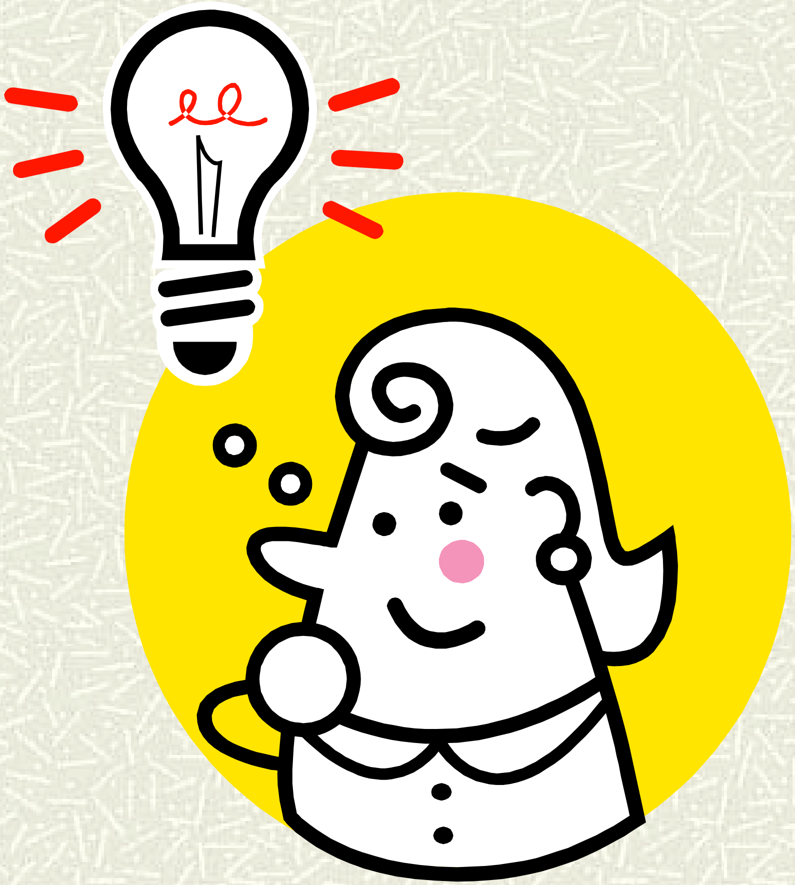
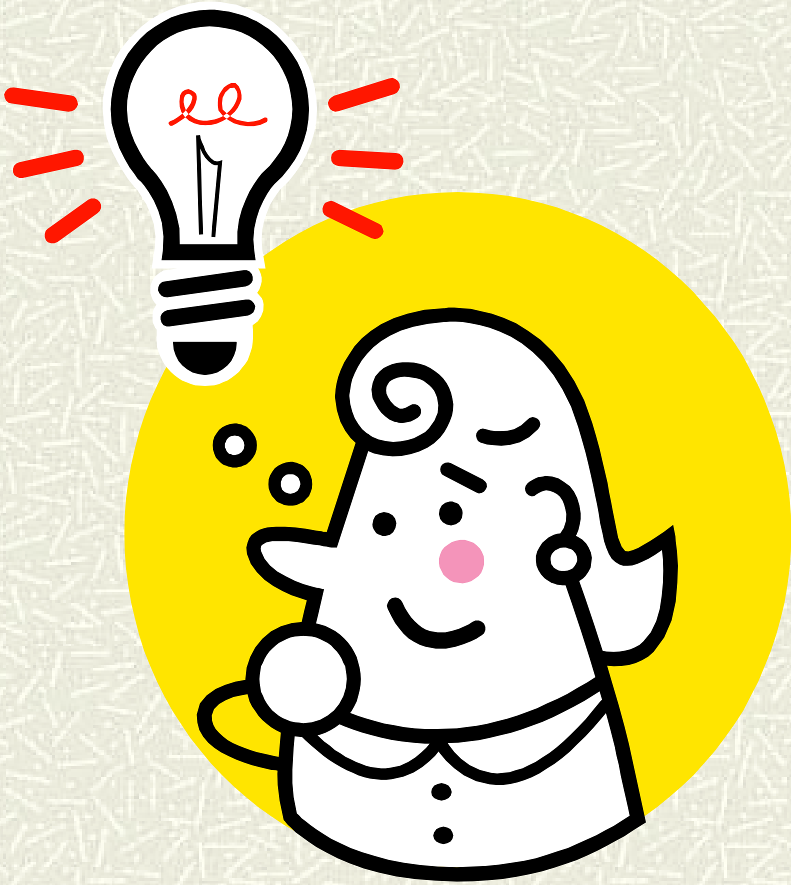
Your Suggestions



- Sample problems and examples in lecture.
- Download recitation problems before recitation.
- Complete exercises in recitations.
- Reorganize web site.
- Have power point slides available earlier.
- Overview class at the beginning.

