

Languages of designs: from known to new

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Abstract. A procedure for defining new languages of designs from known or given ones is presented. It is specified in terms of *shape equivalence rules* or *shape equivalence rule schemata* which allow shapes in spatial relations given or inferred from existing design languages to be replaced with other shapes. The new spatial relations so defined can be used to determine a wide variety of new, original languages of designs. The possibility of using shape equivalence rules or rule schemata for characterizing formal compositional aspects of historic styles or languages of designs and relationships between them is also discussed.

Introduction

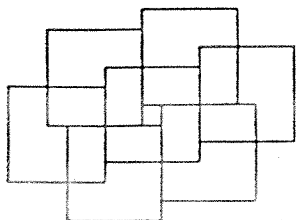
A basic familiarity with the formal definitions of *shapes*, *spatial relations*, *shape rules*, *shape grammars*, and their applications is assumed on the part of the reader. Readers unfamiliar with these definitions are referred to Stiny (1980a; 1980b). Readers satisfied with their understanding of the above terms may wish to pass over the following section on languages of designs.

Languages of designs

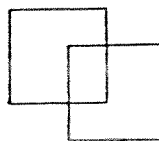
Any design can be understood, in a syntactic or compositional sense, as a complex of shapes and relationships between these shapes. In formal terms, this kind of understanding corresponds to viewing a design in terms of a *vocabulary of shapes* occurring in the design and a set of *spatial relations* between these shapes. Here, a *shape* is defined as a finite collection of lines, a *vocabulary* as a set of shapes no two of which are similar, and a *spatial relation* as a collection or arrangement of shapes.

Consider, for example, the design shown in figure 1(a). One way of describing this design would be to say that it is composed of squares and that these squares are arranged according to the spatial relation between overlapping squares illustrated in figure 1(b).

A more complete description of a design can be given by specifying exactly how spatial relations recur in it or how it may be constructed in terms of these spatial relations. This is accomplished by means of *shape rules* defined in terms of spatial relations. Shape rules are applied to an *initial shape*—a shape made up from shapes in a vocabulary—and to shapes produced from it to actually generate designs.



(a)



(b)

Figure 1. (a) A design; and (b) a spatial relation between overlapping squares which occurs in the design shown in (a).

For example, the spatial relation shown in figure 1(b) may be used to define the shape rule given in figure 2(a). This shape rule is applied recursively to an initial shape consisting of a single square [figure 2(b)] to construct the design shown in figure 1(a).

A set of shape rules and an initial shape determine a *shape grammar*. A shape grammar generates a *language of designs*. The designs in a language are syntactically alike in that their compositions are governed by the same spatial relations.

For example, the shape rule and initial shape given in figure 2 define a shape grammar which generates a language of designs containing not only the design shown in figure 1(a) but other designs as well. In figure 3, other members of this language of designs are depicted.

Of course, any one design can be understood in terms of different vocabularies of shapes and different spatial relations between them. Each such vocabulary and set of spatial relations may be used to define a shape grammar which usually generates a different language of designs. Each of these languages contains, at least, the original design.

The shapes in a vocabulary or the set of spatial relations which are used to define shape rules for constructing a design need not actually appear in the design. Parts or all of these shapes may be erased in the final design. For example, the construction of the design shown in figure 4(a) is based on the same spatial relation between overlapping squares used in the construction of the designs shown in figures 1(a) and 3.

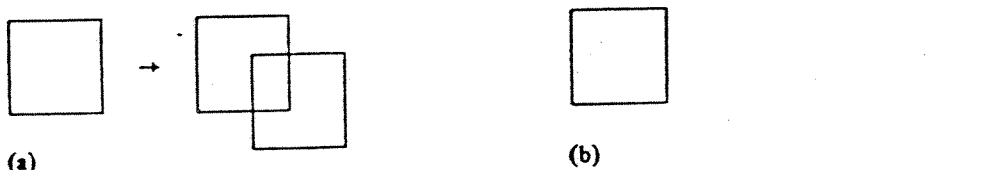


Figure 2. (a) A shape rule defined in terms of the spatial relation shown in figure 1(b); and (b) the initial shape to which the shape rule given in figure 2(a) is recursively applied to construct the design shown in figure 1(a).

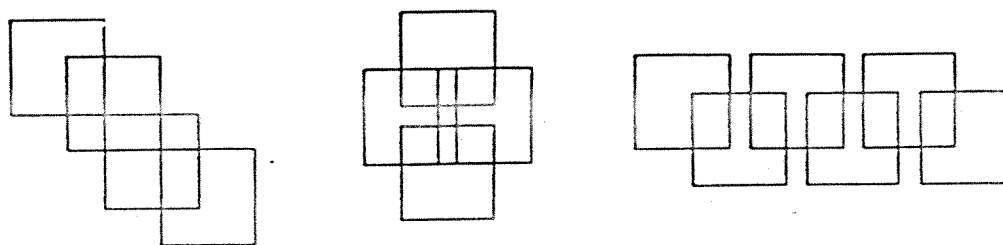


Figure 3. Additional members of the language of designs generated by recursively applying the shape rule given in figure 2(a) to the initial shape shown in figure 2(b).

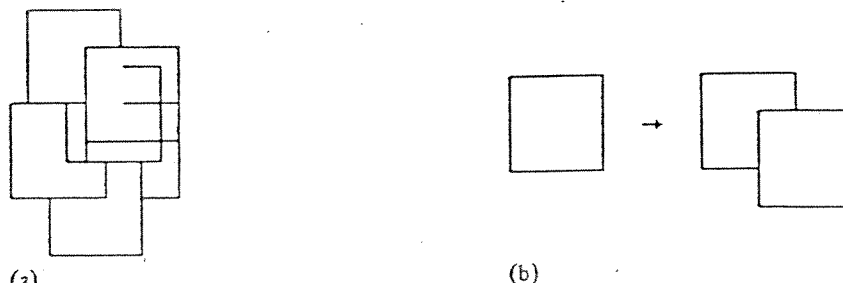


Figure 4. (a) A design whose construction is based on the spatial relation between overlapping squares shown in figure 1(b); and (b) a shape rule which generates the design shown in (a).

The shape rule which generates this design by applying it recursively to an initial shape consisting of a single square is shown in figure 4(b). Each time it is applied part of a shape in the spatial relation is erased.

Often, when a group or corpus of designs is said to characterize a particular 'style', it is because the syntax or formal composition of these designs is alike in some way. In this case, one can usually distinguish a vocabulary of shapes and set of spatial relations common to all or almost all the designs. Shape rules based on these spatial relations fix uniform recurrences of these spatial relations within the designs. A shape grammar, specified in terms of these shape rules, generates a language of designs which includes, at least, some or all of the designs in the corpus. Different syntactic interpretations of a corpus of designs determine different shape grammars which, in turn, usually produce different languages of designs.

Examples of shape grammars defined from a given corpus of designs include traditional Chinese lattice designs (Stiny, 1977), Palladian villa plans (Stiny and Mitchell, 1978), Mughul gardens (Stiny and Mitchell, 1980), Hepplewhite chair-back designs (Knight, 1980), Japanese tearoom plans (Knight, 1981), the architecture of Giuseppe Terragni (Flemming, 1981), and the prairie houses of Frank Lloyd Wright (Koning and Eizenberg, 1981). The vocabulary of shapes and set of spatial relations which are used to construct designs in these languages are not given explicitly in any of these grammars. They are, however, implied in the rules of these grammars. The rules of any shape grammar could, in fact, be rewritten in some 'normal form' which expresses exactly the spatial relations on which they are based. One such normal form would stipulate that all shape rules be of types 3 and 4 as given in Stiny (1980b) and explained later in this section. A normal form for shape grammars would permit shape grammars and the languages they define to be analyzed, compared, and manipulated in some uniform and systematic manner.

Shape grammars which generate languages of designs may also be defined from scratch. In this case, they are defined by starting with some vocabulary of shapes and then defining a set of spatial relations between the shapes in the vocabulary. These spatial relations may be determined according to heuristics which, for example, define spatial relations having particular syntactic or semantic properties. Shape rules specified in terms of these spatial relations control the way in which these spatial relations will recur in a design.

Four fundamental types of shape rules which can be defined from any spatial relation between two shapes s and t are identified in Stiny (1980b). Shape rules of types 1 and 2 are the most basic:

$$\text{type 1: } \langle x, \emptyset \rangle \rightarrow \langle s+t, \emptyset \rangle, \quad \text{type 2: } \langle s+t, \emptyset \rangle \rightarrow \langle x, \emptyset \rangle,$$

where x is the shape s or the shape t and \emptyset is the empty set indicating that no labels are attached to any of the shapes in these shape rules. Type 1 and type 2 shape rules correspond, respectively, to adding and subtracting shapes in accordance with the spatial relation between s and t . Given any spatial relation between two shapes s and t , two type 1 and two type 2 shape rules may be defined from it corresponding to when x is equal to s and when x is equal to t .

Shape rules of types 3 and 4 restrict the way in which type 1 and 2 shape rules are applied by labelling the shapes occurring in them:

$$\text{type 3: } \langle x, P \rangle \rightarrow \langle s+t, Q \rangle, \quad \text{type 4: } \langle s+t, P \rangle \rightarrow \langle x, Q \rangle,$$

where x is the shape s or the shape t and P and Q are sets of labelled points. Depending on the way these shapes are labelled, designs with particular syntactic or semantic properties can be generated.

Languages of designs defined by means of type 3 and 4 shape rules are subsets of the languages of designs defined by means of corresponding unlabelled type 1 and 2 shape rules. Numerous examples of languages of designs produced by shape grammars defined by means of type 1, 2, 3, and 4 shape rules are given in Stiny (1980b).

Languages of designs defined either in terms of an existing corpus of designs from which a vocabulary of shapes and spatial relations between them are inferred or in terms of an arbitrary vocabulary of shapes from which spatial relations are defined represent a 'constructive' approach to design. This approach is elaborated on in detail in Stiny (1980b).

Defining new languages of designs

Languages of designs defined constructively contain designs which are formal variations on some compositional 'theme'. Although these languages may contain a rich diversity of designs, all the designs in a language share a common syntactic structure. Any such language may eventually be exhausted; that is, all its variations are known and suitably employed or not employed. When this occurs a new design language is demanded. Changing external events, new ideas, or needs may also necessitate development of a new language of designs. In either event, the designer is faced with the task of creating a language new or different from those currently used.

