Transformations of languages of designs: part 3

T Weissman Knight¶
Royal College of Art, Kensington Gore, London SW7 2EU, England
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Abstract. The model for defining transformations of languages of designs presented in part 2 of this paper is illustrated using two well-known architectural styles of Frank Lloyd Wright. The Usonian houses of Wright's later career are characterized as a transformation of his earlier prairie houses by changing a grammar defining the earlier style into a grammar defining the succeeding one. General conclusions concerning the use of the model for comparing and constructing languages of designs are also given.

In part 2 (Knight, 1983), a formal model for defining transformations of languages of designs was described. Examples showing how new languages of designs could be constructed by transforming the grammar defining a given language accompanied the presentation of the model. Here, the model is applied to an analysis of the relationship between two well-known styles of architectural design.

From butterflies to polliwogs: the language of the prairie transformed

A deceptively simple but largely unrecognized example of a transformation of a style of designs into a new style can be found in the architectural works of Frank Lloyd Wright. The basic compositions of the L-shaped Usonian houses of his later career can be characterized formally as a transformation of the basic compositions of his earlier prairie-style houses by changing a grammar which defines the early style into a grammar which defines the later one.

A grammar which defines Wright's language of prairie-style houses, often described as butterfly-shaped in plan, has already been given (Koning and Eizenberg, 1981). This grammar, based on Wright's compositional theories and produced works, generates basic compositional forms which are subsequently elaborated and ornamented to produce complete designs. Basic compositional forms are built up in terms of simple spatial relations between three-dimensional parameterized blocks. Blocks are distinguished functionally as either living zones or service zones. Living zones include living rooms, dining rooms, libraries, and so on; service zones include kitchens, servants' quarters, bedrooms, and so on. A generation of a basic composition begins with a fireplace—the logical center of a plan. A living zone is then added to the fireplace and a service zone is added to the living zone to form the rectangular core unit of a prairie house. To complete a basic composition, the core unit is extended by adding smaller living and service zones (the wings of the butterfly) to it.

The basic compositions of prairie houses provide the basis for basic compositions of the later Usonian houses, in particular the L-shaped or, what Wright called 'polliwog' (tadpole) plans most typical of this style. Like a prairie house, a polliwog Usonian consists of a two-zone core unit—an open-plan living zone or 'body' which incorporates both kitchen and dining areas, and a bedroom zone or 'tail' which includes bedrooms and bathrooms. The core unit is sometimes extended by adding smaller bedroom or living zones to the body or tail. The logical center of a plan

¶ Present address: School of Architecture and Urban Planning, University of California, Los Angeles, CA 90024, USA

is what Wright termed the 'work space'—a small area located in the hinge of the L-plan. The work space is always contained within the living zone and consists of a kitchen area with an adjacent fireplace facing out towards the living room[1].

The differences between prairie houses and Usonian houses are, to a certain extent, reflections of changes in living standards between the times in which they were built— prairie houses during the early 1900s and Usonian houses during the years before and after World War 2. The Usonian house was conceived partially in response to the urgent need for low-cost housing during the 30s. Hence, the core unit of the prairie house which formed only a part of an expanded final design became, in the case of many Usonians, the whole of the design. Extensions to the core unit in the Usonian house were the exception (for larger or more affluent families) and not the rule. Since servants were no longer a common fixture of households, the kitchen, which was segregated from the living and social areas in the prairie house, became integrated into the living area in the Usonian. Thus, the hostess or host could prepare meals as well as entertain and socialize from the new, centralized work space. The adjacent fire-place was still a focal point of a plan.

The precise relationship between the composition of prairie houses and the composition of Usonian houses can be understood as changes to the spatial relations and ordering of the subset of the rules of the prairie grammar which generates basic compositions. However, these rules must first be reexpressed in normal form so that the spatial relations which determine them and the way they are ordered are made explicit. Readers should refer to this subset, here called G_P^*, [the initial shape and rules (schemata) 1–18] of the original prairie grammar as well as the accompanying text (Koning and Eizenberg, 1981) to see how it is translated into the normal form grammar G_P depicted in figure 1(a). The set of spatial relations and the set of pairs of state labels associated with G_P are shown in figures 1(b) and 1(c), respectively.

It should be observed that G_P does not define a language of basic compositions exactly identical to the language defined by G_P^*. Some rules in G_P^* such as those for generating a double-hearth fireplace, for interpenetrating blocks, and for changing a service zone into a living zone or a split living/service zone are not included in G_P as they are not relevant to subsequent transformations. (However, because transformations may be partial, including rules in G_P which correspond to these or other rules in the original prairie grammar would not affect transformations of G_P since transformations of these rules would not be defined by the sets of change rules given below.) Additionally, G_P generates a few possibilities for basic compositions not generated by G_P^*. These new possibilities are allowed since they appear to satisfy the same stylistic criteria satisfied by other basic compositions.

To simplify the presentation of rules, the three-dimensional blocks or zones in the rules of G_P^* are depicted in G_P in plan form only. Since the height of all blocks is fixed, no information is lost in a two-dimensional (length and width) representation of them. Parameters on blocks in G_P differ from those in G_P^* in the following way. In G_P^*, any side of a block as depicted in the rules is between one to four times as long as the adjacent side. In G_P, the length or longer side of a block as depicted in the rules is between one to four times as long as the width.

[1] In the words of Wright, Usonian plans are summarized thus:
"A Usonian house if built for a young couple, can, without deformity, be expanded, later, for the needs of a growing family. As you can see from the plans, Usonian houses are shaped like polliwogs—a house with a shorter or longer tail. The body of the polliwog is the living room and adjoining kitchen—or work space—and the whole Usonian concentration of conveniences. From there it starts out, with a tail: in the proper direction, say, one bedroom, two bedrooms, three, four, five, six bedrooms long …. The size of the polliwog's tail depends on the number of children and the size of the family budget" (Wright, 1954, pages 167–168).
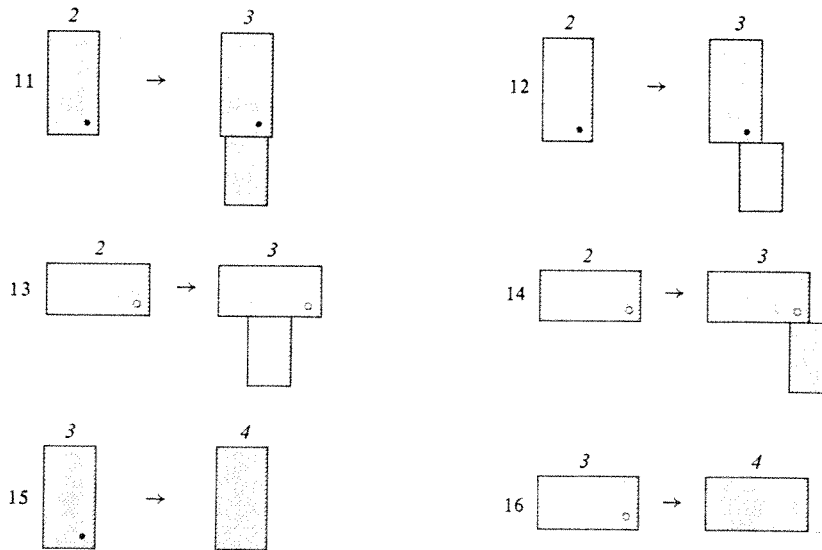
G_P:
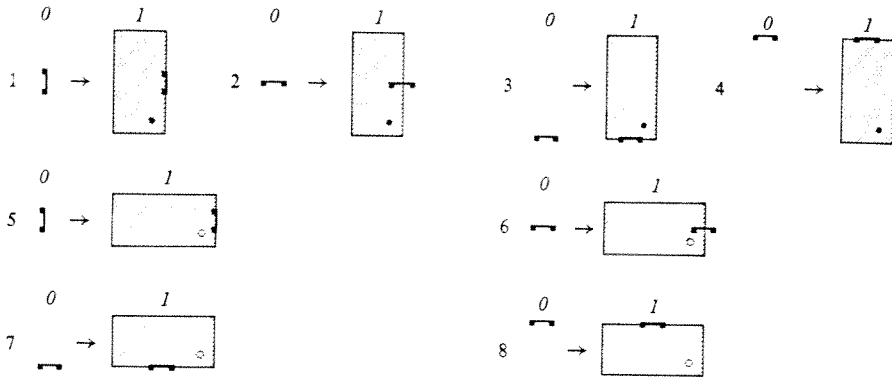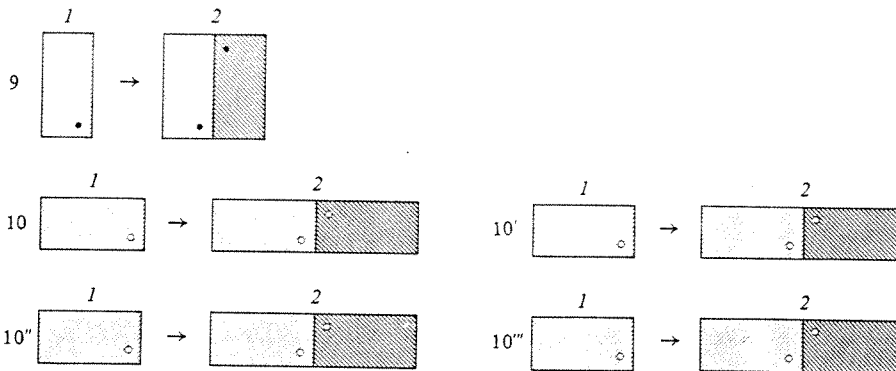
(a)   rules for obligatory extensions