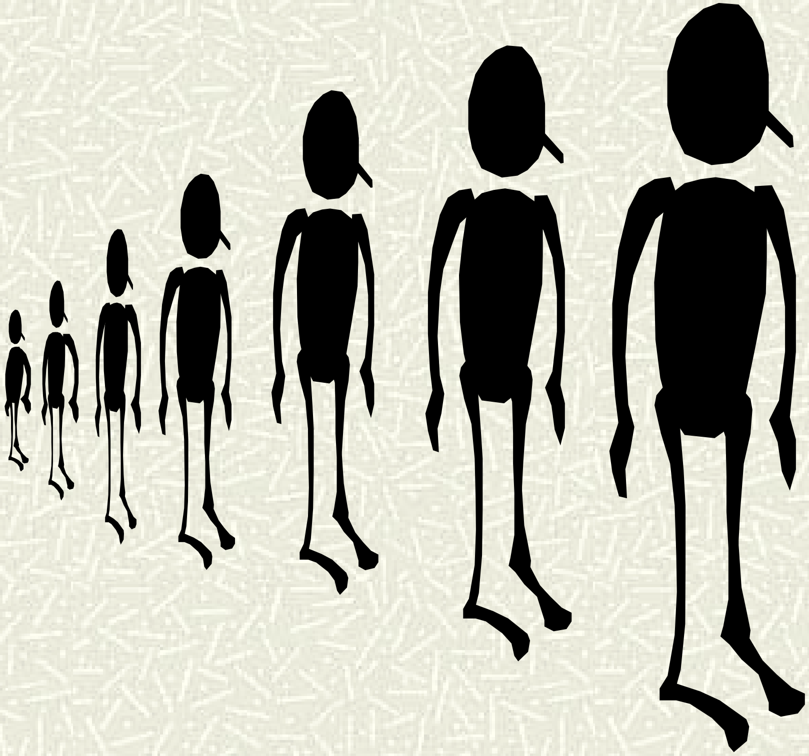
Your Suggestions



- Board/slides.
- Too fast/too slow.
- Book does not have enough examples.

Market Demand

- From individual to market demand.
- Price elasticity of demand.
- Income elasticity of demand.
- An example: the Laffer curve.



From Individual to Market demand

- Individual i 's demand function for good 1:

$$x_{1i} = x_{1i}(p_1, p_2, m_i)$$

- Aggregate demand (market demand) function for good 1:

$$X_1(p_1, p_2, m_1, m_2, \ldots, m_n) = \sum_{i=1}^{n} x_{1i}(p_1, p_2, m_i)$$

Market Demand: Example

- Consider 2 consumers of CDs: $i = 1,2$

- Each consumer has the demand function:

$$x_i = m_i - p$$

- Consumers have different incomes:

$m_1 = \$100$ $m_2 = \$200$

Market Demand: Example

- Individual demand functions:

$$x_1 = \$100 - p \qquad x_2 = \$200 - p$$

- Market demand:

$$X = \$300 - 2p \quad \text{for} \quad p \le \$100$$

$$X = \$200 - p \quad \text{for} \quad \$100 \le p \le \$200$$

Inverse Market Demand: Example

- Market demand:

$$X = \$300 - 2p \quad \text{for} \quad p \le \$100$$

$$X = \$200 - p \quad \text{for} \quad \$100 \le p \le \$200$$

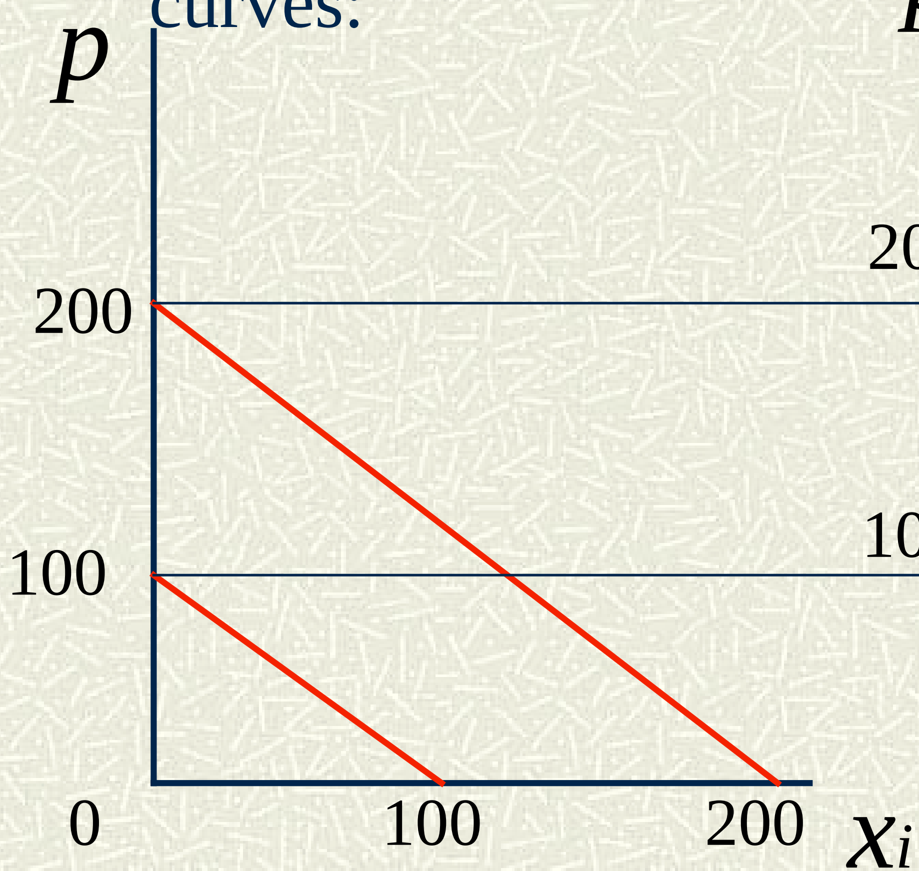
- Inverse demand:

$$p = \$150 - X/2 \quad \text{for} \quad p \le \$100$$

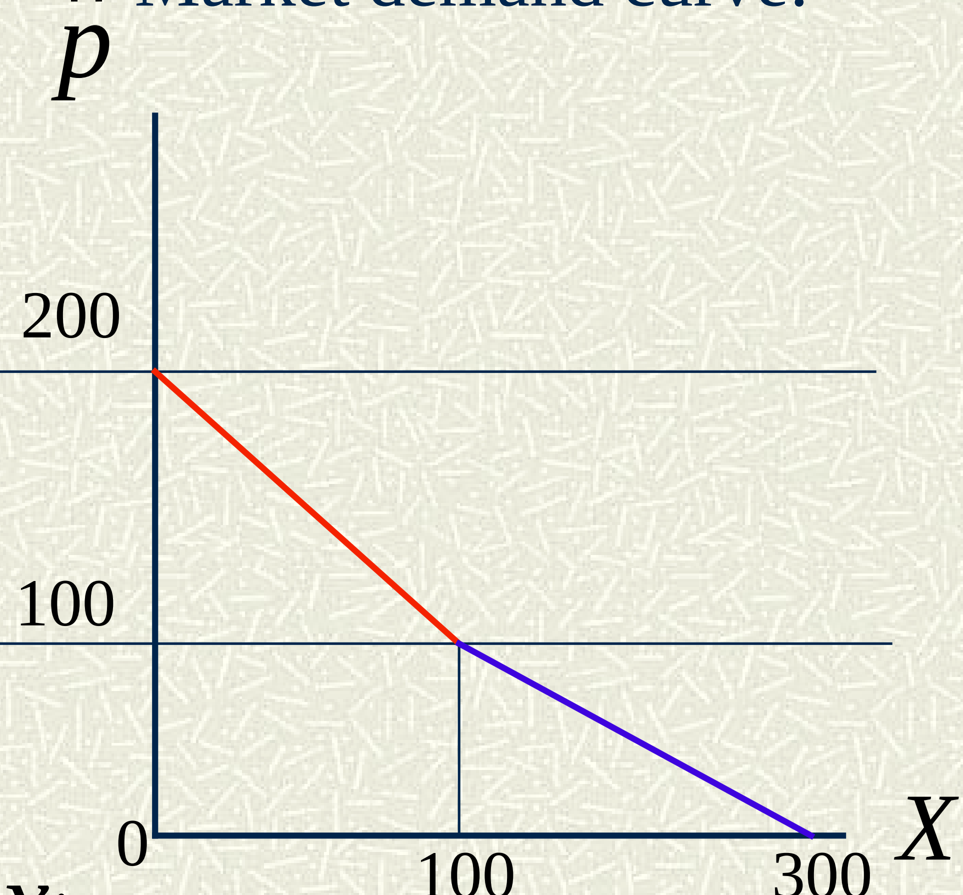
$$p = \$200 - X \quad \text{for} \quad \$100 \le p \le \$200$$