One way out of this predicament is to revive a design style of the past and more fully explore its potentialities. In the history of Western art and architecture, this remedy was often used to goad the progress of an art or architecture suffering from inertia. This is clearly exemplified in nineteenth century Europe when the cry for a new style of architecture resulted in the revival of various past styles, such as the 'Gothic' and 'Classical' styles, as well as a sudden interest in the styles of contemporaneous but distinct cultures such as those of China and India. At the same time, innumerable guidebooks or catalogs were published which documented variations of specific forms within these styles—for example, Gothic window tracery and Classical facades. Architects and designers were encouraged either to copy the models given in the guidebooks or to develop new variations based on these models. If renovation of a past style is the answer to an impoverished or inappropriate state of design development, defining a shape grammar in terms of the revived style is certainly the easiest and most profitable means for exploring all possible variations within this style.

Another, perhaps more efficacious, answer is to develop a design language sufficiently different both from past and from present ones to be considered new. This includes languages which are defined from scratch and languages which are defined in terms of ones already known. In the former case, these languages can be defined by use of the constructive methods given in Stiny (1980b). However, even this method requires a certain amount of ad hoc experimentation in order to derive a vocabulary and spatial relations which generate interesting or appropriate languages of designs. In the latter case, new languages of designs are defined by extending, modifying, or incorporating parts of known design languages. Almost all existing styles of design, even those heralded as new or original at their inception, are in fact derived from or embrace some aspects of a past or present style. This method of innovation at least allows the designer to employ select elements of known languages without having to start from scratch. Even so, he or she must rely on intuition and ad hoc methods in order to create new languages using these elements.

When languages of designs are defined constructively, any or all of the elements used to determine them—shapes in a vocabulary, spatial relations, and shape rules—provide a well-defined basis for constructing new languages of designs. What is needed are systematic methods for defining either new shapes in a vocabulary, new spatial relations, or new shape rules from the existing ones. These new elements,

alone or in conjunction with existing ones, can be used constructively to define new design languages.

In Stiny (1980b) one such method is proposed. It allows for a diversity of type 3 or 4 shape rules to be defined from a given type 1 or 2 shape rule. It consists, basically, of labelling the given shape rule in different ways according to the symmetry properties of the shapes occurring in it. All of the shape rules so defined generate different languages of designs. All these languages are subsets of the language produced by the original shape rule. In general, changing the set of labelled points associated with a shape rule defines a new shape rule which generates a language of designs different from that generated by the original one.

New shape rules may also be defined from a given one by parameterizing the shapes occurring in it. The resulting shape rule schema generates shapes or designs which are generalizations of the designs produced by the original shape rule. New shape rule schemata can be defined from a given shape rule schema by changing conditions on the parameters associated with it. Different spatial relations may be derived from a given one in the same way—by parameterizing the shapes occurring in it or by changing conditions on existing parameters. The parameters attached to shapes in a spatial relation can be used either to vary the shapes themselves or to vary the disposition of these shapes with respect to each other. Shape rule schemata defined from these new spatial relations usually generate different languages of designs.

In this paper, an additional technique for defining new languages of designs is proposed. It is given in terms of shapes and spatial relations between them as these provide the most basic and essential information about the spatial organization of a design. In the analysis of existing designs, recognizing spatial relations between shapes occurring in designs provides the foundation for a rigorous, intelligible discussion of their formal characteristics and structure. In the synthesis of designs, spatial relations are the foundation for constructing designs whose formal characteristics and structure are known. Defining new spatial relations from ones given in an existing language of designs is, therefore, the simplest, most direct and informative way of producing new design languages.

Shape replacement in spatial relations

The following simple yet productive technique for defining new spatial relations from a given one is specified in terms of the shapes occurring in the given spatial relation. Basically, it allows shapes in a spatial relation, containing any number of shapes, to be replaced with other shapes in the spatial relation.

This procedure is similar to a phenomenon in spoken languages known as *metathesis*. Metathesis, the transposition of letters, sounds, or syllables within a word, accounts for much of the change in language systems. For example, the modern English word 'bird' is derived from the Old English word 'brid' by the transposition of the letters 'r' and 'i'. Much of the time, metathesis occurs unconsciously in the speaker as when one accidentally says 'evelate' instead of 'elevate'. With frequent repetition, some of these mistakes are assimilated into the language.

Shape replacement transposes shapes in a spatial relation just as metathesis transposes letters in a word. Consider, for example, the spatial relation in figure 5 which consists of a rectangle and round arch and is a common spatial element of Romanesque architecture and Western architecture in general. If the rectangle and round arch are transposed in this spatial relation a new spatial relation [figure 6(a)] between a rectangle and a round arch results.

The shape replacement procedure, however, extends the concept of metathesis by also allowing one shape in a spatial relation to be replaced by another shape in the relation and not vice versa. For example, given the spatial relation shown in

figure 5, the rectangle can be replaced with a round arch to produce a spatial relation between two round arches [figure 6(b)] or the round arch can be replaced with a rectangle to produce a spatial relation between two rectangles [figure 6(c)]. In other words, the steps performed in a metathesis can be separated and performed individually in a shape replacement operation. The exact manner in which this is done is given in the next section. Notice that each of the spatial relations produced from the original one is also frequently used in Romanesque and other styles of design.

Like metathesis in a word, shape replacement in a spatial relation is an easy and natural innovative process. Unlike metathesis, shape replacement can be performed consciously and deliberately to produce results which are employed consciously and deliberately in a design or design language.

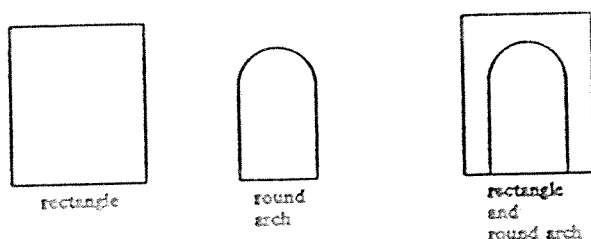


Figure 5. A spatial relation between a rectangle and a round arch.

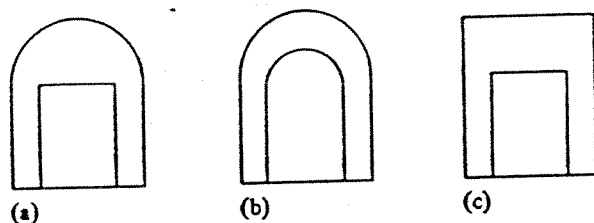


Figure 6. (a) A spatial relation between a rectangle and a round arch which is derived from the spatial relation depicted in figure 5 by transposing the rectangle and round arch; (b) and (c) show the spatial relations between a round arch and a round arch and between a rectangle and a rectangle which are derived from the spatial relation shown in figure 5 by replacing the rectangle with a round arch and by replacing the round arch with a rectangle, respectively.

Shape equivalence rules

The shape replacement procedure described in the previous section can be specified precisely and unambiguously by a *shape equivalence rule* $\alpha \leftrightarrow \beta$, where α and β are the shapes that can be transposed or replaced in some spatial relation. It is applied recursively to a spatial relation to derive new spatial relations in much the same way that a Thue equivalence rule⁽¹⁾ is applied recursively to a word in an associative calculus to produce new words in the calculus. For example, suppose that *pot* is a word and $p \leftrightarrow t$ is a Thue rule in some associative calculus. The Thue rule permits occurrences of the letter *p* to be replaced by the letter *t* and/or occurrences of the letter *t* to be replaced by the letter *p* in any known word in the calculus. By applying this rule to the word *pot* the following new words are produced: *tot*, *top*, and *pop*.

Unlike letters of an alphabet, however, shapes are defined in two, three, or more dimensions. Whereas a letter may be replaced with another letter in only one reasonable way, a shape can be replaced by another shape in an infinite number of different ways. Depending on the shapes involved, some ways of replacing these

⁽¹⁾ A Thue equivalence rule is also called an undirected substitution or Thue production.

shapes may seem more intuitive or correct. In any case, the exact location of the shape replacing another shape must be specified with respect to the Cartesian coordinate system in which the original shape is defined. This may be accomplished by defining a shape equivalence rule which indicates exact coordinates of each of the shapes occurring in the rule. When no ambiguity can arise, the locations of these shapes can be specified informally by identifying at least one corresponding fixed or distinguished point of each shape.

For example, suppose that s is a square and t is a right triangle which have the spatial relation $s+t$ ⁽²⁾ shown in figure 7(a). In figure 7(b), one possible shape equivalence rule defined in terms of the shapes s and t is shown. In this rule, the point p_1 is the distinguished point of the shapes on either side of the rule. It indicates the exact location of the two shapes with respect to each other.

A shape equivalence rule $\alpha \leftrightarrow \beta$, such as the one above, is applied to a set of shapes specifying a spatial relation. If there exists some Euclidean transformation τ such that either $\tau(\alpha)$ or $\tau(\beta)$ is identical to a shape in the spatial relation, then this shape may be replaced by $\tau(\beta)$ or $\tau(\alpha)$, respectively, to define a new spatial relation. This process may be repeated until no new, nonequivalent spatial relations can be generated from any previously generated spatial relation. All the spatial relations so defined are members of the class of spatial relations defined by the shape equivalence rule. The transformations under which a shape equivalence rule is applied may be restricted in order to limit the class of spatial relations defined by the rule.

For example, if the shape equivalence rule depicted in figure 7(b) is applied to the original or initial spatial relation $s+t$ in both directions (\leftrightarrow), four new spatial relations are produced (figure 8). (Notice that applying a shape equivalence rule in both directions (\leftrightarrow) to a spatial relation is analogous to the operation of metathesis in a word.)

Spatial relation 2 is equivalent to the initial spatial relation. It therefore defines equivalent type 1, 2, 3, and 4 shape rules and equivalent languages using these rules. Spatial relations 1, 3, and 4 are not equivalent to each other or to the initial one. Different languages are determined by each of these spatial relations using type 1, 2, 3, or 4 shape rules. If the shape equivalence rule is applied to the initial spatial relation in the direction (\rightarrow) only, four new spatial relations are generated (figure 9).