Figure 1. (a) A normal form grammar G_P which generates basic compositions of Wright's prairie-style houses; (b) the set of spatial relations; and (c) the set of pairs of state labels associated with G_P. Spatial relations and pairs of state labels are numbered to correspond with rules which they specify.
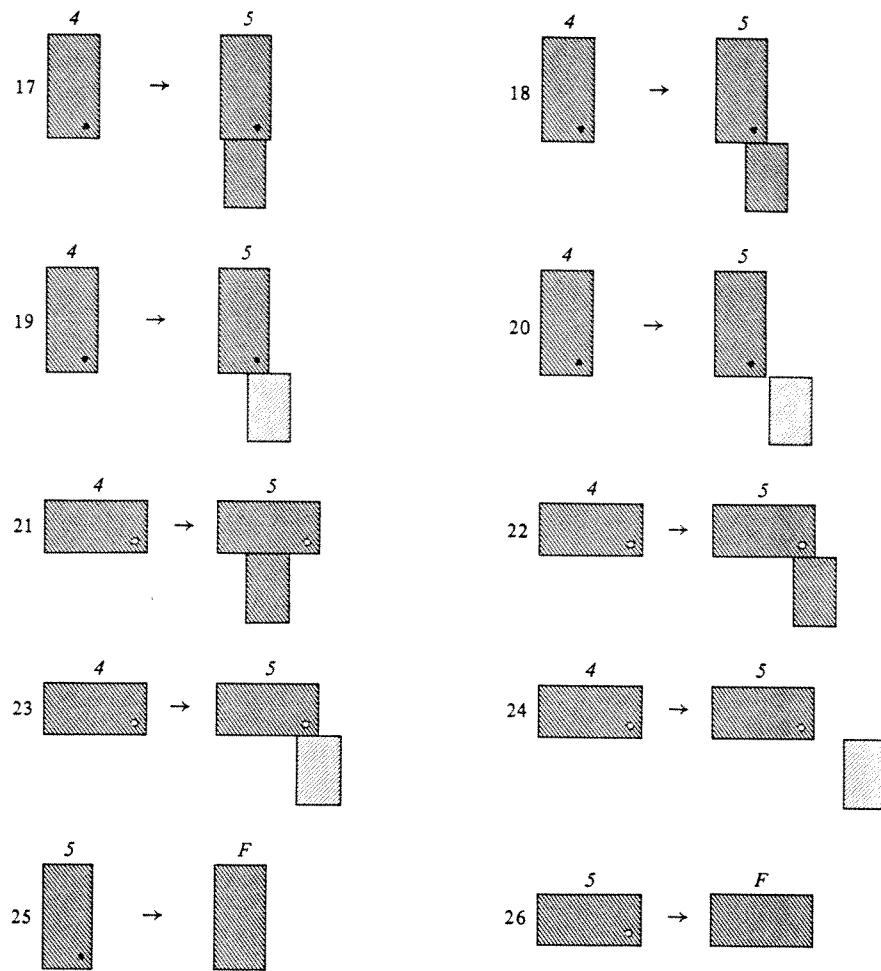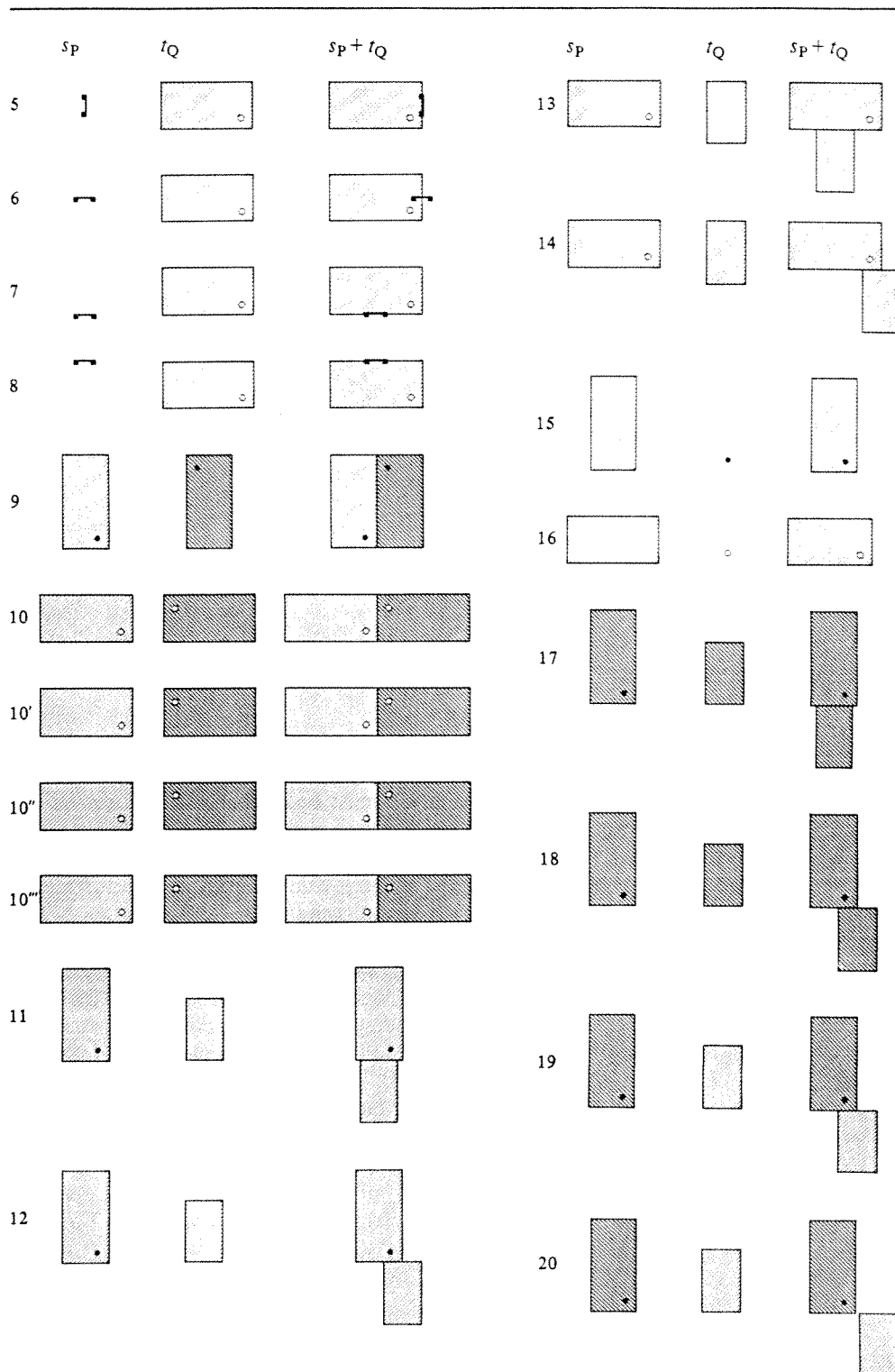
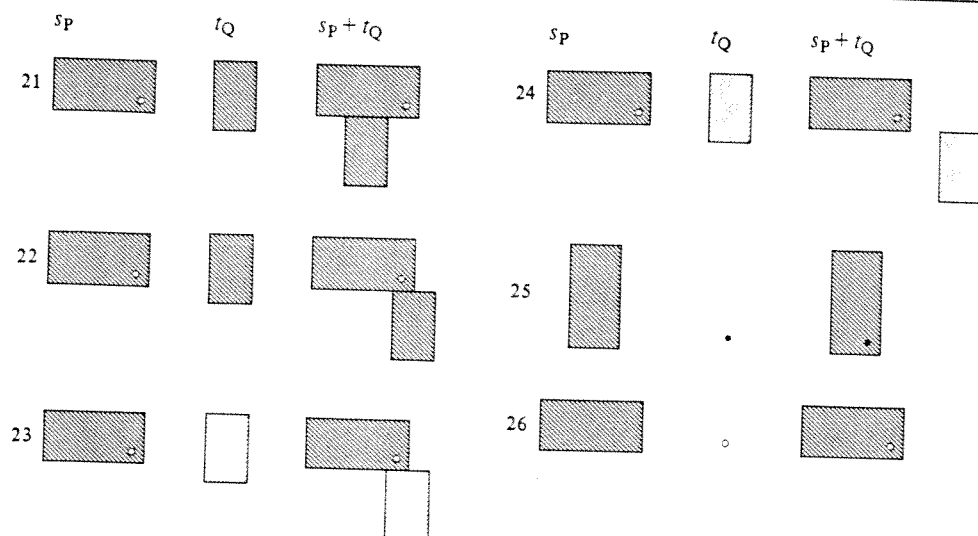rules for obligatory extensions (continued)
final state: F

(a) continued
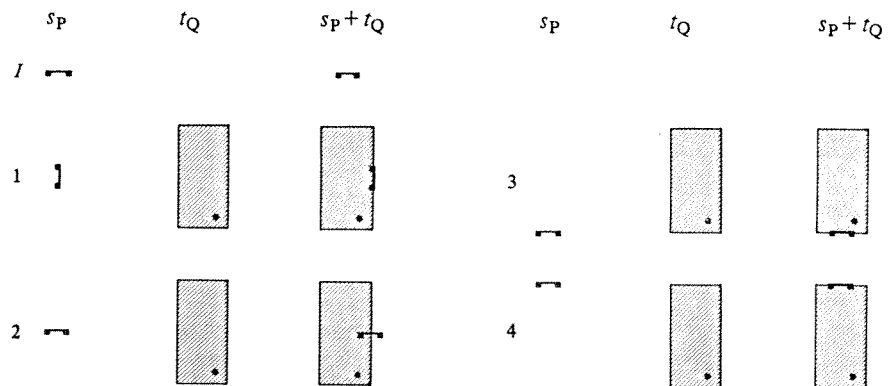
(b)

Figure 1 (continued)

(b) continued

Figure 1 (continued)

s_P	t_Q	$s_P + t_Q$		s_P	t_Q	$s_P + t_Q$
21			24			
22			25			
23			26			

(b) continued

I	$(\#, 0)$	6	$(0, 1)$	10″	$(1, 2)$	15	$(3, 4)$	21	$(4, 5)$		
1	$(0, 1)$	7	$(0, 1)$	10‴	$(1, 2)$	16	$(3, 4)$	22	$(4, 5)$		
2	$(0, 1)$	8	$(0, 1)$	11	$(2, 3)$	17	$(4, 5)$	23	$(4, 5)$		
3	$(0, 1)$	9	$(1, 2)$	12	$(2, 3)$	18	$(4, 5)$	24	$(4, 5)$		
4	$(0, 1)$	10	$(1, 2)$	13	$(2, 3)$	19	$(4, 5)$	25	$(5, F)$		
5	$(0, 1)$	10′	$(1, 2)$	14	$(2, 3)$	20	$(4, 5)$	26	$(5, F)$		

(c)

Figure 1 (continued)

Therefore, for some rules in G_P^*, two rules in G_P must be defined to produce the same results. This change in parameters allows parameterized blocks in the prairie grammar to correspond exactly to parameterized blocks in the Usonian grammar. Functions assigned to blocks in G_P are denoted by grey tones as in G_P^*. The lighter tone indicates a living zone; the darker tone a service zone.

