Cost Minimization

- An alternative approach to the decision of the firm
- Long run and short run costs
- Returns to scale and the cost function
- Different types of costs

Alternative Approach

- Thus far: firm chooses inputs in order to maximize profits
- Alternative approach:

1. Firm chooses inputs in order to **minimize** the **cost** of producing **given** level of **output**
2. Firm **chooses** level of **output** that **maximizes** profits

Cost Minimization

Given factors of production K and L, with rental prices w_K and w_L, find cheapest way to produce a given level of output y:

$$\min_{L,K}\left(w_L L + w_K K\right)$$

such that:

$$y = F(L, K)$$

Cost Function

- Solution to minimization problem is cost function:

$$c(w_L, w_K, y) =$$

$$w_L L^*(w_L, w_K, y) + w_K K^*(w_L, w_K, y)$$

Finding the Cost Function

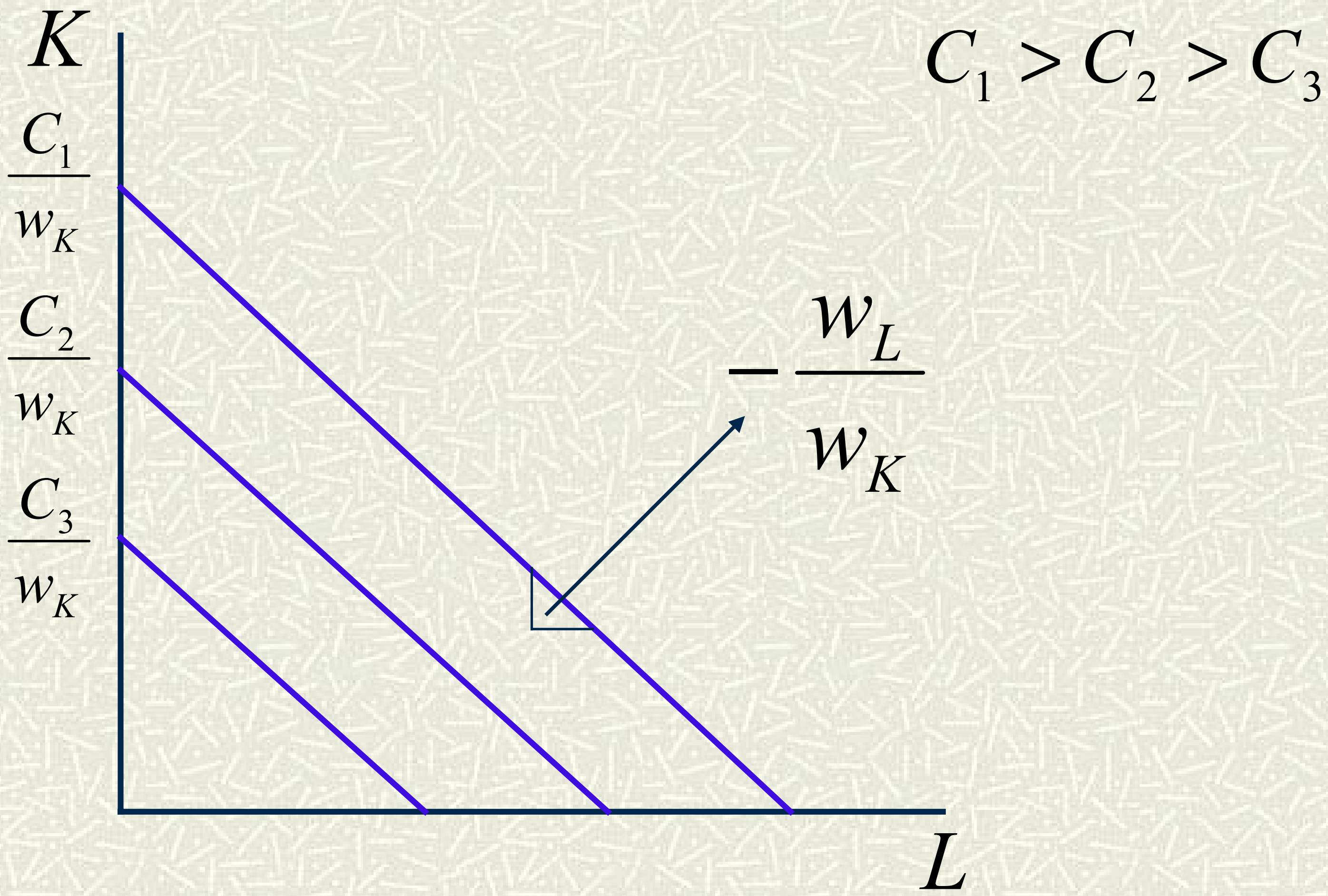
- Cost of using K and L :