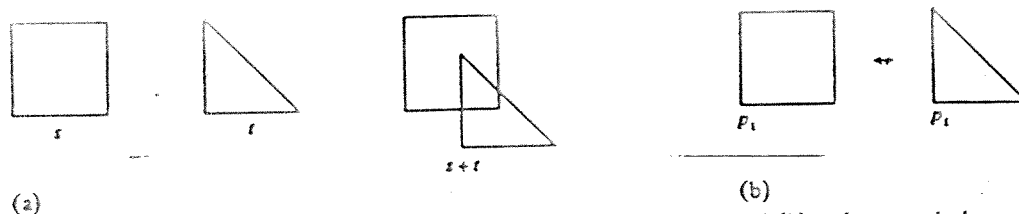


Figure 7. (a) A spatial relation between a square and a right triangle; and (b) a shape equivalence rule defined in terms of the shapes in the spatial relation shown in (a).

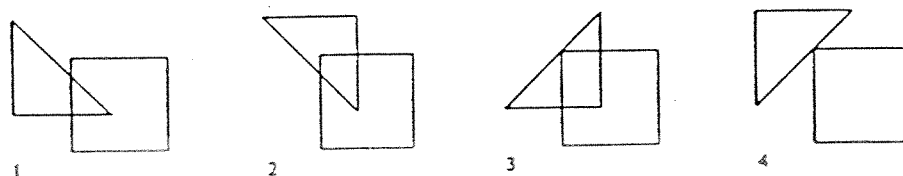


Figure 8. Spatial relations produced by applying the shape equivalence rule given in figure 7(b) to the spatial relation shown in figure 7(a) in both directions (\leftrightarrow).

(2) A spatial relation is sometimes denoted informally in this paper by the shape union of the shapes in the set which specifies it.

None of these spatial relations are equivalent to each other or to the initial spatial relation. They therefore determine different languages. The shape equivalence rule applied to the initial spatial relation in the direction (\leftrightarrow) produces only one new spatial relation (figure 10) which determines yet another different language.

The shape equivalence rule can now be applied, in any direction, to any of the spatial relations produced from the initial one to define still more spatial relations. All of the possible new, nonequivalent spatial relations which can be derived from previously generated ones are illustrated in figure 11. All are produced by recursively applying the shape equivalence rule either to the spatial relations shown in figure 8, the spatial relations shown in figure 9 or the spatial relation shown in figure 10. In figure 12, the complete class of spatial relations generated by the shape equivalence rule is shown. Shapes in the languages produced by use of type 1 shape rules defined from each of these spatial relations are shown in figure 13. For the sake of simplicity, only type 1 shape rules are used to generate the designs shown in this example and following ones.

The precise manner in which shape equivalence rules are defined and applied to spatial relations is given in the following formal definitions:

A shape equivalence rule is a rule of the form $\alpha \leftrightarrow \beta$ where α and β are any shapes not identical to the empty shape s_0 .

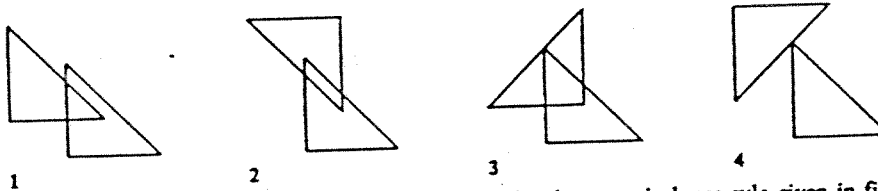


Figure 9. Spatial relations produced by applying the shape equivalence rule given in figure 7(b) to the spatial relation shown in figure 7(a) in the direction (\leftrightarrow).

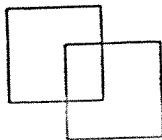


Figure 10. The spatial relation produced by applying the shape equivalence rule given in figure 7(b) to the spatial relation shown in figure 7(a) in the direction (\leftrightarrow).

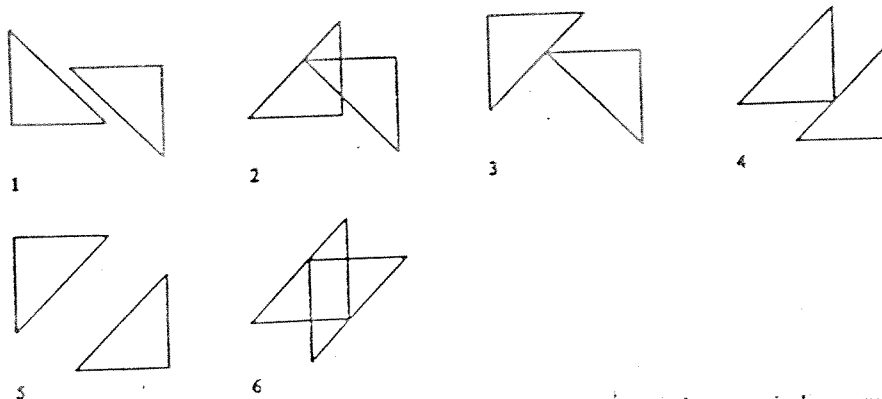


Figure 11. Spatial relations produced by recursively applying the shape equivalence rule given in figure 7(b) to the spatial relations shown in figures 8, 9, or 10.

Given a spatial relation S , specified by a finite set of shapes, new spatial relations may be defined from it by applying an appropriate shape equivalence rule to S .

A shape equivalence rule $\alpha \leftrightarrow \beta$ is applicable to a spatial relation S if and only if there exists a shape $s \in S$ and a Euclidean transformation τ such that either $\tau(\alpha) = s$ or $\tau(\beta) = s$.

If a shape equivalence rule is applicable to a spatial relation S , a new spatial relation S' may be produced by replacing s with $\tau(\beta)$ if $\tau(\alpha) = s$ or by replacing s with $\tau(\alpha)$ if $\tau(\beta) = s$.

The new spatial relation S' is specified by the set:

$$[S - \{\tau(\alpha)\}] \cup \{\tau(\beta)\} \quad \text{if } \tau(\alpha) = s,$$

or

$$[S - \{\tau(\beta)\}] \cup \{\tau(\alpha)\} \quad \text{if } \tau(\beta) = s.$$

As noted previously, one shape can be replaced by another in an infinite number of different ways. This implies that for any two shapes an infinite number of different shape equivalence rules can be defined in terms of them. In order to restrict the overabundant number of shape equivalence rules definable in principle from any two shapes, the following heuristic is suggested for defining these rules. It includes a variety of ways of replacing one shape with another which seem reasonable or intuitive and which produce interesting results as well as other ways which are not as obvious or easily conceived:

Locate one shape with respect to the other so that a maximal line of one shape overlaps a maximal line of the other shape and an endpoint of one of these maximal

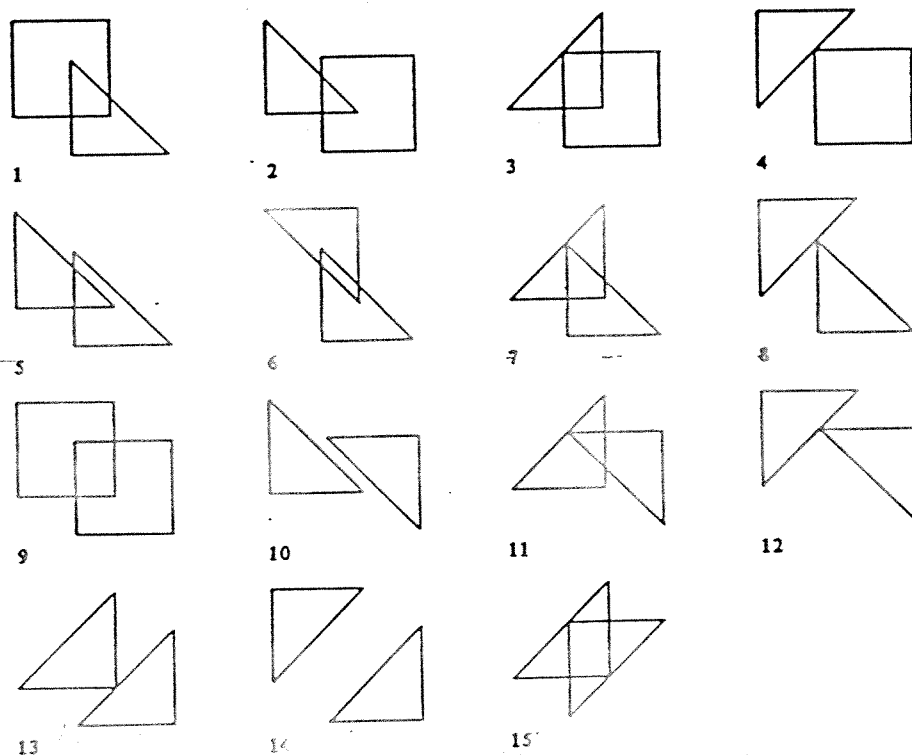
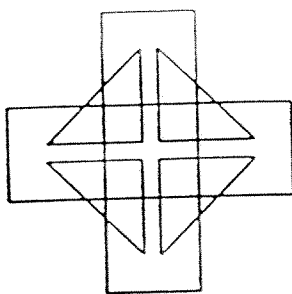
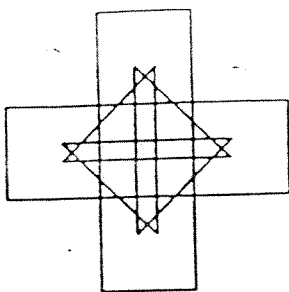


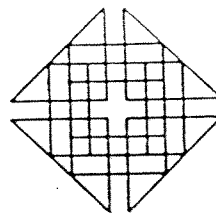
Figure 12. The complete class of spatial relations defined by recursively applying the shape equivalence rule given in figure 7(b) to the given or *initial* spatial relation shown in figure 7(a).