The initial shape of G_P consists of a single-hearth fireplace in a state 0 and is equivalent to rule 1 of G_P^*. Rules 1–8 depict all possible configurations of fireplace and living zone. Rules 1 and 5, 2 and 6, 3 and 7, and 4 and 8 correspond to rules 3, 4, 5, and 6 of G_P^*, respectively. Rules 9 and 10 show the two ways a service zone can be added to a living zone to complete a core unit. These two rules together are equivalent to rule 7 of G_P^*. Rules 10′, 10″, and 10‴ are redundant rules—copies of rule 10 which are meaningful only with respect to later transformations.

Extensions to the core unit are determined by rules 11–26. Rules in this group add functionally distinguished blocks to the living and service zones of the core unit. They are analogous to rules 8–11 and rules 12–18 of G_P^* which first add functionally undistinguished blocks to the core unit and then assign functions to them. In G_P, rules 11–14 fix all possible options for adding a smaller living zone to the living zone of the core unit; rules 17–24 fix all possible options for adding either a smaller service zone or a smaller living zone to the service zone of the core unit. Rules 15, 16, 25, and 26 erase labels associated with the living and service zones of the core unit after they have been extended. Notice how extensions to the core unit are made obligatory by changes in state labels associated with rules.

G_P contains one final state denoted by the symbol F.

Figure 2 shows how rules in G_P apply to derive a basic composition.
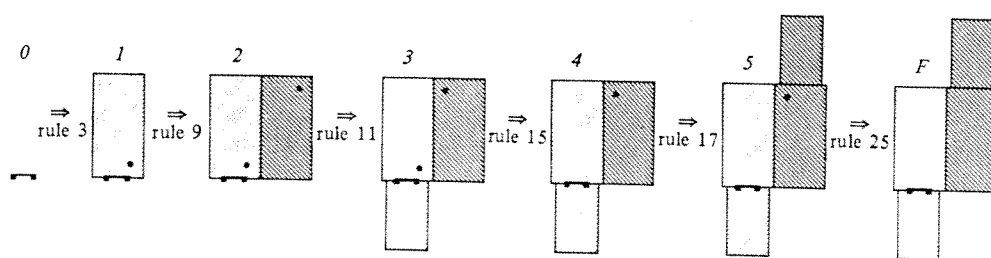
Figure 2. A derivation of a basic composition of a prairie house. This composition is number 59 in the catalogue given by Koning and Eizenberg (1981) and is the basic composition of Wright's Robie house.
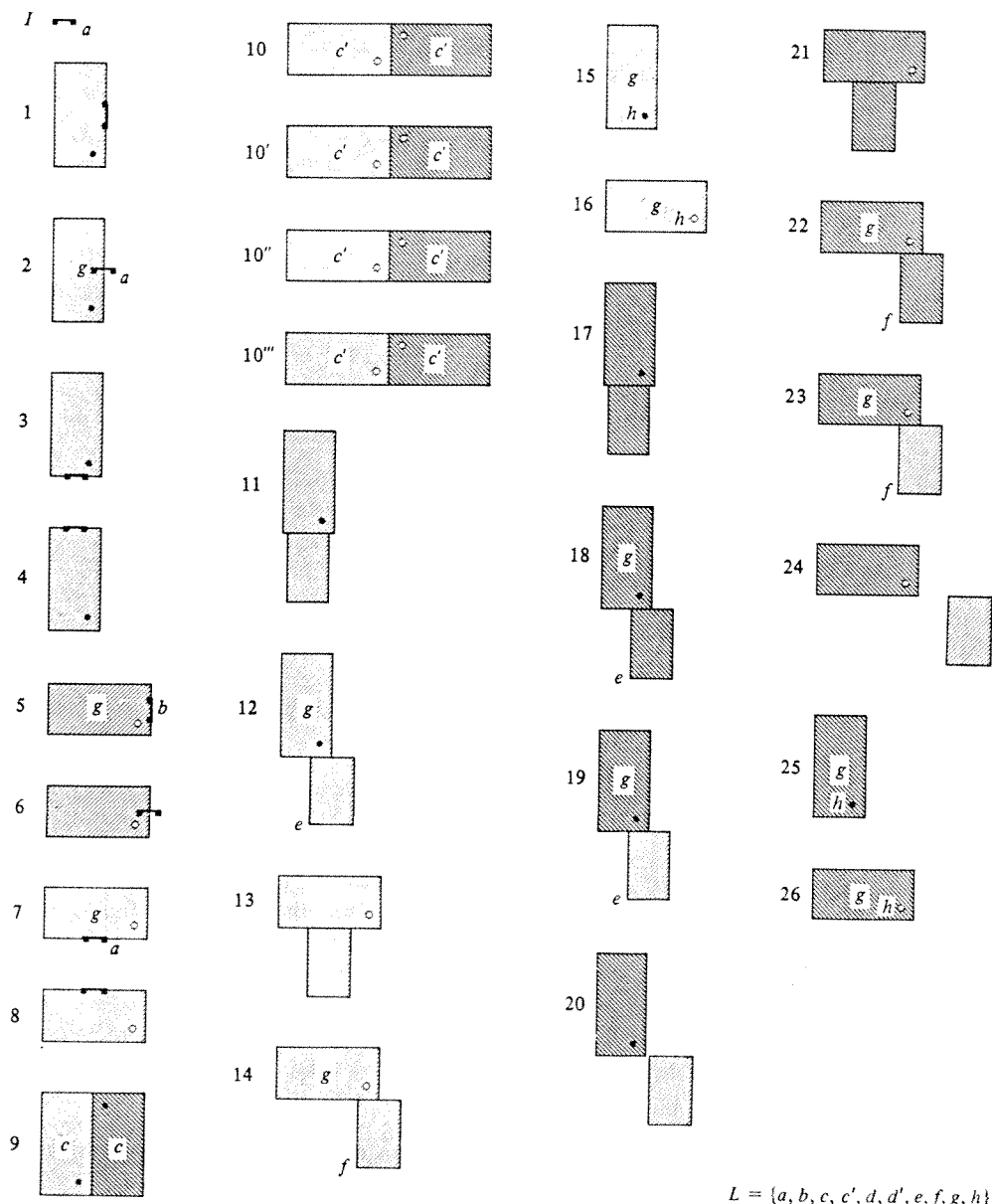

$$L = \{a, b, c, c', d, d', e, f, g, h\}$$

Figure 3. The initial set of spatial relations. Each spatial relation is numbered to correspond with the rule in G_P which it specifies. A set L of nonterminal labels is also given.

A family of transformations of G_P is now defined. This family contains only partial transformations of G_P. Thus, only a subset of G_P is transformed to determine new grammars. This subset contains the initial shape and rules 2, 5, 7, 9, 10, 10', 10", 10'", 12, 14, 15, 16, 18, 19, 22, 23, 25, and 26. In figure 3, the initial set of spatial relations together with a set L of nonterminal labels is illustrated. The initial set of pairs of state labels is the same as the set shown in figure 1(c).

The individual changes to spatial relations and state labels in these sets necessary to produce a polliwog Usonian grammar as well as other new grammars can be summarized roughly as follows.

Spatial relations
(1) The spatial relations between fireplaces and living zones in core units are transformed by moving the fireplace from the border of a living zone to the interior of a living zone and by changing the fireplace into a fireplace/kitchen (work space).
(2) The spatial relations between living zones and service zones in core units are transformed by changing their locations to form various different L-shaped and other spatial relations. An L-shape is produced with a simple 90° rotation of either the living or the service zone. This change is probably the most notable one in the spatial relations which specify G_P and is closely related to the change in state labels described below. Transforming the rectangular core unit of a prairie house into an L-shape allows extensions to be added to it optionally, that is, to one, both, or neither of the zones in it, without disturbing the integrity of the plan ('without deformity', as Wright says). Any extension to a core unit just elongates either the body or the tail of the polliwog. Optional extensions to a prairie core unit, however, would produce a lopsided, imbalanced plan whenever only one zone is extended—a plan analogous in form to a butterfly with only one wing.

In addition to the change in location, a service zone in a prairie core unit (which may contain bedrooms and bathrooms) changes functionally to become a bedroom zone in a Usonian core unit (which contains only bedrooms and bathrooms).
(3) The spatial relations between living or service zones and their extensions are transformed by moving extensions so that they are always aligned with a corner of the shorter side of a zone.

State labels
(1) The state labels associated with rules for extending the core unit are transformed so that these rules are no longer obligatory but optional.

These changes to spatial relations and state labels are given explicitly by the set A of shape change rules and the set B of state change rules shown in figures 4(a) and 4(b), respectively.

In set A, rules 1 and 2 apply to the spatial relation specifying the initial shape and to spatial relations between fireplaces and living zones. [As in the example in figure 7 in part 2 (Knight, 1983), the exact position of a labelled shape in these and other rules in A is indicated by showing the cartesian coordinate system in which the labelled shape is located. These rules each change a labelled shape denoting a fireplace into a labelled shape denoting a fireplace/kitchen while at the same time changing its location. The lines extending from either side of a fireplace on the right-hand side of each rule indicate a wall of the adjoining kitchen area. Since the size and shape of the kitchen area vary from one Usonian plan to another, the exact perimeter of this area is not given in either rule. The nonterminal label a in rule 1 allows this rule to be applied only to spatial relations I, 2, and 7 in the initial set; the nonterminal label b in rule 2 allows this rule to be applied only to spatial relation 5 in the initial set. Both these labels also restrict the euclidean transformations under which a rule can be applied.