$$C = w_L L + w_K K$$

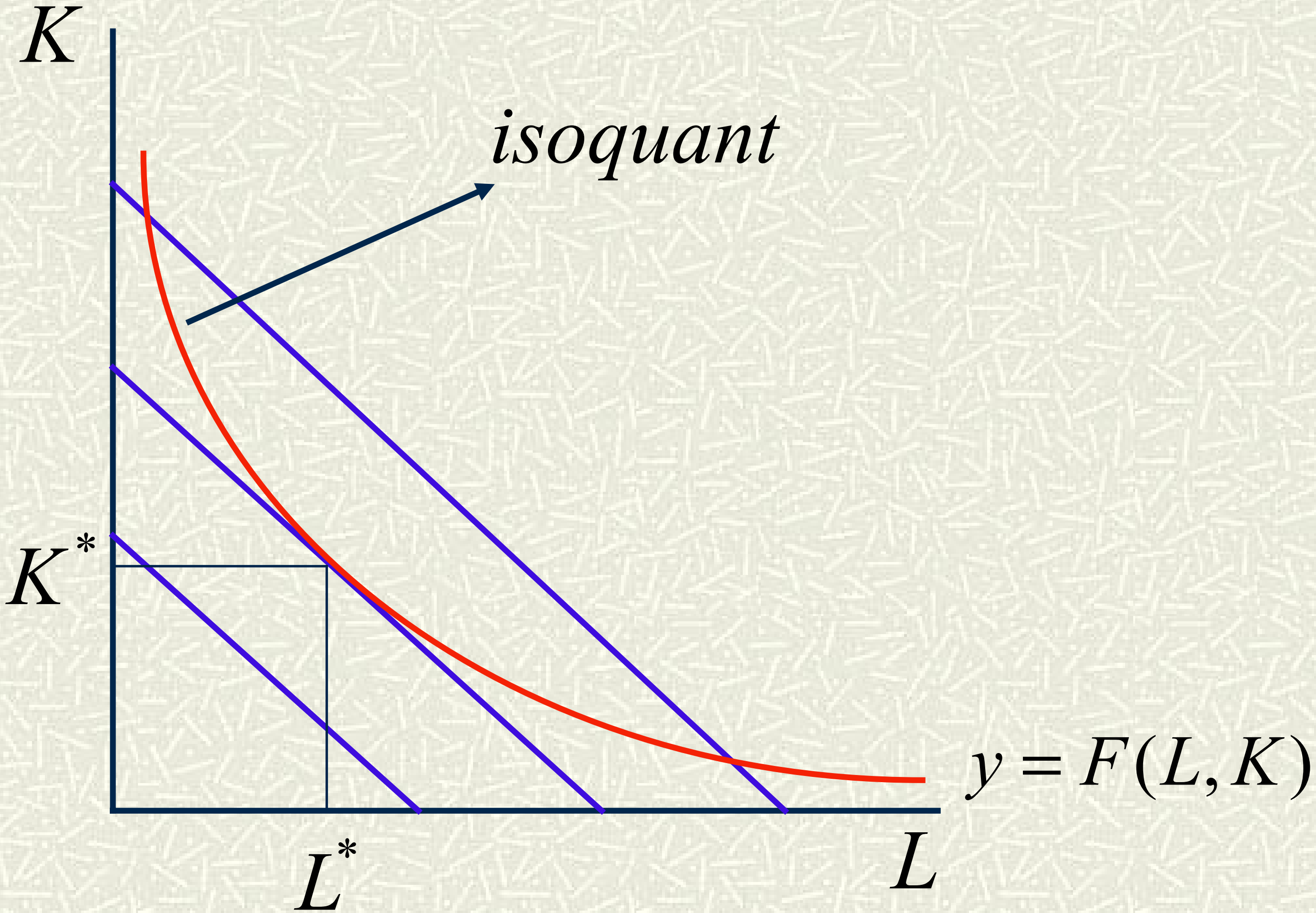
- Isocost lines:

$$K = \frac{C}{w_K} - \frac{w_L}{w_K} L$$

Isocost Lines: $K = \frac{C}{w_K} - \frac{w_L}{w_K} L$



Cost Minimization



Cost Minimization:

Optimal choice:

$$TRS(L^*, K^*) = -\frac{w_L}{w_K}$$

where

$$TRS(L^*, K^*) = -\frac{MP_L(L^*, K^*)}{MP_K(L^*, K^*)}$$

Cost Minimization

To find solution use optimality condition plus production function (2 equations in 2 unknowns):

$$TRS(L^*, K^*) = -\frac{w_L}{w_K}$$

$$y = F(L^*, K^*)$$

Short-Run and Long-Run Cost Functions

- In the **short run** some factors of production are **fixed**: short-run cost function gives the minimum cost to produce a given level of output, **only** adjusting the variable factors of production.
- In the **long run** all factors are **variable**: long run cost function gives the minimum cost to produce a given level of output, adjusting **all** factors of production.

Example: Short Run

- Find the **short run** cost function in the example of consulting firm:

$$w_L = 70 \qquad y = (3000)^{0.2}(x_l)^{0.6}$$

- Quantity of labor used to produce y:

$$x_l = \left(\frac{y}{(3000)^{0.2}}\right)^{\frac{1}{0.6}}$$

Example: Short Run

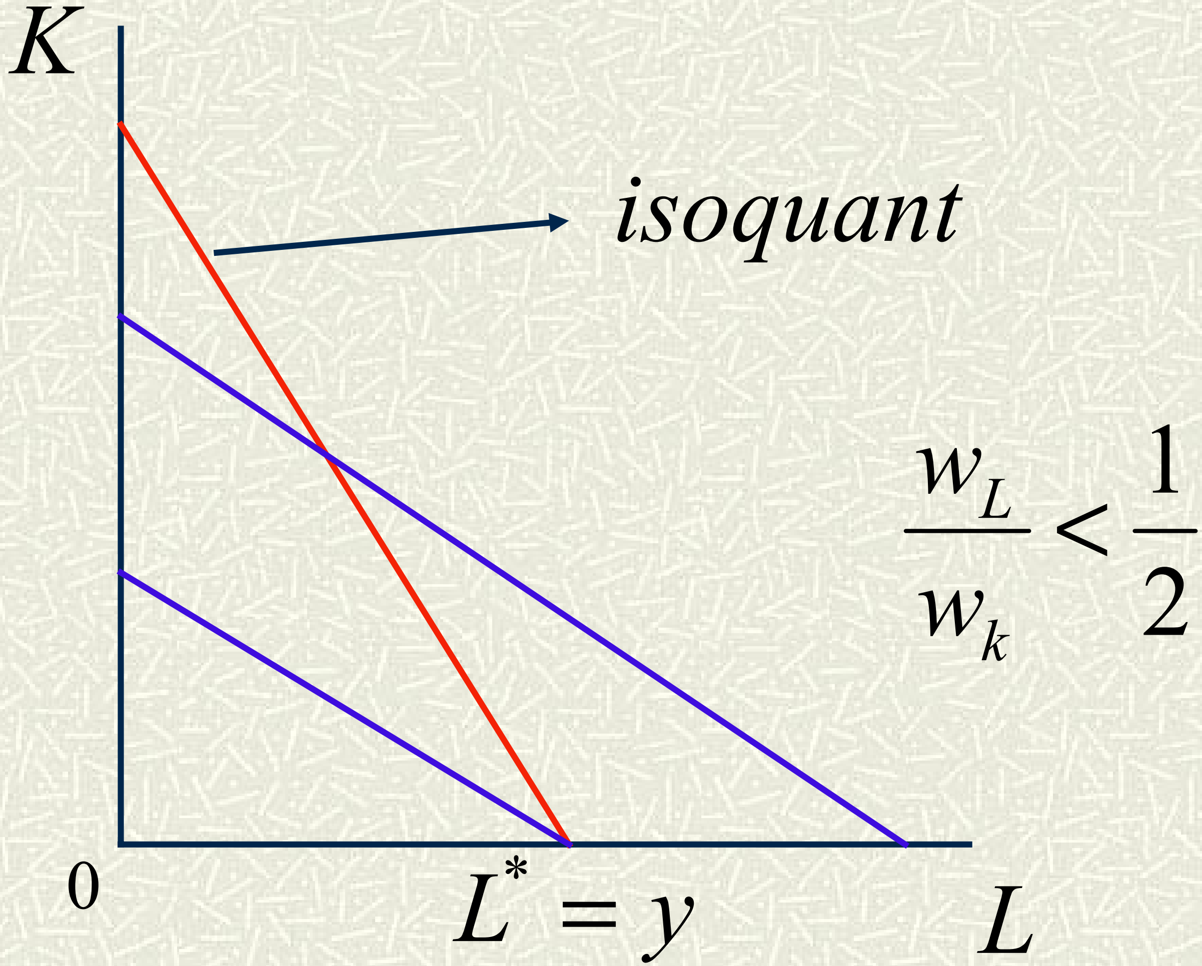
- Quantity of labor used to produce y :

$$x_l = \left(\frac{y}{(3000)^{0.2}} \right)^{\frac{1}{0.6}}$$

- Short run cost function:

$$c(y) = 70 \left(\frac{y}{(3000)^{0.2}} \right)^{\frac{1}{0.6}}$$

Example: Inputs are Perfect Substitutes $y = L + 2K$



Perfect Substitutes

- If $\dfrac{w_L}{w_k} < \dfrac{1}{2}$ labor only input:

$$L^* = y$$

- Cost function is:

$$c(w_L, w_K, y) = w_L y$$

Perfect Substitutes

- If $\frac{w_L}{w_k} > \frac{1}{2}$ capital only input:

$$K^* = \frac{y}{2}$$

- Cost function is:

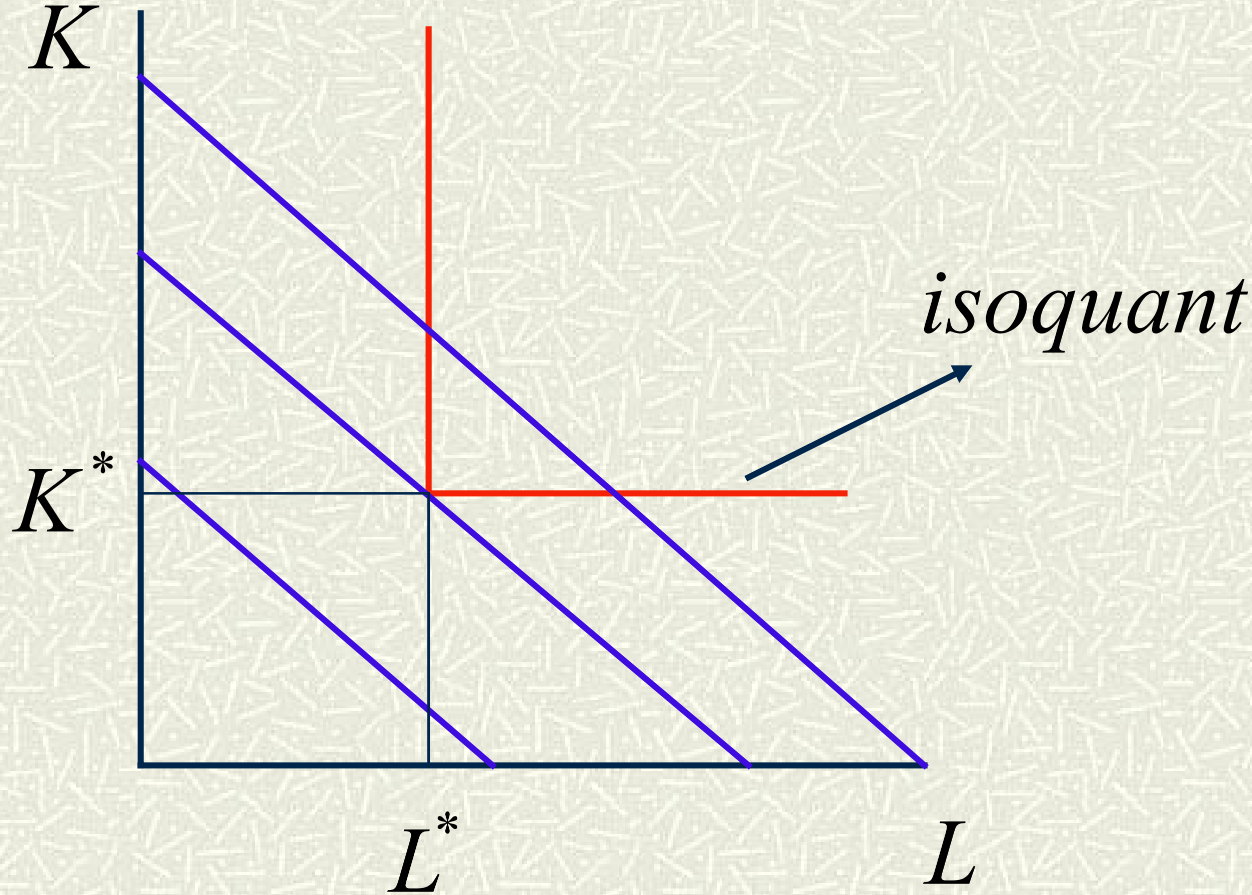
$$c(w_L, w_K, y) = \frac{w_K}{2} y$$

Perfect Substitutes

Summarizing, cost function is:

$$c(w_L, w_K, y) = \min\left(w_L, \frac{w_K}{2}\right)y$$

Fixed Proportions Production Function: $y = \min(L, K)$



Fixed Proportions

- No matter what input prices are:

$$K^* = L^*$$

$$y = \min(K^*, L^*) = L^* = K^*$$

- Cost function:

$$c(w_L, w_K, y) = w_L L^* + w_K K^* = (w_L + w_K) y$$

Cost Function and Returns to Scale

- Constant returns: to double output, need to double all inputs $\longrightarrow$ double cost
- Decreasing returns: to double output, need to **more** than double inputs $\longrightarrow$ **more** than double cost
- Increasing returns: to double output, need to **less** than double inputs $\longrightarrow$ **less** than double cost

Cost Function and Returns to Scale

- Constant returns:

$$c(w_L, w_K, 2) = 2c(w_L, w_K, 1)$$

- Decreasing returns:

$$c(w_L, w_K, 2) > 2c(w_L, w_K, 1)$$

- Increasing returns:

$$c(w_L, w_K, 2) < 2c(w_L, w_K, 1)$$