Market Demand Curve

- Individual demand curves:



- Market demand curve:



Aggregation

- Q:Is the sum of our demands (aggregate demand) for a good always equal to the demand of one individual whose income is given by the sum of our incomes?
- In other words is aggregate demand equal to the demand of some **representative consumer** who has income equal to the sum of all individual incomes?

Aggregation

A: **No**, for two reasons:

1. Individuals have different preferences
2. Even if individuals had the same preferences, some goods are necessary goods, and others are luxury goods.

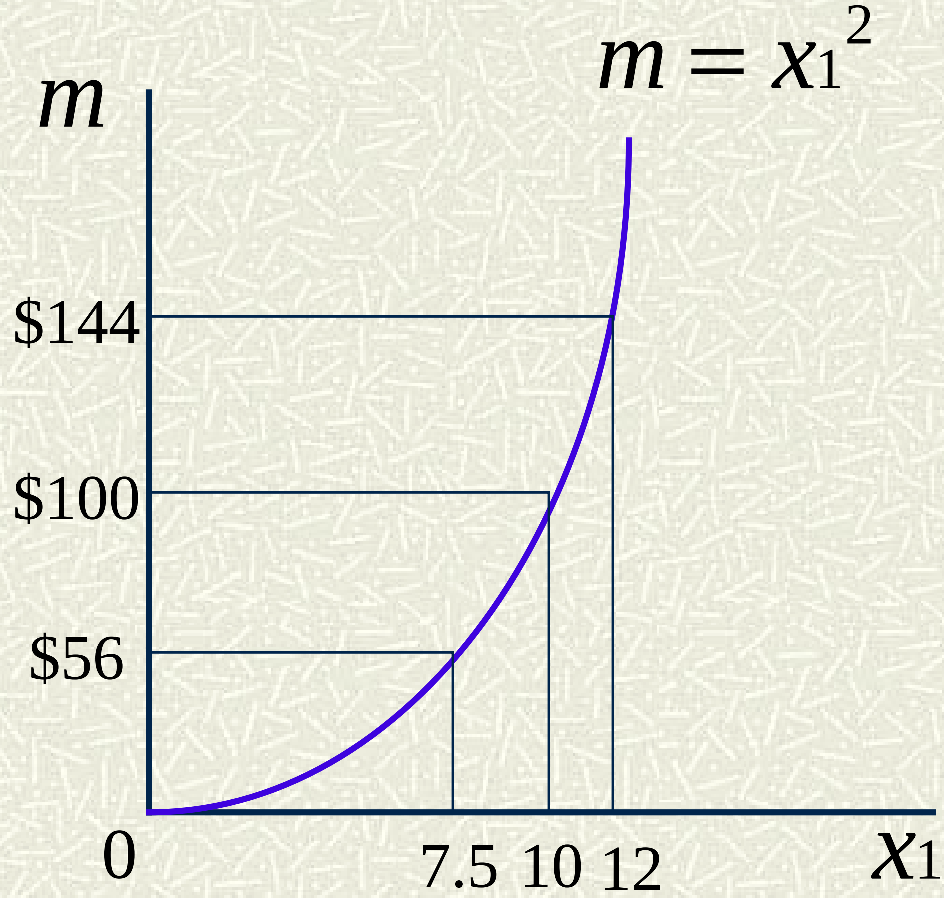
Aggregation: Example with a Necessary Good