A:
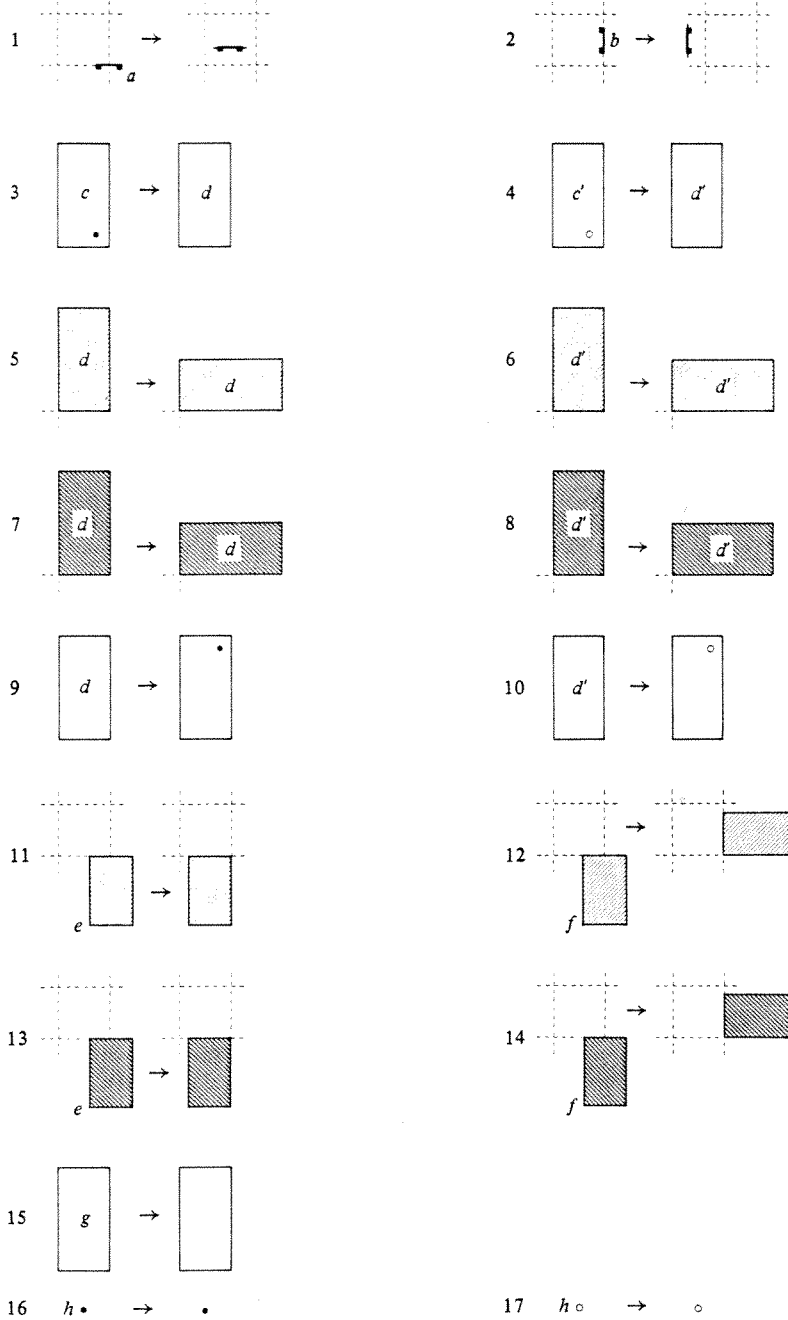
(a)

B:

$r_1, r_2, r_5, r_7, r_9, r_{10}, r_{10'}, r_{10''}, r_{10'''}$: identity state change rules

r_{12}, r_{14}:	$(2, 3) \rightarrow (2, 4)$
r_{15}, r_{16}:	$(3, 4) \rightarrow (\#, 3)$
$r_{18}, r_{19}, r_{22}, r_{23}$:	$(4, 5) \rightarrow (3, F)$
r_{25}, r_{26}:	$(5, F) \rightarrow (\#, F)$

(b)

Figure 4. (a) A set A of shape change rules, and (b) a set B of state change rules.

Rules 3-10 apply to spatial relations between living and service zones in core units (spatial relations 9, 10, 10′, 10″, and 10‴). Rules 3 and 4 erase labels associated with living or service zones; rules 5-8 rotate living or service zones to produce L-shaped and other spatial relations; and rules 9 and 10 add labels back to living or service zones. The nonterminal labels c, $c′$, d, and $d′$, which are attached to the centroids of zones in these rules, control the order in which they are applied. Note that rules 5-8 are optional. It is thus possible to leave the position of a living or service zone unchanged by applying rule 9 directly following rule 3 or rule 10 directly following rule 4.

Rules 11, 12, 13, and 14 apply to spatial relations between living or service zones and their extensions (spatial relations 12, 14, 18, 19, 22, and 23). These rules change the locations of extensions. The nonterminal labels e and f indicate which spatial relations these rules apply to and restrict the euclidean transformations under which they can be applied.

Rules 15, 16, and 17 are identity rules. They apply to spatial relations between living or service zones and their extensions (spatial relations 12, 14, 18, 19, 22, and 23) and to spatial relations between service or living zones and labels associated with them (spatial relations 15, 16, 25, and 26). The latter spatial relations always remain the same since the only rules which apply to them are identity rules.

All rules in set A apply to spatial relations between parameterized labelled shapes. Therefore, parameters must be specified for all labelled shapes in spatial relations produced by these rules. All zones or blocks in new spatial relations have parameters identical to those on blocks in the initial spatial relations. The labels ● and ○ are also parameterized as in the initial spatial relations. They always label one corner of a living or service zone. (When a zone is square, these labels must of course be located so that they distinguish consistently between the length and width of the zone.) Extensions must always be aligned with the corner of a living or service zone labelled by the symbol ● or ○. Fireplace/kitchens are always located within and not on the border of the upper half of living zones.

In set B, rules r_{12}, r_{14}, r_{15}, r_{16}, r_{18}, r_{19}, r_{22}, r_{23}, r_{25}, and r_{26} apply to change the state labels associated with rules for extending the core unit and for erasing labels. All other rules are identity rules.

Figure 5(a) shows a derivation of a final set of spatial relations from the initial set of spatial relations using shape change rules in A. Many other final sets are possible. Figure 5(b) shows the final set of pairs of state labels which is derived from the initial set of pairs of state labels by applying state change rules in B. Here, only one final set is possible.
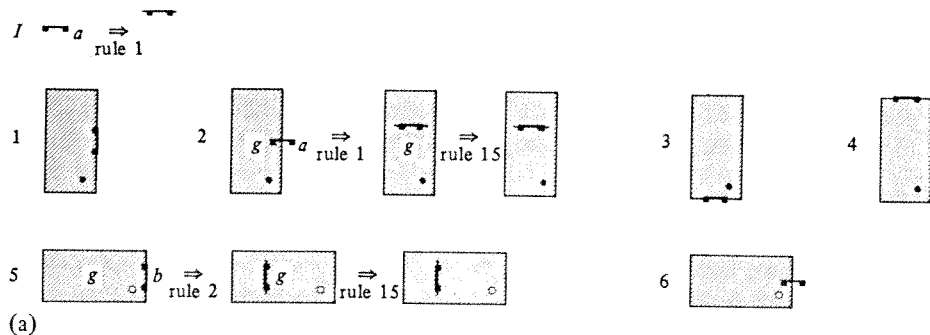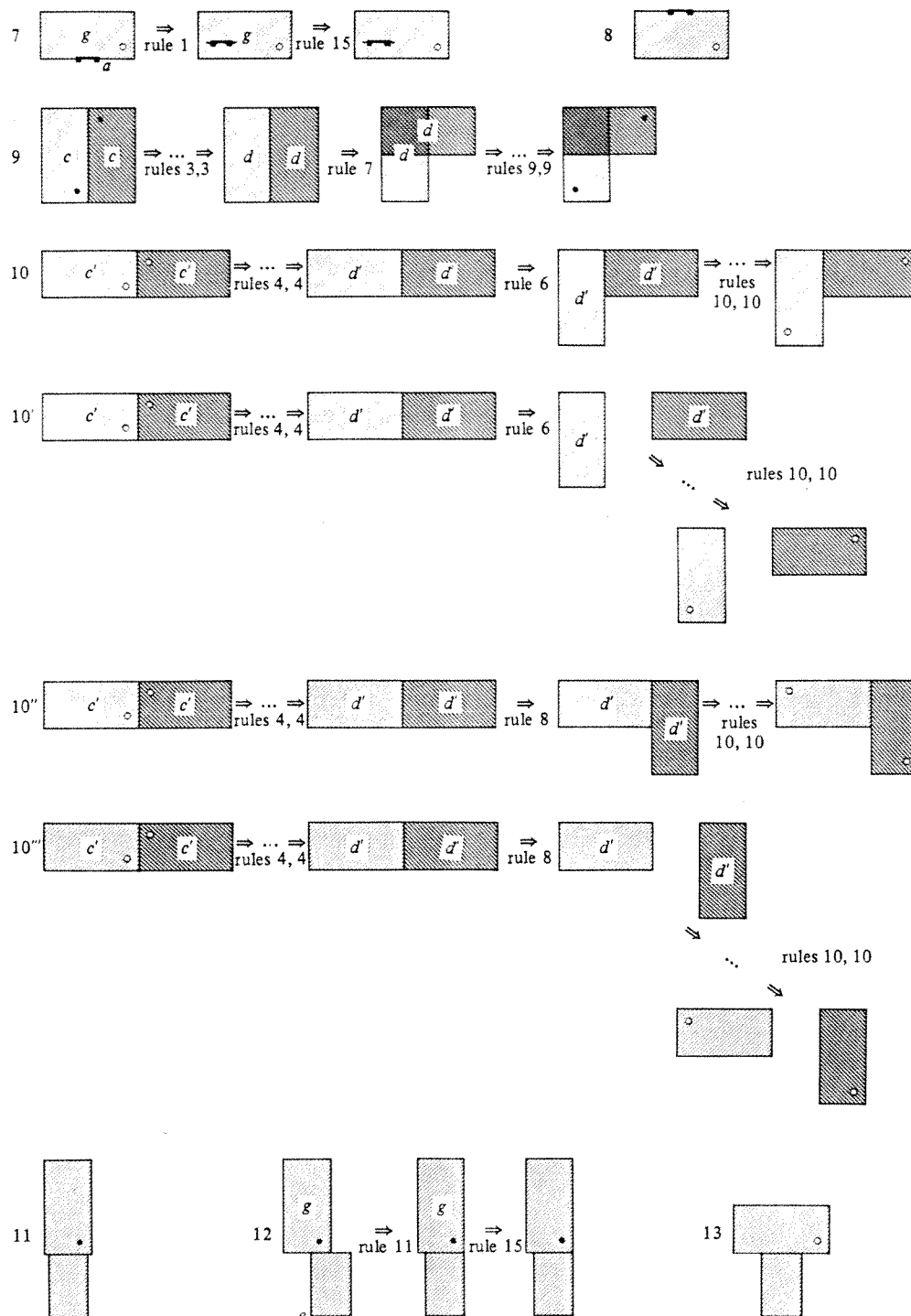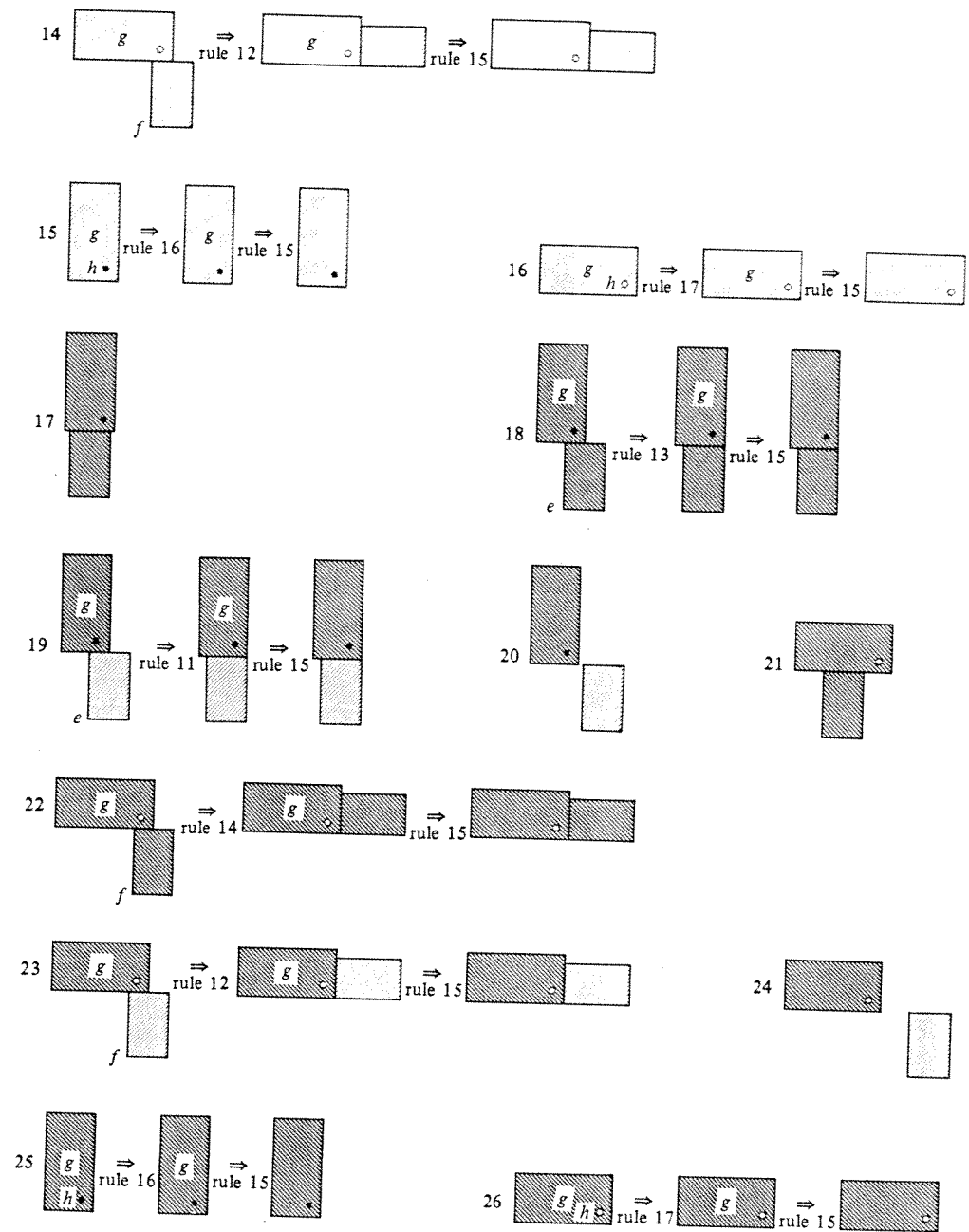