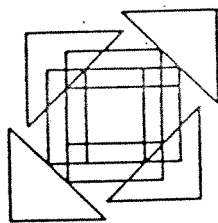
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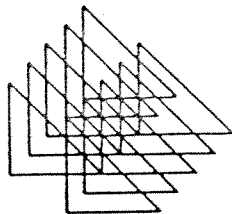
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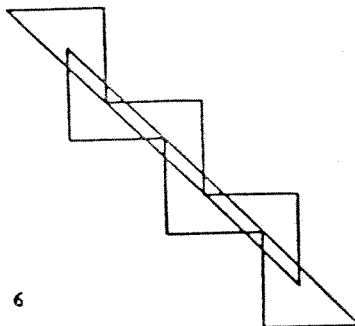
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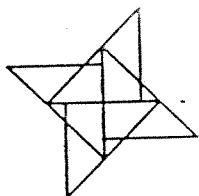
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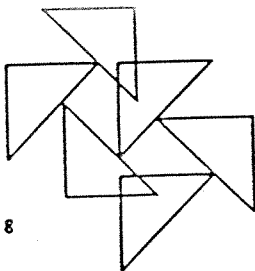
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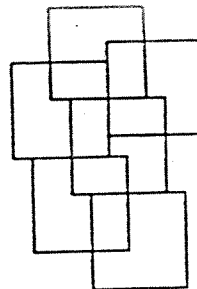
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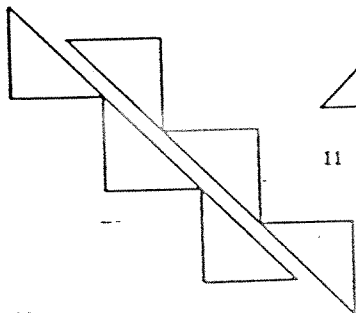
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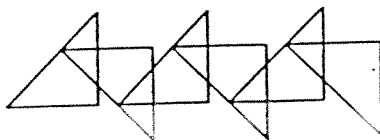
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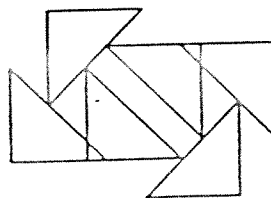
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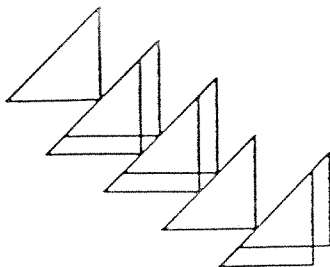
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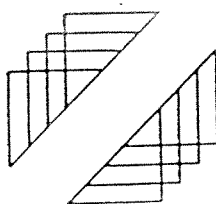
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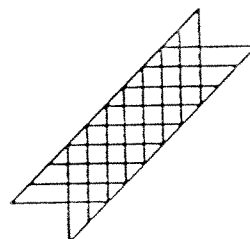
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14



15

Figure 13. A design in each of the languages produced using type 1 shape rules defined from each of the spatial relations shown in figure 12. The number given beneath each design corresponds to the number of the spatial relation used to construct it.

lines is coincident with an endpoint of the other maximal line. The shared endpoint in each shape is the distinguished point in the shape equivalence rule which indicates the location of the two shapes with respect to each other.

Defining a shape equivalence rule in this manner allows the rule to be easily applied. One shape is replaced by another by using a maximal line and its endpoint in one shape as a guide for locating the new shape.

Shape equivalence rule schemata

Consider now another example of a spatial relation $s + t$ where s is a rectangle and t is a trapezoid [figure 14(a)]. One shape equivalence rule defined in terms of the shapes s and t according to the previously given heuristic is shown in figure 14(b). In figure 15, some of the spatial relations in the infinite class of spatial relations defined by applying this shape equivalence rule to the initial spatial relation are given.

Although the new spatial relations are interesting, they appear to have no immediate relationship to the original spatial relation other than that they consist of the same shapes. One might expect that given the spatial relation $s + t$ which consists of a trapezoid embedded in a rectangle one could at least derive the inverse relation—a rectangle embedded in a trapezoid. However, no one shape equivalence rule will produce this spatial relation when applied to the initial one. What is needed is a more general version of a shape equivalence rule which allows any rectangle to be replaced with a trapezoid of commensurate size and vice versa. This generalized version of a shape replacement operation can be specified by a *shape equivalence rule schema*.

A *shape equivalence rule schema* $\alpha \leftrightarrow \beta$ consists of parameterized shapes α and β . Like a shape equivalence rule, it is applied recursively to a set of shapes specifying a spatial relation to define a class of spatial relations. A shape equivalence rule schema

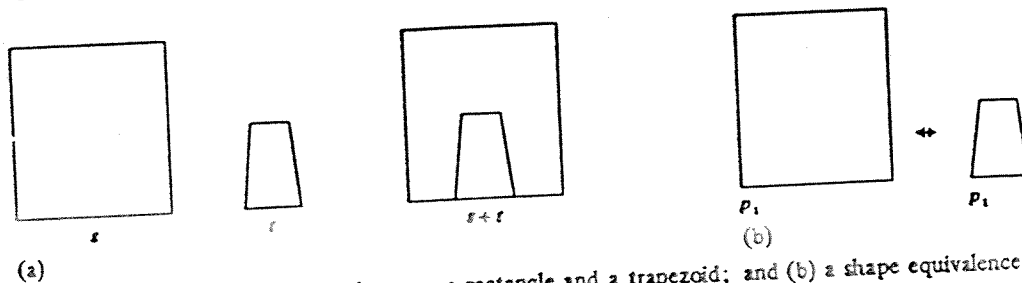


Figure 14. (a) A spatial relation between a rectangle and a trapezoid; and (b) a shape equivalence rule defined in terms of the shapes in the spatial relation depicted in (a).

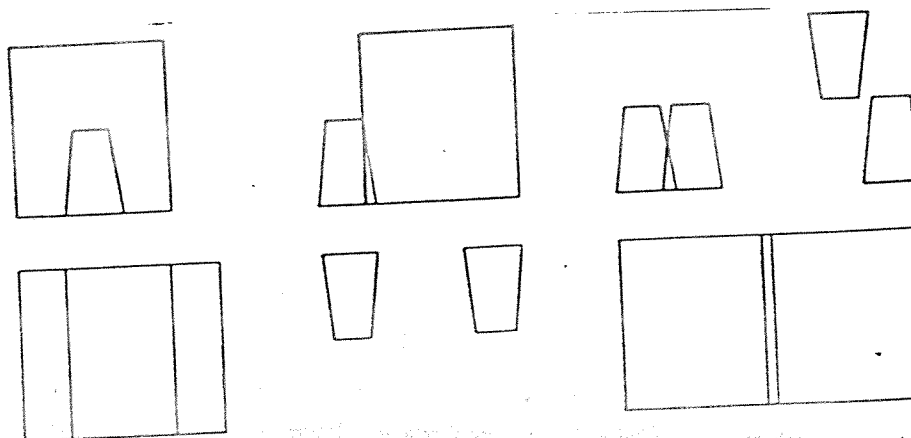


Figure 15. Some members of the infinite class of spatial relations generated by recursively applying the shape equivalence rule given in figure 14(b) to the spatial relation shown in figure 14(a).

applies to a spatial relation when there exists an assignment g to all variables in α and β and a Euclidean transformation τ such that either $\tau[g(\alpha)]$ or $\tau[g(\beta)]$ is identical to a shape in the spatial relation. This shape may then be replaced by $\tau[g(\beta)]$ or $\tau[g(\alpha)]$, respectively, to define a new spatial relation. The transformations under which a shape equivalence rule schema is applied may be restricted in order to limit the spatial relations defined by the schema. The parameters in a shape equivalence rule schema specify the family of shapes to which α and β belong. They also indicate corresponding points of α and β which show how these shapes are located with respect to one another.

For example, a shape equivalence rule schema which replaces any rectangle with a trapezoid and vice versa is depicted in figure 16. Values assigned to the parameters in this schema must satisfy the following conditions:

- (1) The points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and (x_4, y_4) are the vertices of a rectangle such that the distance between (x_1, y_1) and (x_4, y_4) or between (x_2, y_2) and (x_3, y_3) is greater than or equal to the distance between (x_1, y_1) and (x_2, y_2) or between (x_3, y_3) and (x_4, y_4) .
- (2) The points (x_1, y_1) , (x_2, y_2) , (x_5, y_5) , and (x_6, y_6) are the vertices of a trapezoid such that the points (x_5, y_5) and (x_6, y_6) are coincident with the line with endpoints (x_3, y_3) and (x_4, y_4) and are equidistant from the endpoints of this line.

Some members of the class of spatial relations defined by applying this shape equivalence rule schema to the original spatial relation $s + t$ are shown in figure 17. Additional spatial relations, similar to those shown, may be defined by assigning different values to the variables associated with the trapezoid. A spatial relation in this class consists of either a trapezoid embedded in a rectangle, a rectangle embedded in a trapezoid, a trapezoid embedded in a trapezoid, or a rectangle embedded in a rectangle. All are intuitively sensible derivatives of the initial spatial relation. Notice that the spatial relations shown in figures 5 and 6 consisting of rectangles and round arches can be derived by applying a shape equivalence rule schema similar to the one given in figure 16.

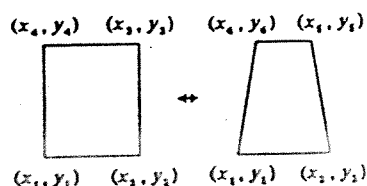


Figure 16. A shape equivalence rule schema which replaces a rectangle with a trapezoid and vice versa.

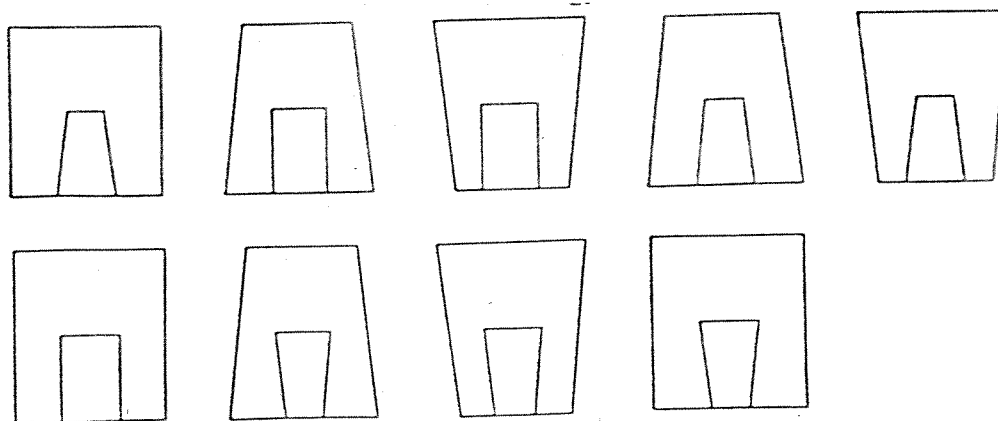


Figure 17. Some members of the infinite class of spatial relations defined by recursively applying the shape equivalence rule schema given in figure 16 to the spatial relation depicted in figure 14(x).

A useful heuristic for defining a shape equivalence rule schema in terms of two shapes in a spatial relation is similar to the one given for defining a shape equivalence rule:

Define the family of shapes to which each shape belongs. This entails parameterizing each of the shapes. Choose a representative member of each family. Locate one with respect to the other so that a maximal line in one shape overlaps a maximal line in the other shape and every endpoint of a maximal line in one shape is coincident with a point in the other shape. (For example, if the two shapes are polygons, this would require inscribing one shape inside the other.)

