{p_g}\right) = \frac{1250}{p_g}$$

When LFF have an income equal to \$150, their individual demand is:

$$x_g = \frac{1}{4} \frac{150}{p_g} = \frac{37.5}{p_g},$$

and their aggregate demand is

$$X_g^{LFF} = 50\left(\frac{37.5}{p_g}\right) = \frac{1875}{p_g}.$$

Aggregate demand for football games by the Pittsford population is then

$$X_g = X_g^{EFF} + X_g^{LFF} = \frac{3125}{p_g}.$$

Thus, at any given price, the demand for football games is lower because individuals who like football less have more income with respect to the situation of point (c).

**Exercise #2.** (a) Equilibrium is determined by equating supply and demand:

$$Q_D = Q_S\left(P\right)$$

or

$$6000 = 4000P.$$

This gives  $P^* = 1.5$ , and  $Q_D = Q_S(P^*) = 6000$ .

(b) We now have two prices:  $P_S$  and  $P_D$ , where  $P_S$  denotes the price received by suppliers and  $P_D$  the price paid by consumers. The two prices are related in the following way:

$$P_D = P_S + 0.15$$

The equilibrium condition becomes:

$$Q_D = Q_S\left(P_S\right).$$

Substituting we get that

$$P_S^* = 1.5.$$

The price that consumers pay is instead:

$$P_D^* = 1.5 + 0.15 = 1.65.$$

The number of gallons of gasoline sold remains constant at 6000. What is going on here is that consumers' demand for gasoline is perfectly inelastic, i.e., consumers want to consume 6000 millions of gallons of gasoline no matter what the gasoline price is. Suppliers are willing to supply that amount of gasoline only if they receive the price 1.5 (check this using the supply function). Therefore the tax is completely passed along to the consumers.

(c) The government tax revenue is: 0.15(6000) = 900 millions. There is no deadweight loss from the tax. As we have seen in class, the deadweight loss of the tax is the loss in consumer's and producer's surplus due to the fact that after the tax some units of the good are not bought/sold anymore. In this exercise, the quantity of gasoline that is bought/sold is not affected by the tax. Thus there is no deadweight loss. See Figure 2 for the plot.

(d) Let  $P_D^{EU}$  denote the gasoline price paid by European consumers. We want to find the quantity tax  $t^{EU}$  such that the price paid by Europeans is three times higher than the price paid by Americans. The latter is \$1.65, as we have seen from (b). Therefore, we want to find a  $t^{EU}$  that explains why

$$P_D^{EU} = \$1.65\,(3) = \$4.95.$$

Since we are supposing that gasoline demand in Europe is also  $Q_D = 6000$ , the equilibrium condition (with quantity tax) in the European market is

$$6000 = 4000 P_S.$$

Therefore, producers in Europe get the same amount as producers in the US:  $P_S^{EU} =$ \$1.5. European consumers pay the full amount of the tax (just like American ones):

$$P_D^{EU} = P_S^{EU} + t.$$

Therefore,

$$t^{EU} = P_D^{EU} - P_S^{EU} = \$4.95 - \$1.5 = \$3.45.$$

European governments revenue from the gasoline tax is 3.45(6000) = 20,700 millions.

**Exercise #3.** Price elasticities:

• (a) D(p) = 60 - p.

$$\varepsilon = -\frac{p}{60-p}$$

• (b) D(p) = a - bp.

$$\varepsilon = -\frac{bp}{a - bp}.$$

• (c)  $D(p) = 40p^{-2}$ .

$$\varepsilon = -\frac{80}{p^3} \frac{p}{40p^{-2}} = -2$$

• (d) 
$$D(p) = Ap^{-b}$$
.

$$\varepsilon = -\frac{Ab}{p^{b+1}}\frac{p}{Ap^{-b}} = -b.$$

• (e) 
$$D(p) = (p+3)^{-2}$$
.

$$e^{p^{p+1}}Ap^{-p}$$
  
 $\varepsilon = -2(p+3)^{-3}\frac{p}{(p+3)^{-2}} = -\frac{2p}{p+3}.$ 



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