(a)

Figure 5. (a) A derivation of a final set of spatial relations from the initial set of spatial relations (figure 3) using shape change rules in set A [figure 4(a)]; (b) the final set of pairs of state labels derived from the initial set of pairs of state labels [figure 1(c)] using state change rules in set B [figure 4(b)].
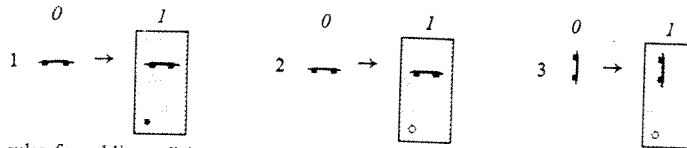
(a) continued

Figure 5 (continued)

14   g ○   ⟹ rule 12 g ○   ⟹ rule 15   ○

f

15   g h •   ⟹ rule 16 g •   ⟹ rule 15   •

16   g h ○   ⟹ rule 17 g ○   ⟹ rule 15   ○

17   •

18   g •   ⟹ rule 13 g •   ⟹ rule 15   •

e

19   g •   ⟹ rule 11 g •   ⟹ rule 15   •

e

20   g •

21   ○

22   g ○   ⟹ rule 14 g ○   ⟹ rule 15   ○

f

23   g ○   ⟹ rule 12 g •   ⟹ rule 15   ○

f

24   ○

25   g h •   ⟹ rule 16 g •   ⟹ rule 15   •

26   g h ○   ⟹ rule 17 g ○   ⟹ rule 15   ○

(a) continued

I	(≠, 0)	9	(1, 2)	10'''	(1, 2)	16	(≠, 3)	23	(3, F)
2	(0, 1)	10	(1, 2)	12	(2, 4)	18	(3, F)	25	(≠, F)
5	(0, 1)	10'	(1, 2)	14	(2, 4)	19	(3, F)	26	(≠, F)
7	(0, 1)	10''	(1, 2)	15	(≠, 3)	22	(3, F)		

(b)

Figure 5 (continued)

$T(G_P) = G_U$:

rules for adding a living zone
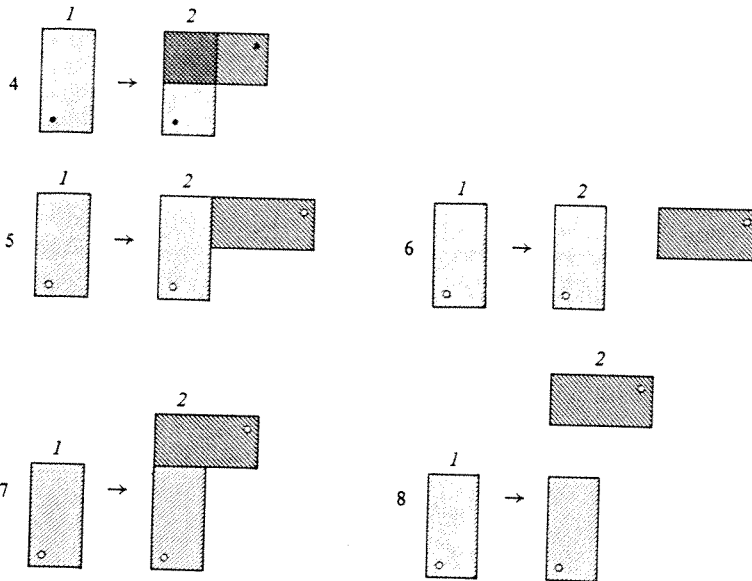
rules for completing the core unit: adding a bedroom zone
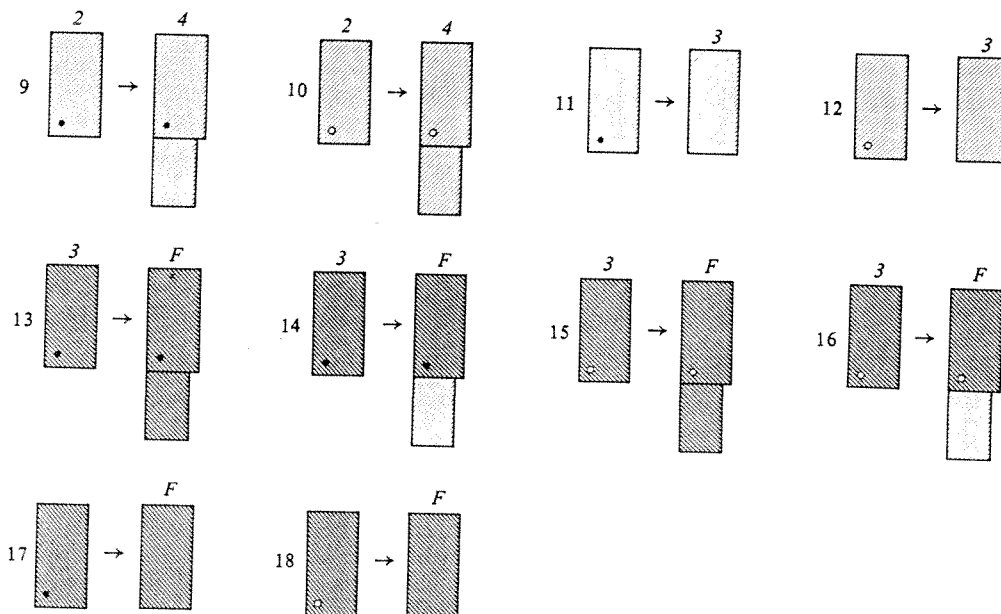
rules for optional extensions
final state: F