As with shape equivalence rules, there will often be more than one way to define a shape equivalence rule schema in terms of two shapes which is in accordance with the above heuristic. Each of the shape equivalence rule schemata so defined will usually generate a different class of spatial relations when applied to the initial spatial relation.

The precise way in which shape equivalence rule schemata are defined and applied to spatial relations is given in the following formal definitions:

A shape equivalence rule schema is a rule schema of the form $\alpha \leftrightarrow \beta$ where α and β are parameterized shapes and no member of the family of shapes specified by α or β is identical to the empty shape s_0 .

A shape equivalence rule schema $\alpha \leftrightarrow \beta$ is applicable to a spatial relation S if and only if there exists a shape $s \in S$, an assignment g to all the variables in α and β , and a Euclidean transformation τ such that either $\tau[g(\alpha)] = s$ or $\tau[g(\beta)] = s$.

If a shape equivalence rule schema is applicable to S , a new spatial relation S' may be produced by replacing s with $\tau[g(\beta)]$ if $\tau[g(\alpha)] = s$ or by replacing s with $\tau[g(\alpha)]$ if $\tau[g(\beta)] = s$.

The new spatial relation S' is specified by the set

$$[S - \{\tau[g(\alpha)]\}] \cup \{\tau[g(\beta)]\} \quad \text{if } \tau[g(\alpha)] = s,$$

or

$$[S - \{\tau[g(\beta)]\}] \cup \{\tau[g(\alpha)]\} \quad \text{if } \tau[g(\beta)] = s.$$

Shape equivalence rule schemata are more or less appropriate than shape equivalence rules as a means for defining a class of spatial relations depending on the shapes in the initial spatial relation. If the spatial relation consists of shapes which are not easily inscribed within one another or which differ greatly in area or form, it may be more expedient to redefine these shapes in terms of families of shapes. These parameterized shapes can then be used to specify a shape equivalence rule schema applicable to the spatial relation.

Shape replacement using new shapes

The concept of shape replacement in spatial relations can be generalized by allowing it to include the replacement of shapes in a given spatial relation with a new shape not in the spatial relation. Here, a new shape is understood to mean any shape not congruent to a shape in the given spatial relation. The choice of this new shape is arbitrary and can be determined in a number of ways. It may, for example, be created from scratch, selected from a host of other known shapes, borrowed from the vocabulary of another design language, or perceived as a subshape of other known shapes or designs.

Consider, for instance, the spatial relations shown in figure 18. All are prevalent in Romanesque and other styles of design. If one or more of the round arches in each of these spatial relations is replaced with a new shape, a pointed arch⁽³⁾, new spatial

(3) The origin of the pointed arch is a perplexing issue of considerable interest to architectural historians. One theory is that the shape was derived from the intersection of two round arches as shown in spatial relation 4 of figure 18.

relations are defined. Figure 19 shows some of the new spatial relations produced by this replacement operation. These new spatial relations together with the original ones are typical of Gothic design. The shape replacement operation represents, in a very elementary way, the transition between spatial elements used in Romanesque design and spatial elements used in Gothic design.

This shape replacement operation, which replaces a shape in a spatial relation with a shape which is not in the spatial relation, can be specified by a shape equivalence rule or rule schema. The shape equivalence rule or rule schema is defined and applied in exact accordance with the formal definitions given in the preceding sections. The heuristics given may also be used to determine how the shapes in the rule or rule schema replace one another.

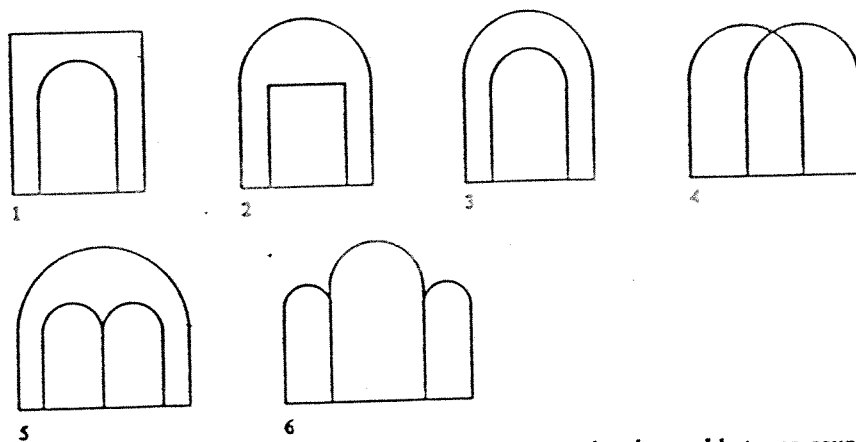


Figure 18. Spatial relations between rectangles and round arches and between round arches and round arches.

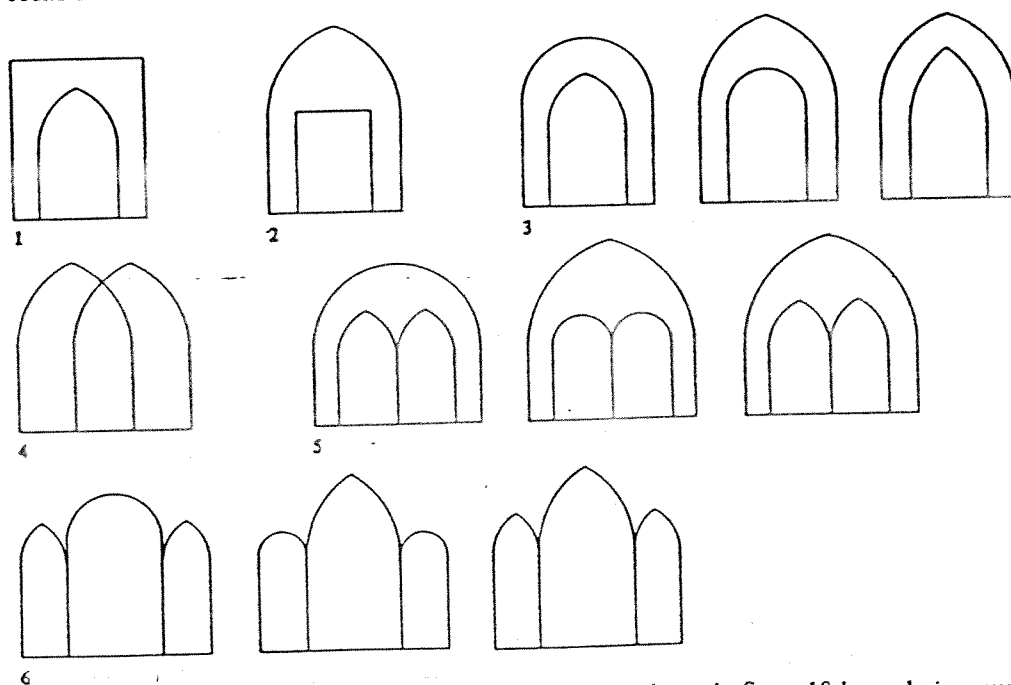


Figure 19. Spatial relations derived from the spatial relations shown in figure 18 by replacing round arches with pointed arches. The numbers given beneath these spatial relations correspond to the numbers of the spatial relations from which they are derived.

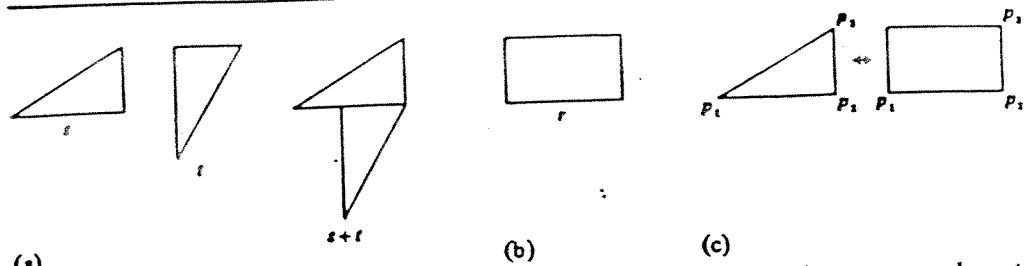


Figure 20. (a) A spatial relation between two right triangles; and (b) a new shape, a rectangle, not in the spatial relation illustrated in (a); and (c) a shape equivalence rule defined in terms of a right triangle and a rectangle.