- 2 consumers with same preferences
- Equal income distribution:

$$X_1 = 10 + 10 = 20$$

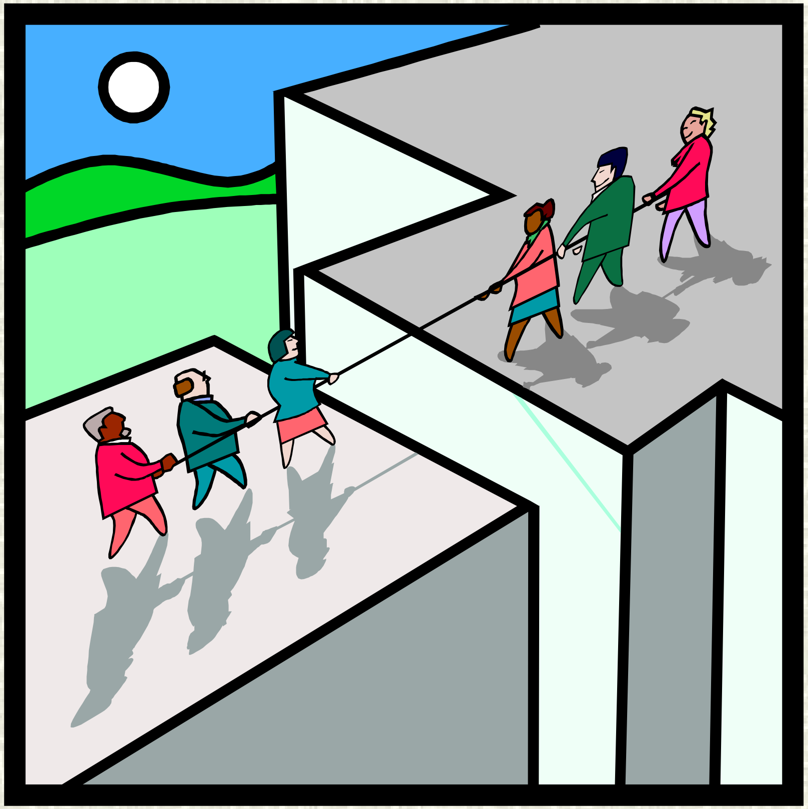
- Unequal income distribution:

$$X_1 = 12 + 7.5 = 19.5$$



Elasticity

- Looking for a measure of how “responsive” individual and aggregate demands are to changes in price and income.
- This measure is important to determine effects of taxes on prices.



One Candidate

- One **candidate** measure of how "responsive" demand is to price changes is the **slope** of the **demand function** (at a given point):

$$\frac{\partial X_1(p_1, p_2, m_1, m_2, \ldots, m_n)}{\partial p_1}$$

Problem with Slope of Demand Function

- Example: $X_G = 100 - p$ where X_1 represents gallons of gasoline and p is the price of one gallon.

- Change units and measure gasoline in quarts (1/4 of gallon).
- Let X_Q represent quarts of gasoline. Demand is:

$$X_Q = 400 - 4p$$

Elasticity

- Instead of using slope, use price elasticity of demand ε :

$$\varepsilon = \left(\frac{\partial X_1(p_1, p_2, m_1, m_2, \ldots, m_n)}{\partial p_1} \right) \left(\frac{p_1}{X_1} \right)$$