Figure 6. A transformation T of the grammar G_P. $T(G_P)$ is the result of composing transformations T_A and T_B determined from the final sets of spatial relations and pairs of state labels shown in figures 5(a) and 5(b), respectively. $T(G_P)$, called G_U, generates basic compositions of Wright's L-shaped Usonian houses.
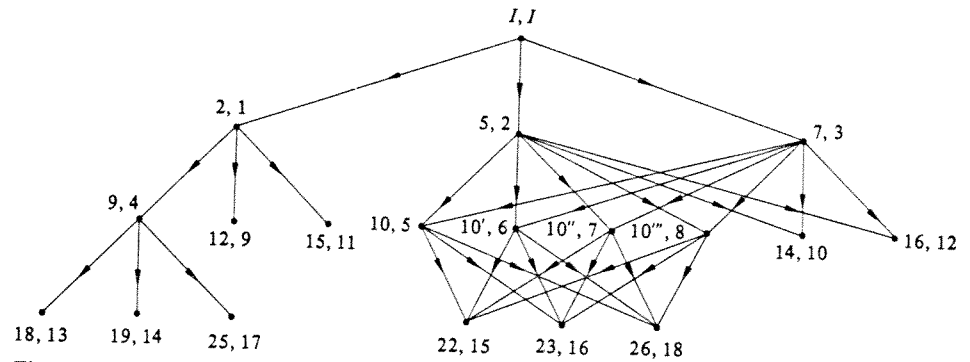
Figure 7. An isomorphism between the recursive structure of the subset of G_P transformed and the recursive structure of G_U. Both structures are represented by the same directed graph. The first number in a pair of numbers labelling a node is the number of a rule in G_P; the second number is the number of the transformation of this rule in G_U.
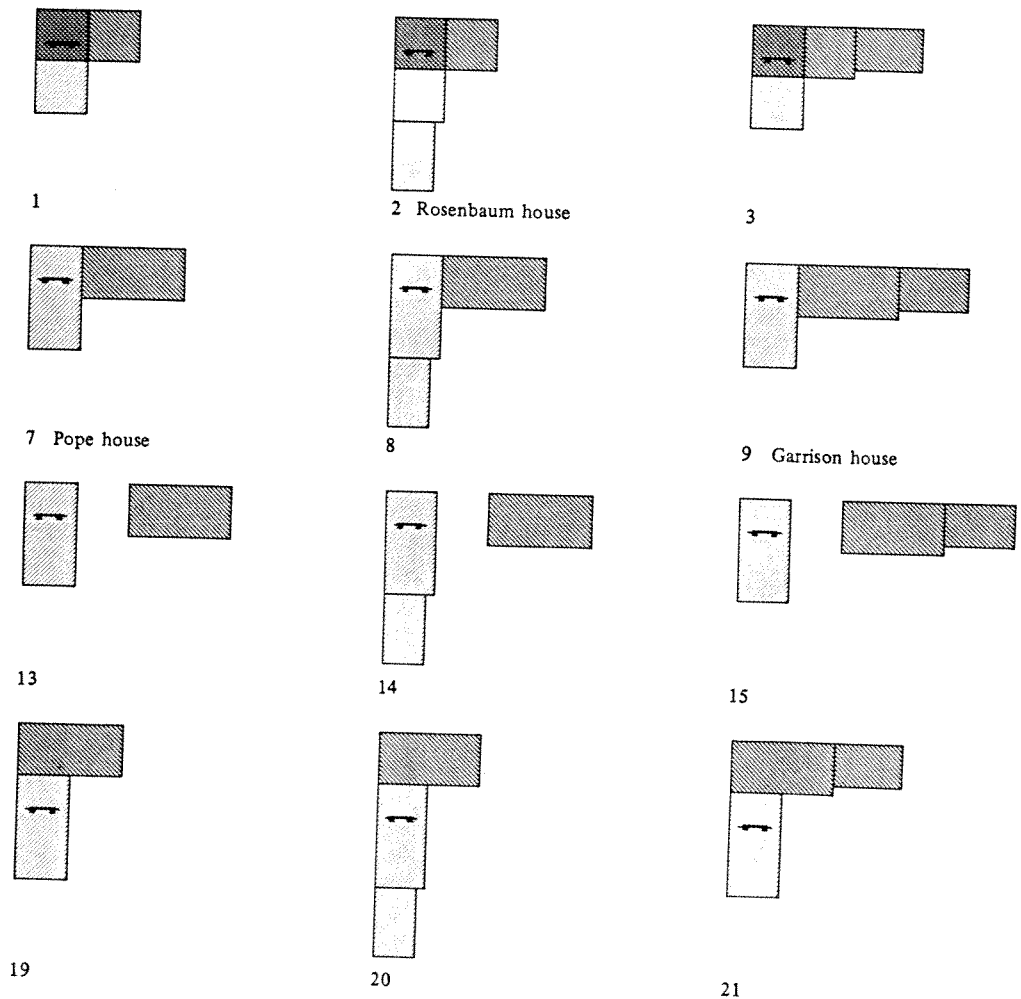
Figure 8. A catalogue of basic compositions generated by G_U. The dimensions of each composition can be varied according to conditions on parameters in the rules to define differently proportioned plans. The basic compositions of the Jacobs, Lusk, Newman, Rosenbaum, Garrison, and Pope houses are identified by name.

These two transformations determine corresponding transformations T_A and T_B which together determine a transformation $T_A \cdot T_B = T$ of the grammar G_P. The new grammar $T(G_P)$ produced is illustrated in figure 6. This grammar is called G_U. The recursive structure of G_U and the recursive structure of the subset of G_P transformed are isomorphic as shown in figure 7. Thus, T is a member of the family of transformations defined by the sets A and B. The close relationship between the rules of G_P and the rules of G_U should be apparent at once. Although spatial relations and state labels are different, the ways these grammars operate are virtually identical.

G_U is a grammar which defines a language of basic compositions of L-shaped Usonian houses. This language is illustrated in figure 8. Included in the language are the basic compositions of houses designed by Wright which typify this style: the Jacobs (1936), Lusk (1936), Newman (1939), Rosenbaum (1939), Garrison (1939), and Pope (1940) houses. These are identified by name in figure 8. All other basic compositions are new.
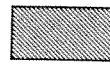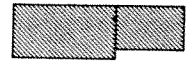

4

5

6

10   Lusk house

11

12
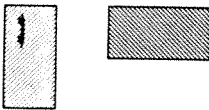
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18

22

23   Jacobs house
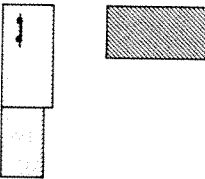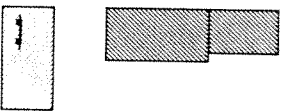
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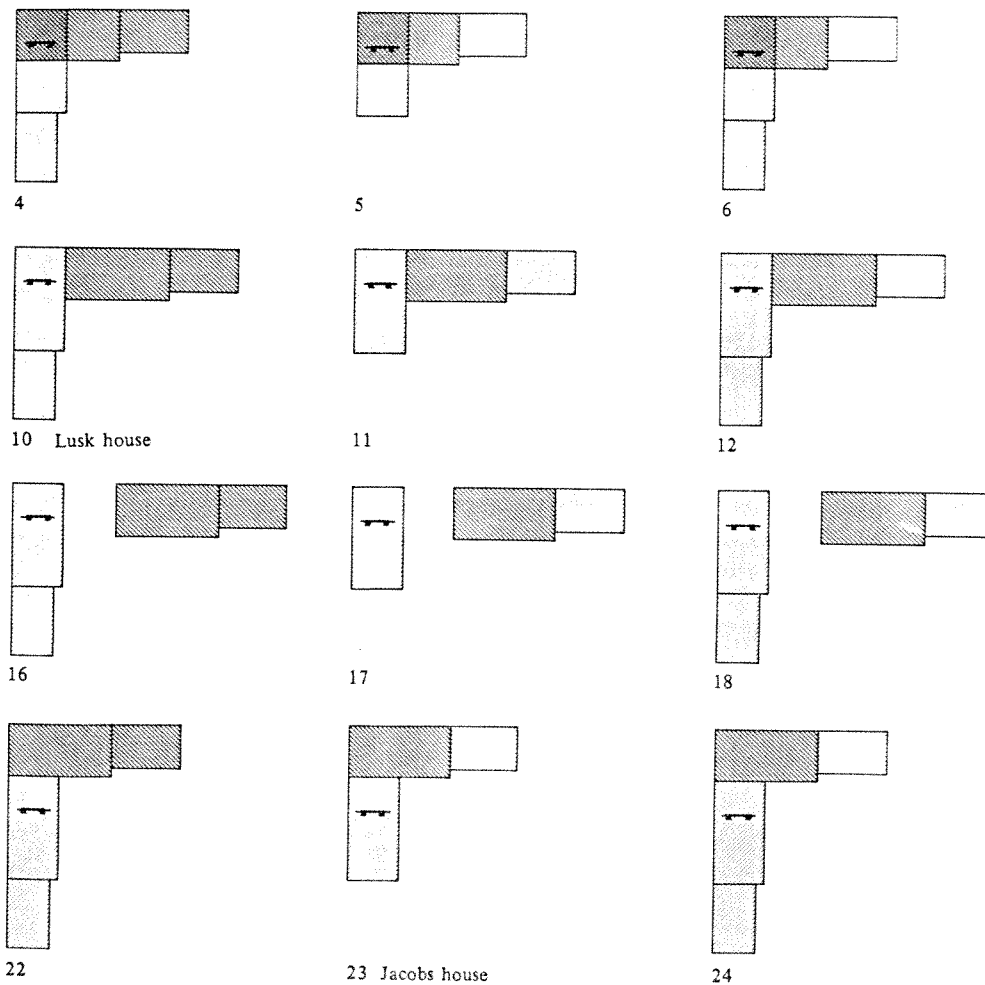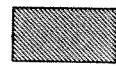
Figure 8 (continued)
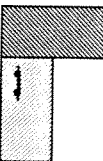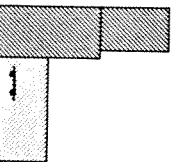
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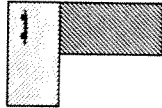
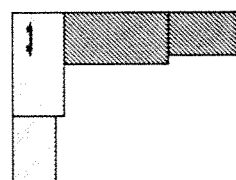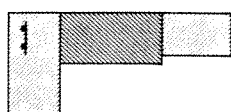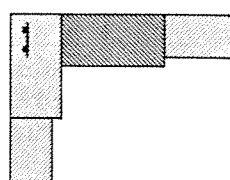31   Newman house
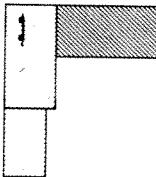
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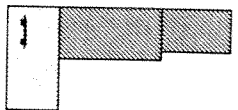
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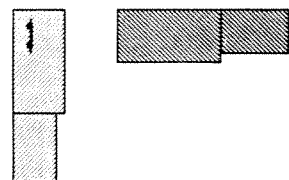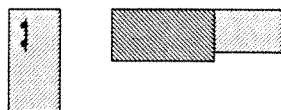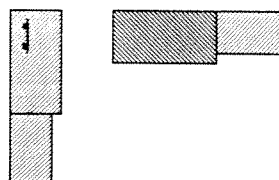
Figure 8 (continued)

Figure 8 (continued)

The living zone in a core unit of a Usonian basic composition includes the work space, dining area, and living room. An extension to it is also a living zone and is used as either a workshop or a study. The bedroom zone in a core unit includes one to three bedrooms and all bathrooms. An extension to it is either a bedroom zone containing one or two additional bedrooms and possibly a bathroom, or a living zone which is used as either a workshop or a study and can be converted into a bedroom in the future. A line between a living zone and a bedroom zone in a composition does not necessarily correspond to a wall or partition in the plan. It only approximates a nonphysical boundary between spaces used for different purposes. A line between a living or service zone and its extension frequently does correspond to a physical boundary such as a wall, partition, or secondary fireplace.