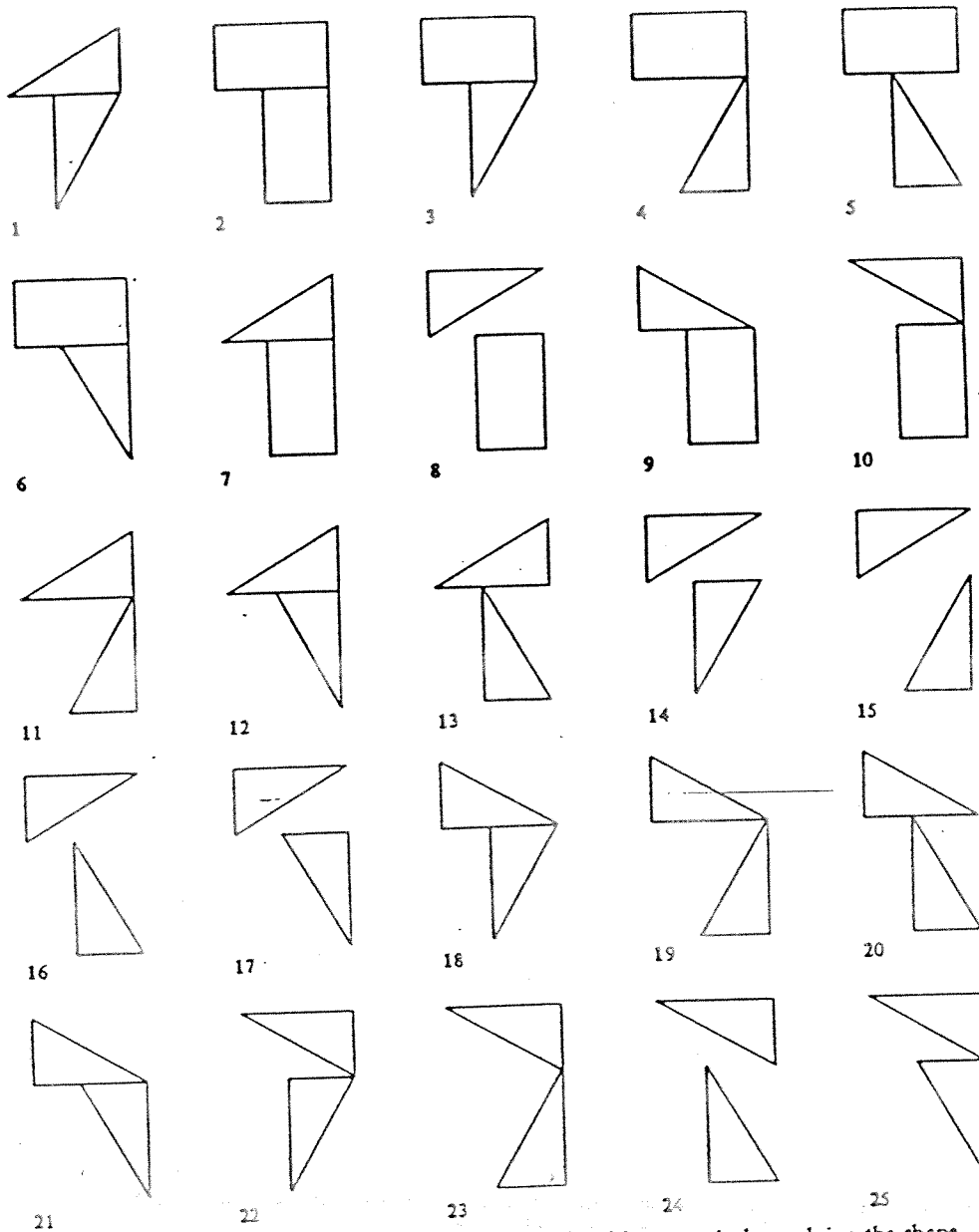
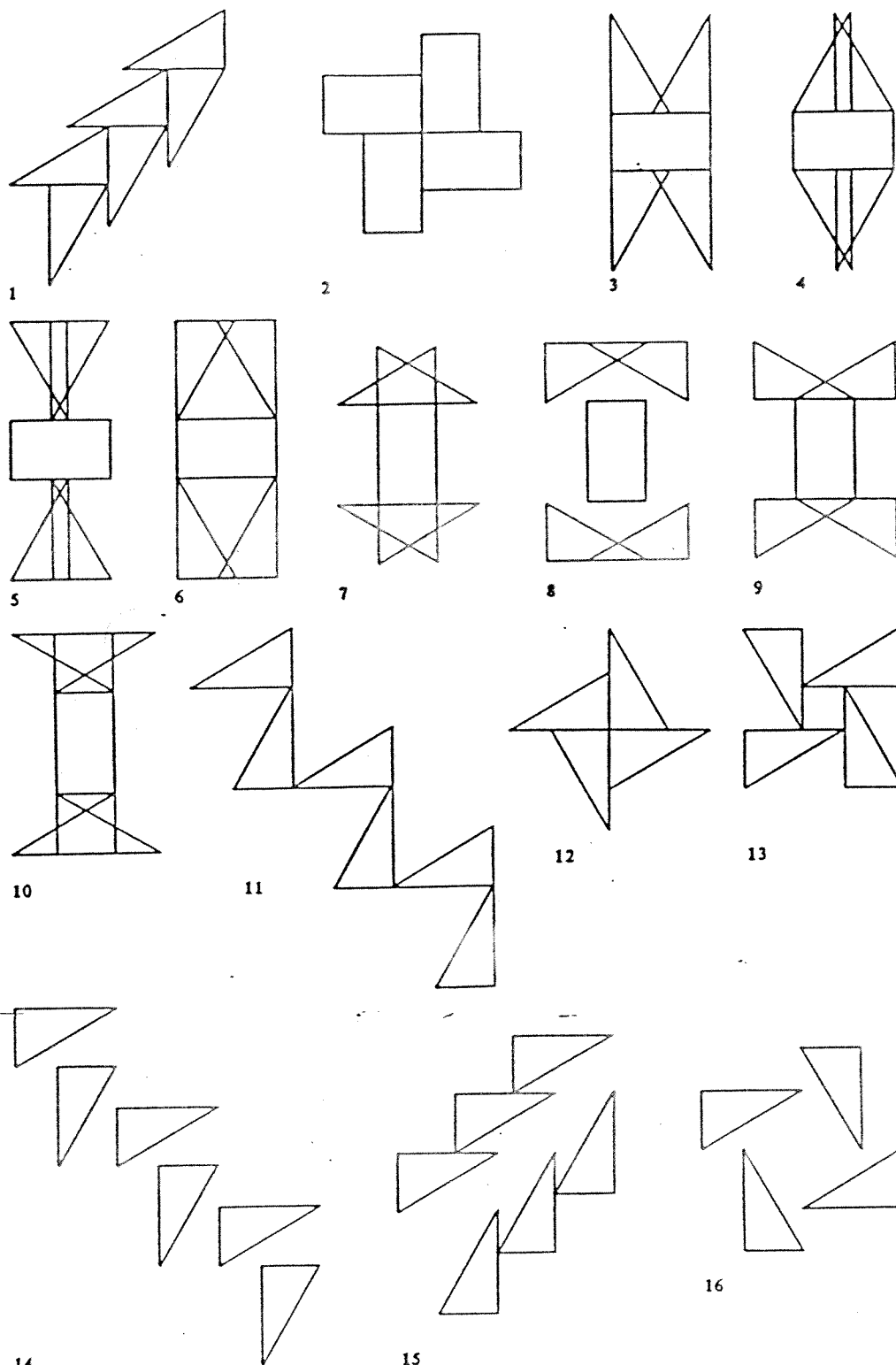


Figure 21. The complete class of spatial relations defined by recursively applying the shape equivalence rule given in figure 20(c) to the spatial relation shown in figure 20(a).



14

15

16

Figure 22. A design in each of the languages produced by means of type 1 shape rules defined from each of the spatial relations shown in figure 21. The number given beneath each design corresponds to the number of the spatial relation used to construct it.

For example, suppose that s and t are right triangles which have the spatial relation shown in figure 20(a). Let r be a new shape, a rectangle [figure 20(b)] not in this spatial relation. One possible shape equivalence rule defined in terms of either of the shapes s or t and the new shape r is given in figure 20(c). The points p_1 , p_2 , and p_3 are the distinguished points of the shapes on either side of the rule. Usually, when one of the shapes in a shape equivalence rule or rule schema is not a shape in the initial spatial relation, more than one distinguished point needs to be given in order to define the new shape completely and unambiguously. In figure 21, the complete class of spatial relations generated by applying this shape equivalence rule to the initial spatial relation is shown. A shape in each of the languages produced using type 1 shape rules defined from each of the spatial relations is shown in figure 22.

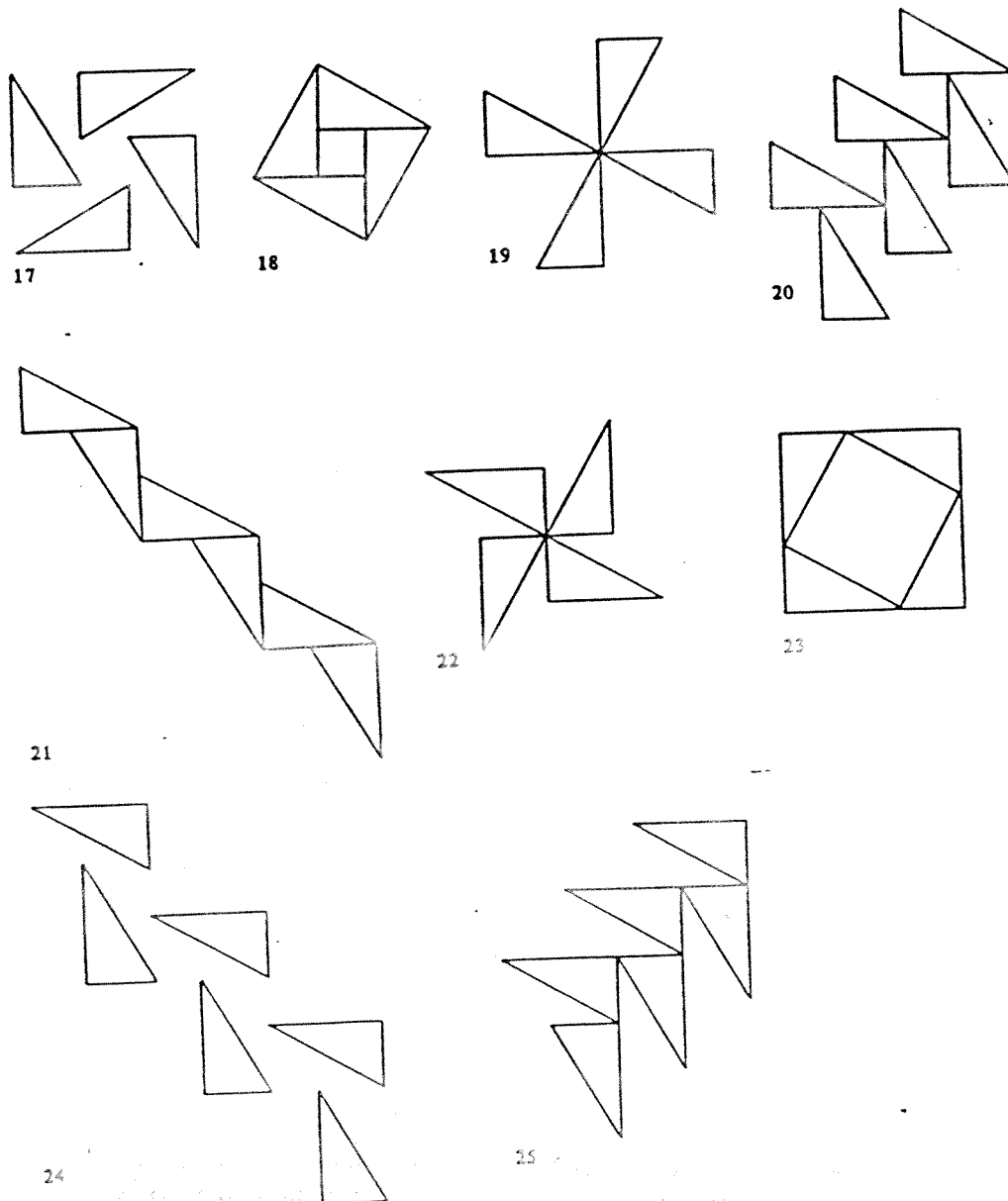


Figure 22 (continued)

Classes of spatial relations

A shape equivalence rule or rule schema applied recursively to a spatial relation generates a class of spatial relations. Each spatial relation in the class can be derived from any other spatial relation in the class by applying the shape equivalence rule or rule schema to any of these spatial relations a finite number of times.