- Advantage: ε independent of units

Example Cont’d

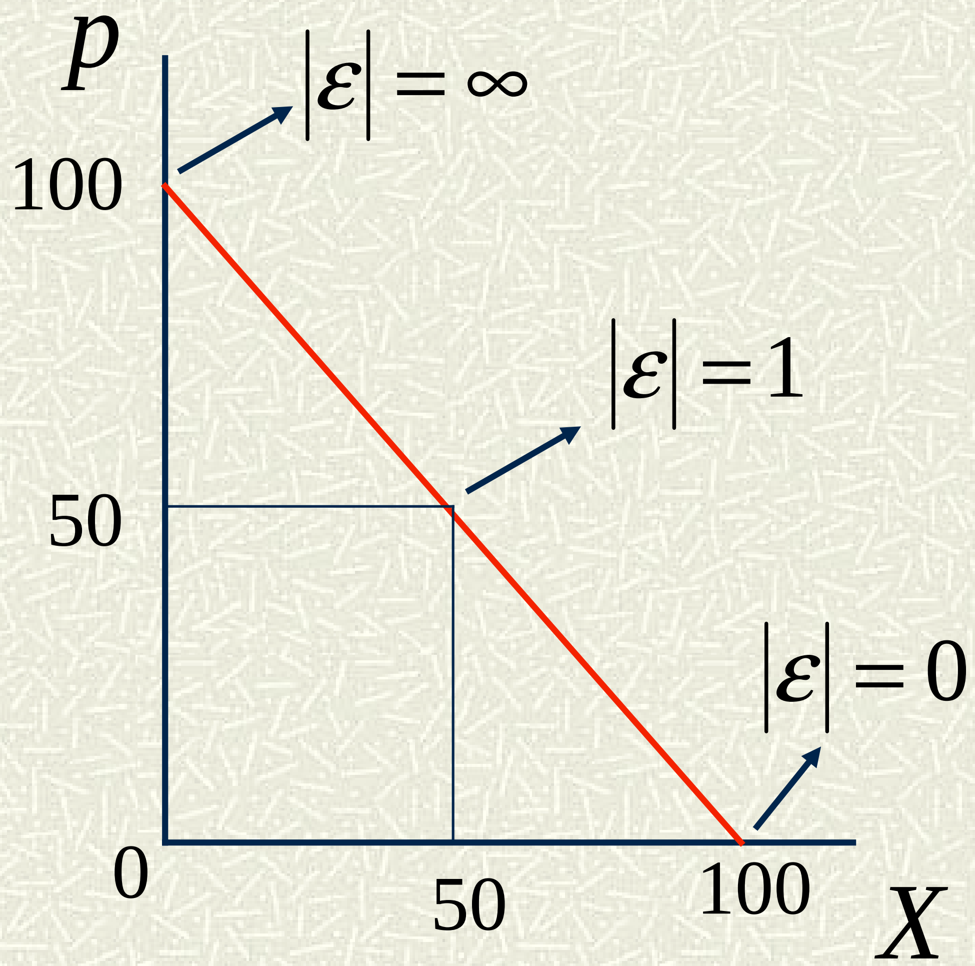
- Demand for gasoline: $X_G = 100 - p$
- Elasticity: $\varepsilon = -1\left(\frac{p}{X_G}\right) = -\frac{p}{100 - p}$
- Demand for gasoline: $X_Q = 400 - 4p$
- Elasticity:

$$\varepsilon = -4\left(\frac{p}{X_Q}\right) = -\frac{4p}{400 - 4p} = -\frac{p}{100 - p}$$

Properties of Elasticity

- Elasticity changes with demand:

$$\varepsilon = -\frac{p}{100 - p}$$



Properties of Elasticity

- A demand function is **elastic** if: $|\varepsilon| > 1$
- A demand function is **inelastic** if: $|\varepsilon| < 1$
- A demand function is **unit elastic** if: $|\varepsilon| = 1$

Example: Cobb-Douglas

- Demand function: $x = c\dfrac{m}{p}$
- Slope: $\dfrac{\partial x}{\partial p} = -c\dfrac{m}{p^2}$
- Elasticity: $\dfrac{\partial x}{\partial p}\dfrac{p}{x} = \left(-c\dfrac{m}{p^2}\right)\dfrac{p^2}{cm} = -1$

Income Elasticity of Demand

- Describes how responsive demand is to changes in individual or aggregate income.
- Defined similarly to price elasticity:

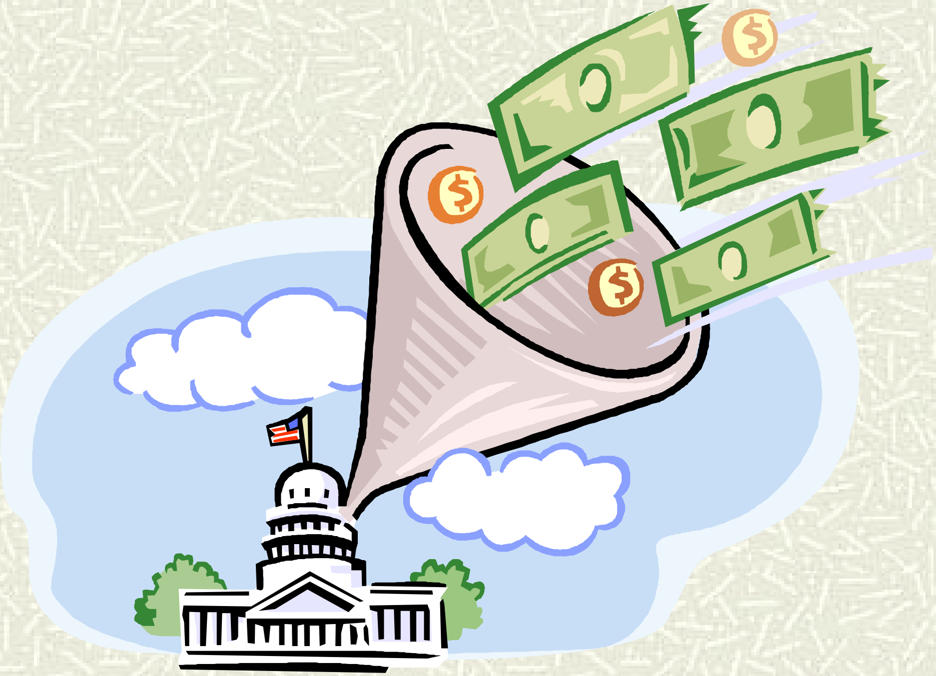
$$\eta = \left(\frac{\partial x_1(p_1, p_2, m)}{\partial m} \right) \left(\frac{m}{x_1} \right)$$