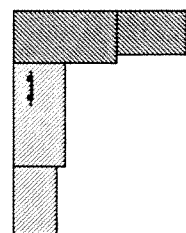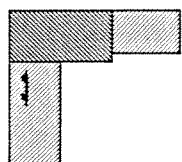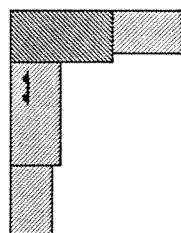When the living zone and bedroom zone in a core unit intersect as in basic compositions 1–6, then a part of the living zone and a part of the bedroom zone are contained within the same space in the hinge of the plan. In the Rosenbaum house, for instance, both the kitchen and master bathroom are contained in this area. When the living zone and bedroom zone do not border each other as in basic compositions 13–18, 25–30, 37–42, and 49–54, then the intermediate space could be either a courtyard, terrace, or other open or semi-enclosed space. (For example, see the Adelman house (1953)—a Usonian house close in plan to an L-shaped Usonian in which two separate areas of the house are linked by an outdoor terrace.)

G_U is not the only grammar definable from the sets A and B of change rules given above. Because A determines several different final sets of spatial relations, several transformations of G_P into new grammars are possible. These grammars differ from each other only in rules for adding a service zone to a living zone (completing a core unit) which are specified by spatial relations between these two zones. These core-unit spatial relations can vary in two ways.

First, the locations of the living zone and service zone with respect to each other can be different. In G_U, all such spatial relations form L-shapes. In figure 9, all
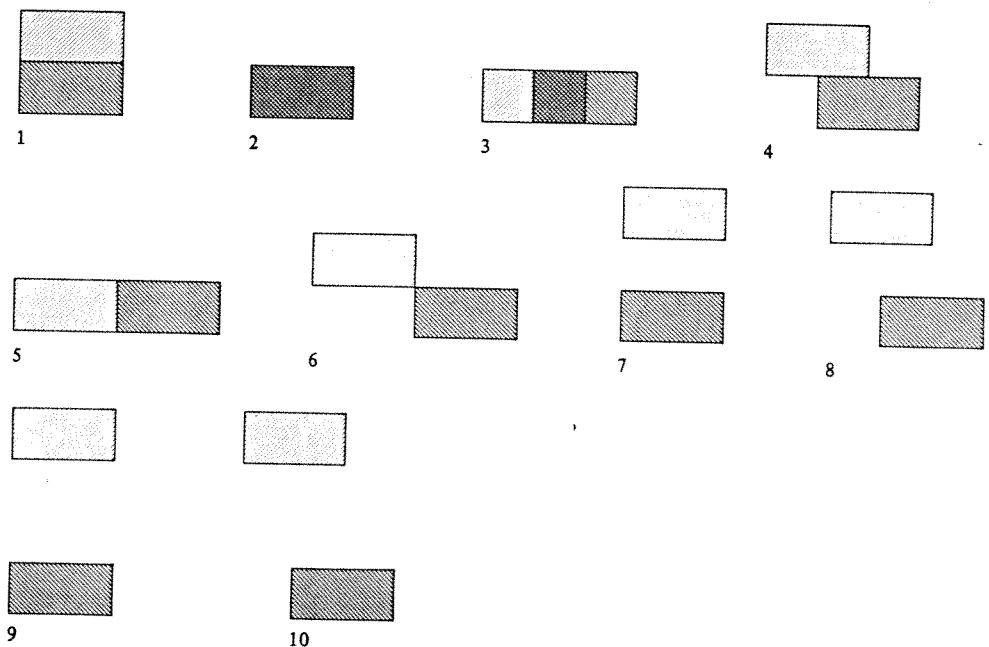

Figure 9. All other spatial relations between living and service zones in a core unit which result from different applications of shape change rules in set A [figure 4(a)].

other possible spatial relations between living and service zones which result from the application of rules in A are shown. Spatial relations 3 and 5 correspond roughly to what Sergeant (1976) calls an "in-line Usonian"; spatial relation 2 to some of Wright's early one-zone Usonians. Other spatial relations can be considered as bases for new Usonian-style compositions.

Second, the labels associated with living and service zones in core units can be placed at different corners of these zones. Thus, extensions can be adjoined to these zones in different places.

With these diverse possibilities for changes, it should be easy to visualize the variety of grammars which can be defined from G_P. Each grammar can be said to define a language of Usonian-style basic compositions. Some grammars, such as G_U, generate designs which resemble a type of Usonian built by Wright, some generate completely new-looking Usonians, and others combinations of the two. Of course, this is a rather open interpretation of a Usonian 'metastyle'. Since the primary concern here is to explicate the relationship between G_P and G_U, other possible grammars resulting from the same rules for transformation are treated in a somewhat loose way. However, the range of possible grammars can easily be narrowed by restricting the application of change rules in A (specifically those rules which produce new core-unit spatial relations) with appropriate nonterminal labelling devices. Similarly, the range of rules in these grammars (particularly those specifying core units) can be controlled by adding or removing redundant rules in G_P.

To return to G_U, the following rules can be added to it to develop and ornament basic compositions. All these rules are optional and are analogous in purpose, but not in form, to ornamentation rules in the original prairie grammar which also develop basic compositions. Although prairie houses and Usonians are strongly related in basic form, their designs rapidly diverge with these further embellishments.

Ornamentation rules for Usonian basic compositions are shown in figure 10. Rules are numbered so that they follow in sequence from rules in G_U. All labelled shapes in these rules are parameterized. As with rules in G_U, parameters are not given explicitly but are explained informally in the text where necessary.

Rules 19 and 20 apply to add a living zone, either a workshop or study, to the hinge of an L-plan. The area of the added space is always between one-quarter to one-half the area of the living or service zone in the core unit. The length of any side of this space is between one to two times that of the adjacent side.

Rules 21–25 each interpenetrate one zone in a plan with another zone. The depth of interpenetration is always less than one-third the length of the interpenetrating zone. If any one of these rules is applied to interpenetrate a zone of a core unit with another zone of the core unit, then one of rules 26, 27, or 28 can be applied to reattach an extension to the interpenetrating zone.

Secondary fireplaces may be added to plans with extensions by applying rules 29–35. In rule 29, the larger space is a zone of the core unit and the smaller space is its extension. The extension may or may not interpenetrate the adjoining zone. The label ♦ can be placed anywhere along the line which separates a zone and its extension, but not at its endpoints. Rule 30 replaces the label ♦ with a single-hearth fireplace; rule 31 erases the label ♦ and adds a single-hearth fireplace to a corner of a zone. Rules 32–35 create different double-hearth fireplaces from single-hearth fireplaces.

Rules 36–39 add spaces to extensions. In rule 36, parameters are identical to those in rule 29 with the exception of the symbol ◄ which always labels a side of an extension as shown in this rule. The area of the added space is always between one to one-half the area of the extension it adjoins. The length of any one of its sides varies between one to two times that of the adjacent side. Notice that a bedroom zone can never be added to a living zone.

19

→

20 →

rules for extending the 'hinge' of the core unit

21 →

22 →

23 →

24 →

25 →

26 →

27 →

28 →

rules for interpenetrating zones

29 →

30 →

31 →

32 →

33 →

34 →

35 →

rules for adding
secondary fireplaces

36 →

37 →

38 →

39 →

rules for extending
extensions

40 →

41 →

42 →

rules for indenting
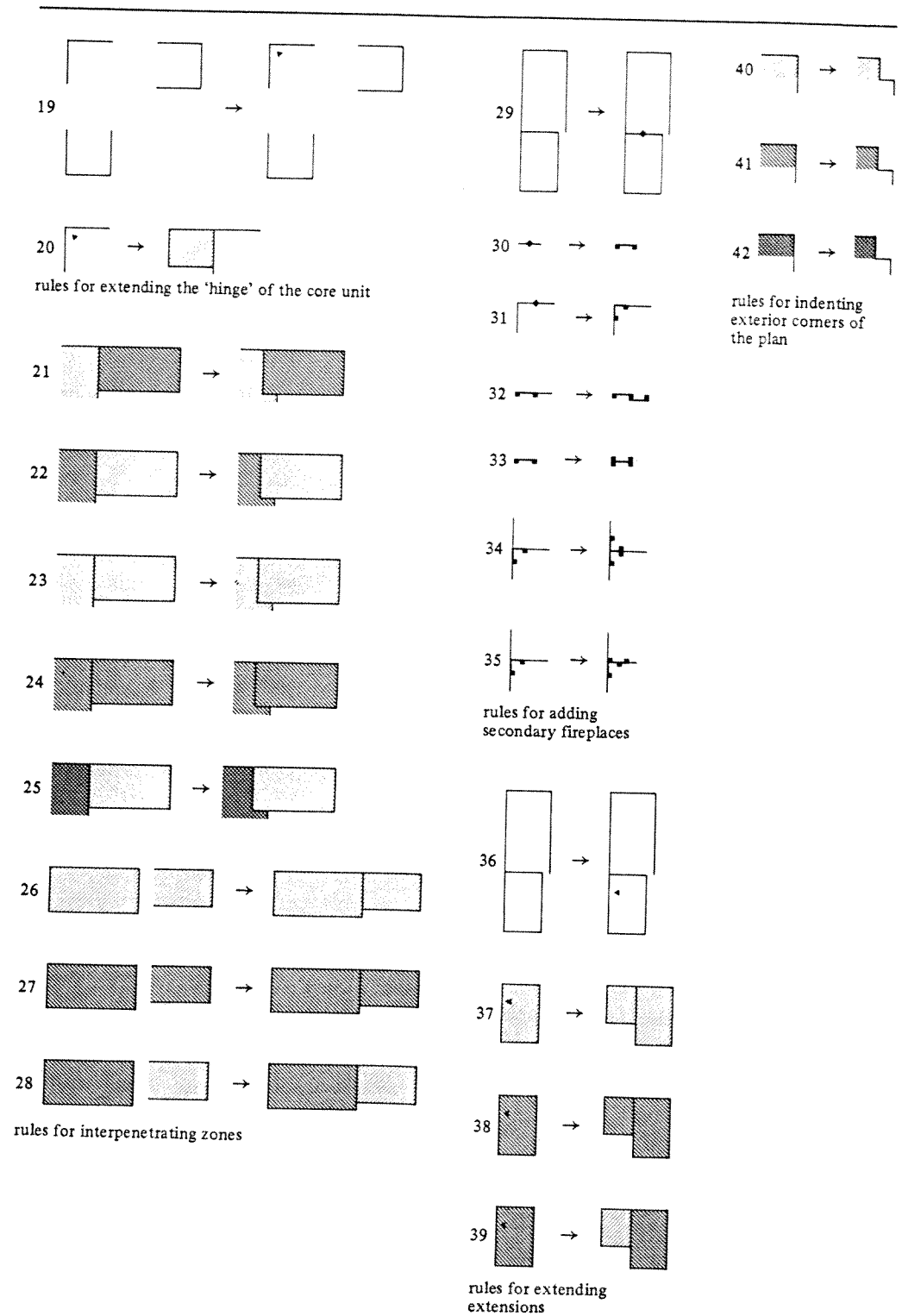exterior corners of
the plan

Figure 10. Ornamentation rules for Usonian basic compositions.