Given a spatial relation between two shapes s and t , a shape equivalence rule defined in terms of these shapes generates a class of spatial relations between $\tau(s)$ and $\psi(t)$, $\tau(s)$ and $\psi(s)$, and $\tau(t)$ and $\psi(t)$, where τ and ψ are Euclidean transformations. A shape equivalence rule schema defined in terms of these shapes generates a class of spatial relations between $\tau[g(s)]$ and $\psi[h(t)]$, $\tau[g(s)]$ and $\psi[h(s)]$, and $\tau[g(t)]$ and $\psi[h(t)]$, where τ and ψ are Euclidean transformations and g and h are assignments to all of the variables in the shape equivalence rule schema.

A shape equivalence rule defined in terms of a shape s in a given spatial relation between two shapes s and t and a new shape r generates a class of spatial relations which contains, at least, the given spatial relation and a spatial relation between $\tau(r)$ and t . A shape equivalence rule schema defined in this way generates a class of spatial relations which contains at least, the given spatial relation and a spatial relation between $\tau[g(r)]$ and t . Depending on various formal properties of the shapes in the given spatial relation and shape equivalence rule or rule schema, additional spatial relations between $\tau(r)$ and $\psi(s)$, $\tau(r)$ and $\psi(r)$, and $\tau(s)$ and $\psi(t)$ or between $\tau[g(r)]$ and $\psi[h(s)]$, $\tau[g(r)]$ and $\psi[h(r)]$, and $\tau[g(s)]$ and $\psi[h(t)]$ may be generated.

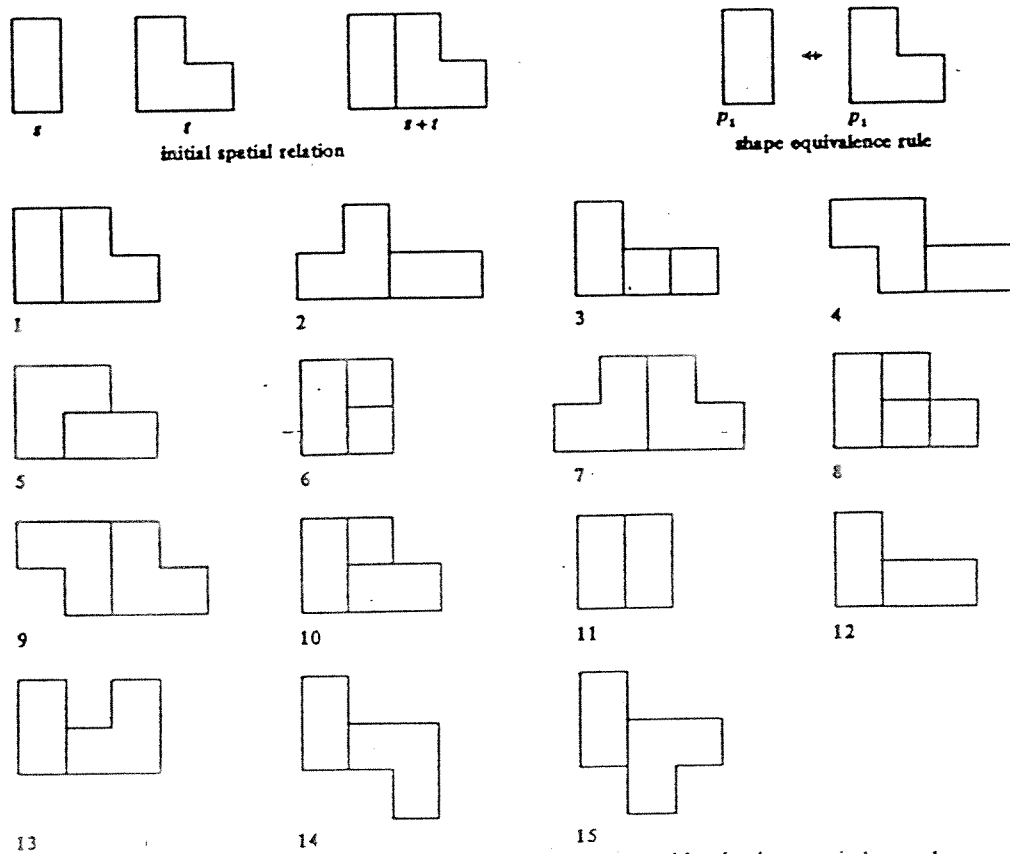
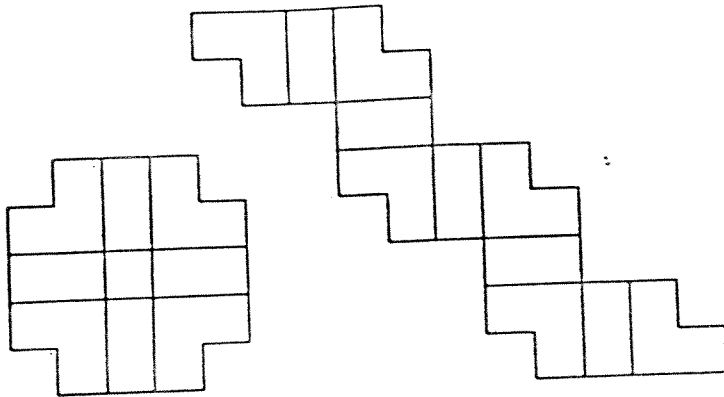
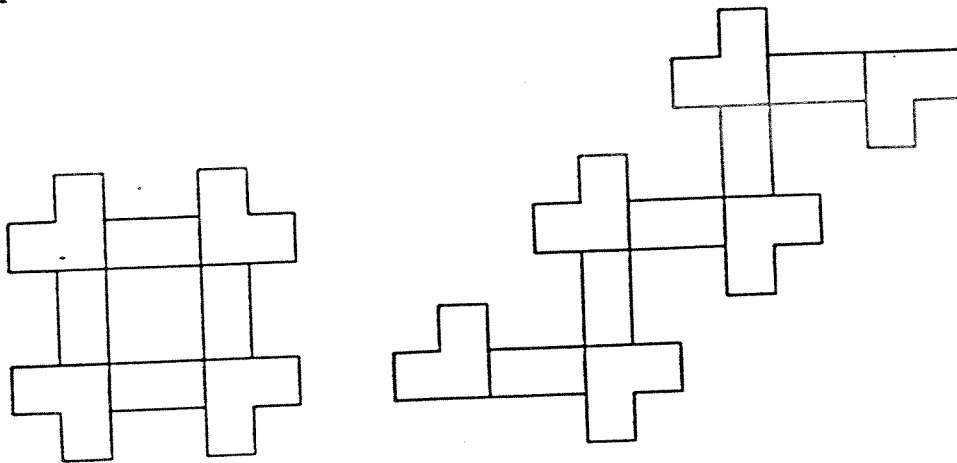


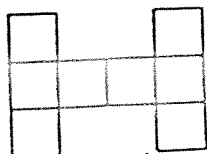
Figure 23. An example of a shape equivalence rule which generates an infinite class of spatial relations when applied to the initial spatial relation.



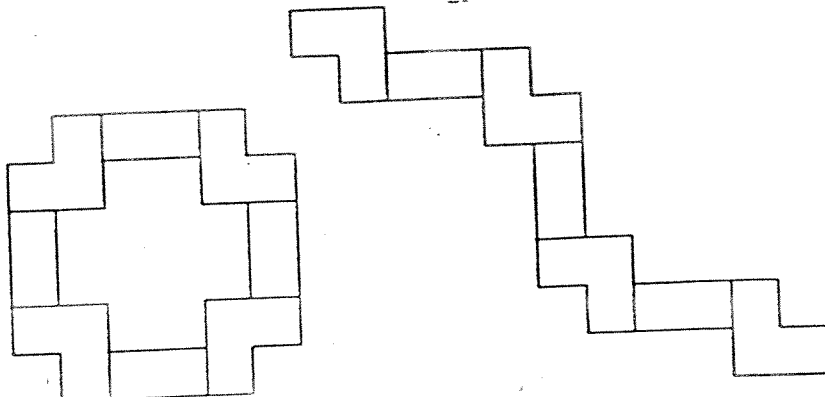
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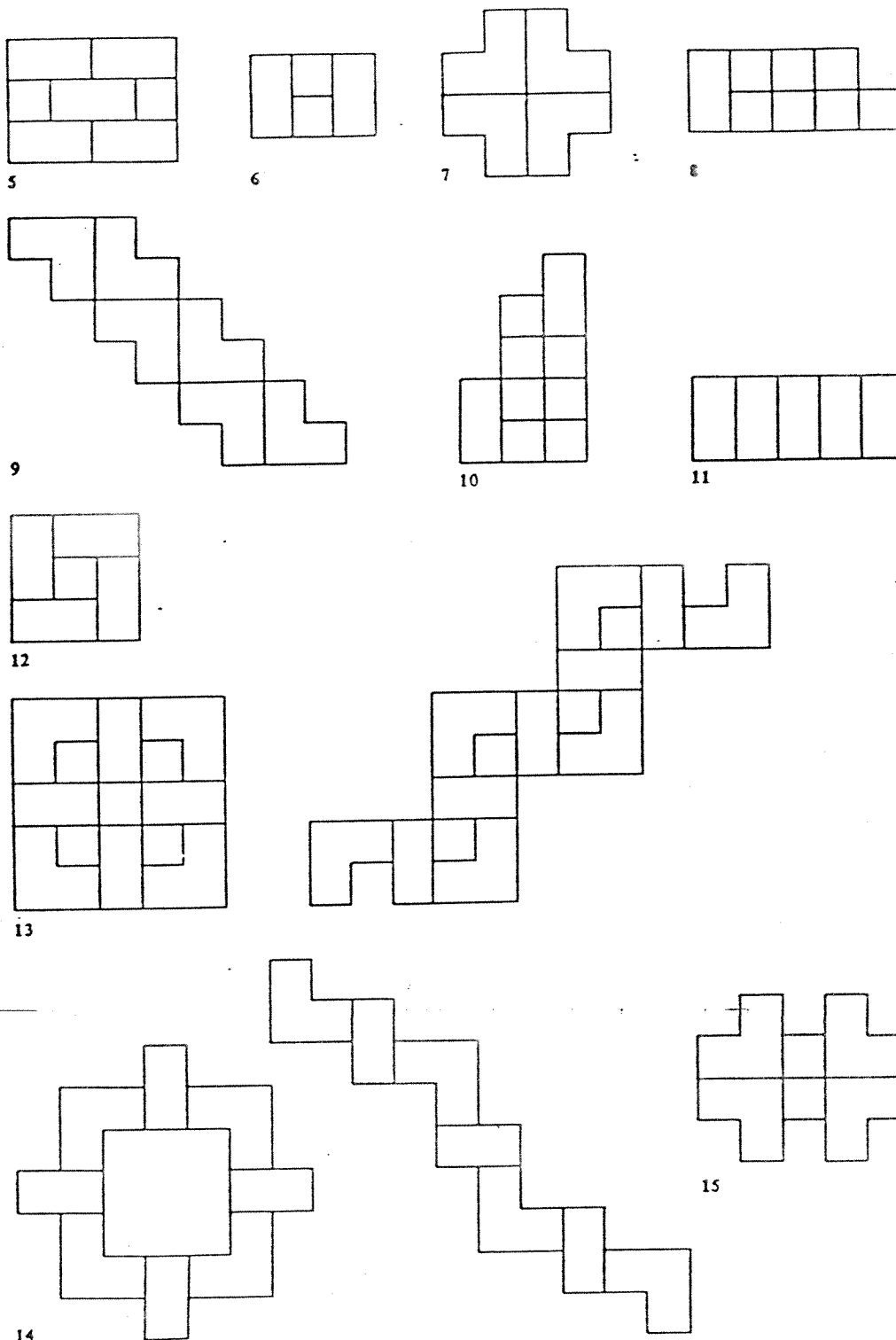