Income Elasticity of Demand

- Normal goods: $$\frac{\partial x_1(p_1, p_2, m)}{\partial m}\frac{m}{x_1} > 0$$
- Inferior goods: $$\frac{\partial x_1(p_1, p_2, m)}{\partial m}\frac{m}{x_1} < 0$$
- Luxury goods: $$\frac{\partial x_1(p_1, p_2, m)}{\partial m}\frac{m}{x_1} > 1$$

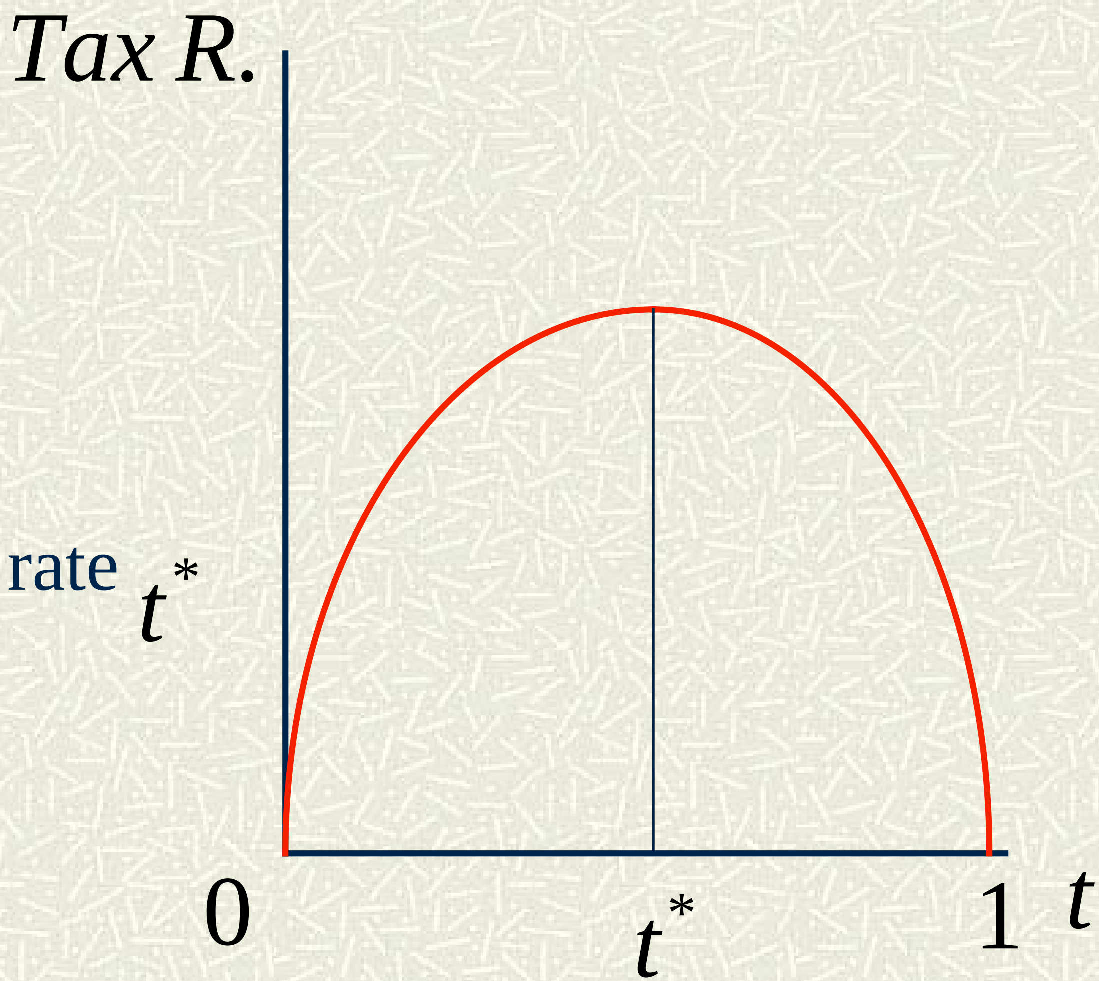
The Laffer Curve

- How do government tax revenue change when the tax rate changes?



The Laffer Curve

- If $t = 0$: zero revenues.
- If $t = 1$: zero revenues.
- There exists a tax rate t^* that maximizes revenues.



The Laffer Curve

- Consider a population of identical workers
- Each worker earns an hourly wage w^*
- Each worker has to pay a tax t on his/her wage
- Thus a worker's net hourly wage is:

$$w = (1 - t)w^*$$

The Laffer Curve

- A worker decides how many hours to work according to the following labor supply function:

$$x_h = w^a = \left((1-t)w^*\right)^a$$

- Tax revenue:

$$T = twx_h$$

The Laffer Curve

- Tax revenue: $T = twx_h$
- How do revenues change with the tax rate:

$$\frac{\partial T}{\partial t} = wx_h + tw\frac{\partial x_h}{\partial t}$$

The Laffer Curve

- How do revenues change with the tax rate:

$$\frac{\partial T}{\partial t} = w x_h + t w \frac{\partial x_h}{\partial t}$$

- Compute:

$$\frac{\partial x_h}{\partial t} = \frac{\partial ((1-t)w)^a}{\partial t} = -a((1-t)w)^{a-1} w$$

The Laffer Curve

Compare
$$\frac{\partial x_h}{\partial t} = -a((1-t)w)^{a-1}w$$

with
$$\frac{\partial x_h}{\partial w} = a((1-t)w)^{a-1}(1-t)$$

so that