Finally, rules 40, 41, and 42 each apply to cut off or indent an exterior corner of any zone in a plan. The dimensions of the indentation are parameterized. The total area of indentations to a zone cannot be greater than one-third the original area of the zone. These rules apply recursively to plans to produce the interior alcoves and niches and undulating exterior facades characteristic of polliwog Usonians.

A design is a member of the language generated by G_U together with the ornamentation rules above whenever it has no labels, other than tone labels and the
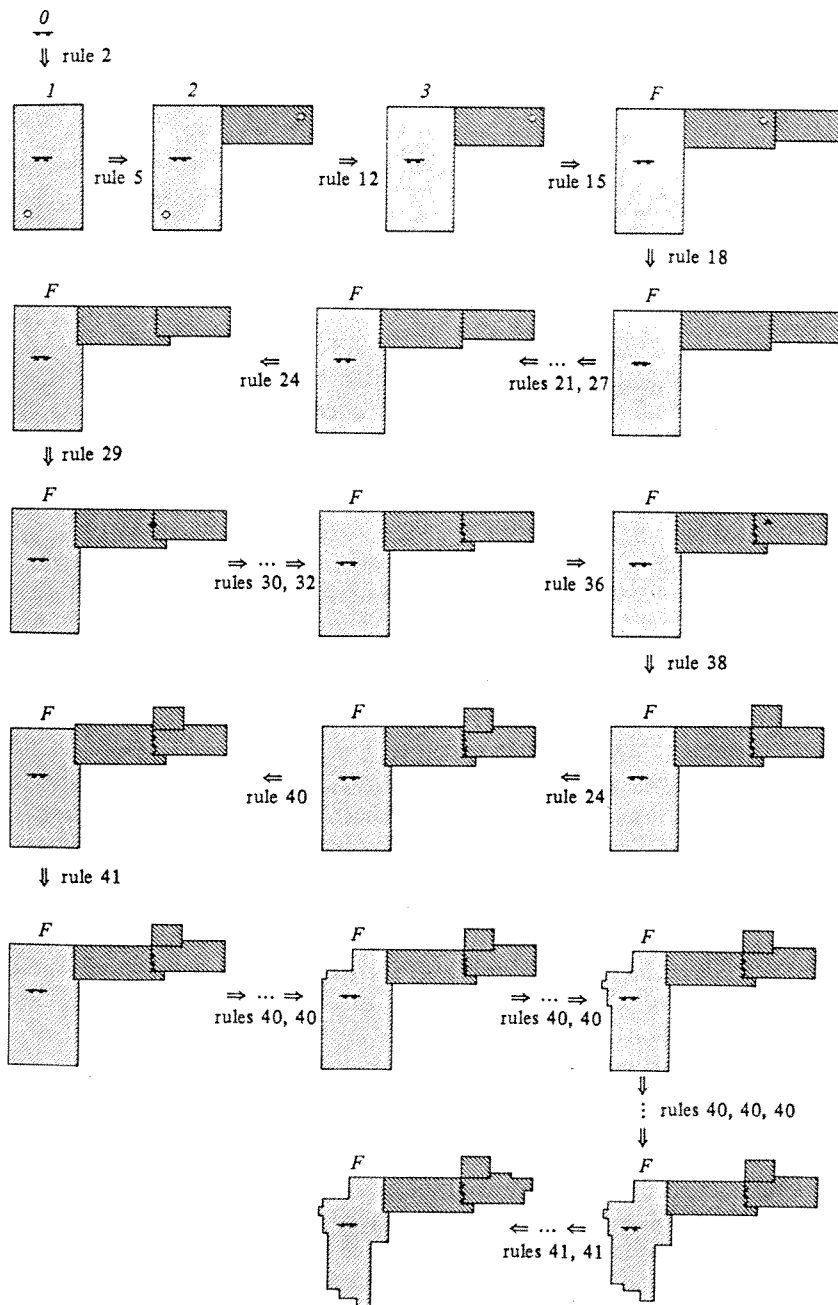
Figure 11. A derivation of the Garrison house using basic composition rules (figure 6) followed by ornamentation rules (figure 10).

final state label F, associated with it and whenever all areas in a design distinguished by tone labels are completely bounded by lines. The latter proviso ensures that designs generated by inappropriate applications of some of the ornamentation rules are not included in the language.

In figure 11, a derivation of the Garrison house is illustrated. Rules in G_U are applied first to generate a basic composition. Ornamentation rules are then applied to produce the final design. The final designs of the Jacobs, Lusk, Newman, Rosenbaum, and Pope houses produced by applying ornamentation rules to their basic compositions (see figure 8) are illustrated in figure 12. Interested readers may wish to compare these designs with the more detailed plans given in Sergeant (1976).
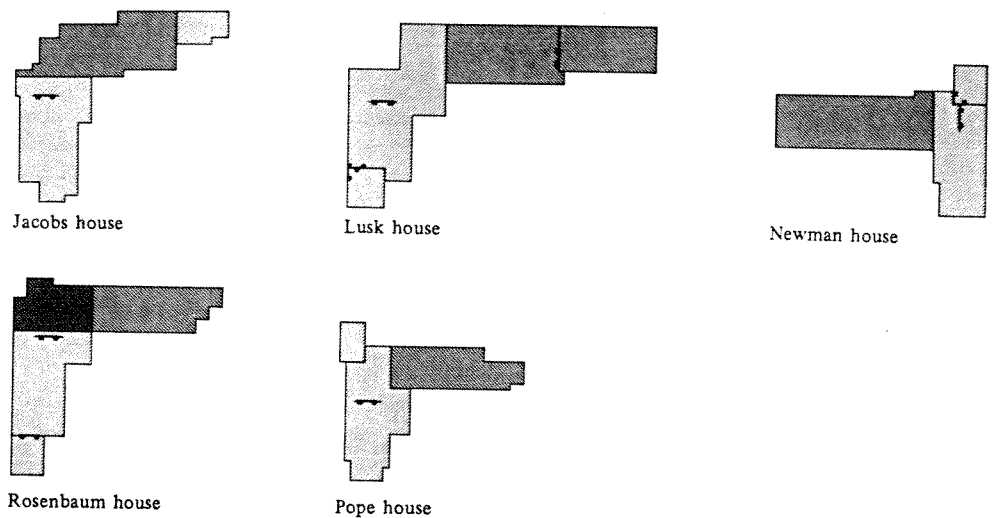
Jacobs house   Lusk house   Newman house

Rosenbaum house   Pope house

Figure 12. The final designs of the Jacobs, Lusk, Newman, Rosenbaum, and Pope houses produced by applying ornamentation rules to appropriately dimensioned basic compositions of these houses.

Conclusions

The examples given here and in part 2 of this paper (Knight, 1983) begin to demonstrate the potential of rules of transformation for comparing and constructing languages of designs. Beyond the requirements of the recursive structure and normal form of a grammar, there are no formal limits to the kinds of shape changes and state changes which can be defined for a grammar. When comparing languages, this often allows for a multiplicity of interpretations of the changes from one language to another. Different understandings of the significant features of designs in a language and different understandings of which features remain invariant and which change can lead to different characterizations of a transformation. Defining a transformation is thus relative to the selection of grammars which define languages as well as to the selection of rules which transform them.

However, if the selection of grammars for specifying languages appears arbitrary, then this impression is considerably weakened when they are placed, via rules of transformation, in the context of one another. The plausibility of a particular grammar as a description of a style is both confirmed and enhanced by linking it to grammars defining styles from which it has emerged or towards which it contributes. The rather unique characterization of Wright's prairie-style houses as arrangements of blocks around a central fireplace, for example, is now even more compelling since it can be shown that these block arrangements are easily transformed into arrangements which specify Usonian-style houses. It would be interesting to see if the more popular but

less comprehensive description of prairie houses as projections of three-dimensional planning grids could be placed as firmly and explicitly in the tradition of Wright's other works. Of course, there is no reason to expect that all transitions in styles, either in the career of an individual designer or in the history of a given culture, be as logically consistent as the transition from prairie to Usonian houses.

In the construction of new languages of designs, the formalisms presented in part 2 (Knight, 1983) may be relaxed somewhat by dispensing with the restrictions set by the recursive structures of grammars. Although working within these restrictions allows for a wide variety, in fact an infinite number, of transformations to be defined for any given grammar, it may seem an unnecessary inhibition for those who wish to experiment more freely with transformations. And dropping these restrictions would in many instances permit considerably more possibilities for changes, in particular changes to the ordering of rules.

Formal definitions may be further simplified by making changes directly to labelled shapes and state labels in rules. The intermediate step of changes to spatial relations and pairs of state labels associated with rules was introduced for pedagogical reasons only—as a formal reminder of these two underlying mechanisms for generating designs.

With these reductions in formal definitions, the normal form is by itself a sufficient framework within which to define new languages. Transformations are still controlled by directly or indirectly changing only spatial relations and their ordering. However, if the recursive structures of grammars are ignored, the results of successive changes may be increasingly difficult to understand as the structures of new grammars begin to differ from the original. And, although a less restricted approach might be feasible for creating new languages, it would not be adequate for analyzing relationships between languages. Without the constraints of recursive structures, it would be possible to transform any given grammar into any other one and thus to deduce a relationship between any two languages of designs. (Briefly, this is because one-to-one mappings can always be defined between grammars with equal numbers of addition rules and equal numbers of subtraction rules. Grammars with unequal numbers of addition and subtraction rules can always be filled in with redundant rules.) More importantly though, isomorphic recursive structures provide intuitive as well as formal justification for claims for the relatedness of grammars and languages.

The study of transformations in design is undeniably complex and difficult. Certainly, the ideas and very schematic examples given in this paper only begin to address the problems involved. But though the answers they provide are partial, it is hoped that they contribute in a clear and understandable way towards our rapidly accumulating knowledge of languages of designs.

References
Knight T W, 1983, "Transformations of languages of designs: part 2" *Environment and Planning B: Planning and Design* **10** 129–154
Koning H, Eizenberg J, 1981, "The language of the prairie: Frank Lloyd Wright's prairie houses" *Environment and Planning B* **8** 295–323
Sergeant J, 1976 *Frank Lloyd Wright's Usonian Houses* (Watson-Guptill Publications, New York)
Wright F L, 1954 *The Natural House* (Horizon Press, New York)