$$\frac{\partial x_h}{\partial t} = -\frac{\partial x_h}{\partial w}\frac{w}{(1-t)}$$

The Laffer Curve

- We know that:

$$\frac{\partial x_h}{\partial t} = -\frac{\partial x_h}{\partial w}\frac{w}{(1-t)}$$

- Then:

$$\frac{\partial T}{\partial t} = wx_h + tw\frac{\partial x_h}{\partial t} = wx_h - tw\frac{\partial x_h}{\partial w}\frac{w}{(1-t)}$$

The Laffer Curve

- We want tax revenues to decrease with the tax rate:

$$\frac{\partial T}{\partial t} = w x_h - t w \frac{\partial x_h}{\partial w} \frac{w}{(1-t)} < 0$$

- This occurs when:

$$x_h < t \frac{\partial x_h}{\partial w} \frac{w}{(1-t)}$$

The Laffer Curve

- This occurs when:

$$x_h < t \frac{\partial x_h}{\partial w} \frac{w}{(1-t)}$$

- Rearrange:

$$\frac{\partial x_h}{\partial w} \frac{w}{x_h} > \frac{(1-t)}{t}$$

The Laffer Curve

- Condition: $$\frac{\partial x_h}{\partial w}\frac{w}{x_h} > \frac{(1-t)}{t}$$

- Compute elasticity of labor supply:

$$\frac{\partial x_h}{\partial w}\frac{w}{x_h} = a((1-t)w)^{a-1}(1-t)\frac{w}{((1-t)w)^a}$$

$$= a$$

The Laffer Curve

- Thus we have that tax revenues **increase** when government **reduces** tax rate if:

$$a > \frac{(1-t)}{t}$$

- Elasticity of labor supply estimated to be at most 0.2
- Tax rate on labor income is at most 0.5

The Laffer Curve

- Elasticity of labor supply estimated to be at most 0.2
- Tax rate on labor income is at most 0.5
- Plug into our condition and check that it is **not** verified:

$$a > \frac{(1-t)}{t} \longrightarrow 0.2 > \frac{1-0.5}{0.5} = 1$$