3



4

Figure 23 (continued)



some designs in each of the languages determined by type 1 shape rules defined from each of the spatial relations above—the numbers given beneath these designs correspond to the numbers of the spatial relations used to construct them

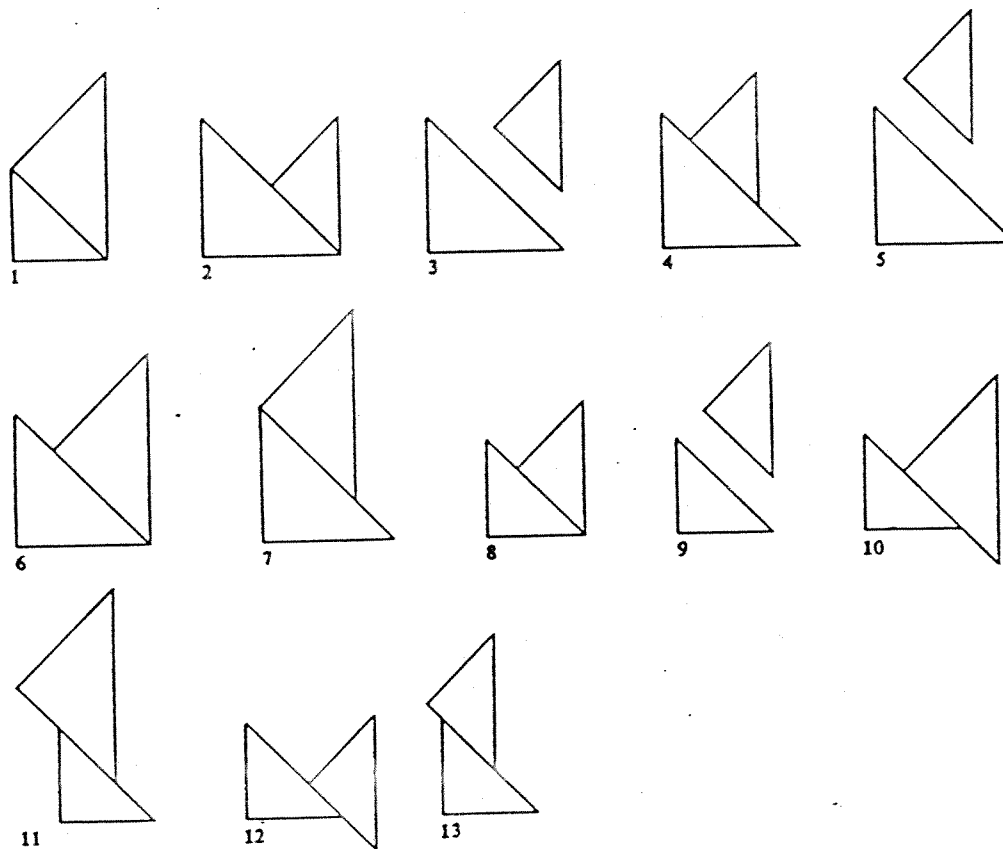
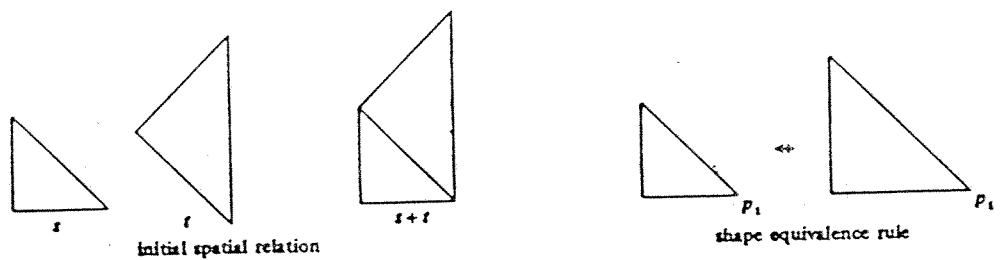
Figure 23 (continued)

A class of spatial relations may be finite or infinite in extent. Again, this depends on various formal characteristics of the initial spatial relation, the shape equivalence rule or rule schema, and the shapes occurring in them. The shape equivalence rules and rule schemata given in the examples in the preceding sections generated both finite and infinite classes of spatial relations. In figures 23-25, three additional examples of shape equivalence rules which generate infinite classes of spatial relations are given.

In the first example (figure 23), the initial spatial relation consists of a rectangle and an L shape. Fifteen members of the infinite class of spatial relations generated by a shape equivalence rule defined in terms of the shapes in the initial spatial relation are shown. Spatial relations 2-6 are generated by applying the shape equivalence rule to the initial spatial relation in both directions (\leftrightarrow). Spatial relations 7-10 are generated by applying the shape equivalence rule to the initial spatial relation in the direction (\rightarrow). Spatial relations 11 and 12 are generated by applying the shape equivalence rule to the initial spatial relation in the direction (\leftarrow). Spatial relations 13, 14, and 15 are generated by applying the shape equivalence rule to spatial relation 12 in the direction (\rightarrow). An infinite number of new spatial relations may be produced by applying the shape equivalence rule to spatial relations 13, 14, and 15 and to spatial relations produced from them. Some members of each of the languages determined by type 1 shape rules defined from each of the fifteen spatial relations are also given.

In the second example (figure 24), the initial spatial relation consists of similar right triangles. Thirteen members of the infinite class of spatial relations generated by a shape equivalence rule defined in terms of these shapes are shown. Spatial relations 2-5 are generated by applying the shape equivalence rule to the initial spatial relation in both directions (\leftrightarrow). Spatial relations 6 and 7 are generated by applying the shape equivalence rule to the initial spatial relation in the direction (\rightarrow). Spatial relations 8 and 9 are generated by applying the shape equivalence rule to the initial spatial relation in the direction (\leftarrow). Spatial relations 10 and 11 are generated by applying the shape equivalence rule to spatial relations 6 and 7, respectively, in the direction (\leftarrow). Spatial relations 12 and 13 are generated by applying the shape equivalence rule to spatial relations 10 and 11, respectively, in the direction (\leftarrow). Infinitely more spatial relations can be produced by applying the shape equivalence rule in different ways to spatial relations 2-13 and to spatial relations produced from them. A member of each of the languages determined by type 1 shape rules defined from each of the thirteen spatial relations shown, is also given.

In the third example (figure 25), the initial spatial relation consists of a rhombus and a square. A shape equivalence rule is defined in terms of the rhombus and a new shape, an equilateral triangle. Six of the spatial relations in the infinite class of spatial relations defined by the shape equivalence rule are shown. Spatial relations 2 and 3 are generated by applying the shape equivalence rule to the initial spatial relation in the direction (\rightarrow). Spatial relation 4 is generated by applying the shape equivalence rule to spatial relation 2 in the direction (\leftarrow). Spatial relations 5 and 6 are generated by applying the shape equivalence rule to spatial relation 3 in the direction (\leftarrow). An infinite number of new spatial relations can be generated by applying the shape equivalence rule to spatial relations 5 and 6 and to spatial relations produced from them. A design in each of the languages generated by type 1 shape rules defined from each of the six spatial relations is also shown.



some members of the infinite class of spatial relations generated by the shape equivalence rule

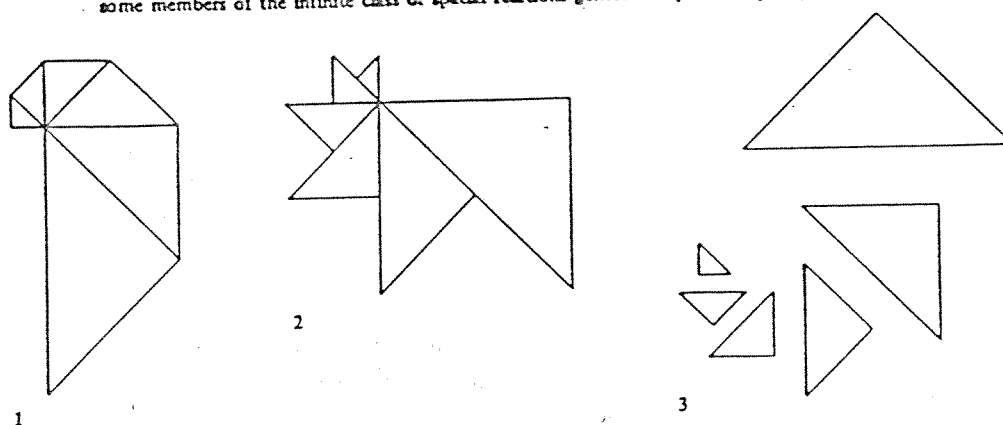
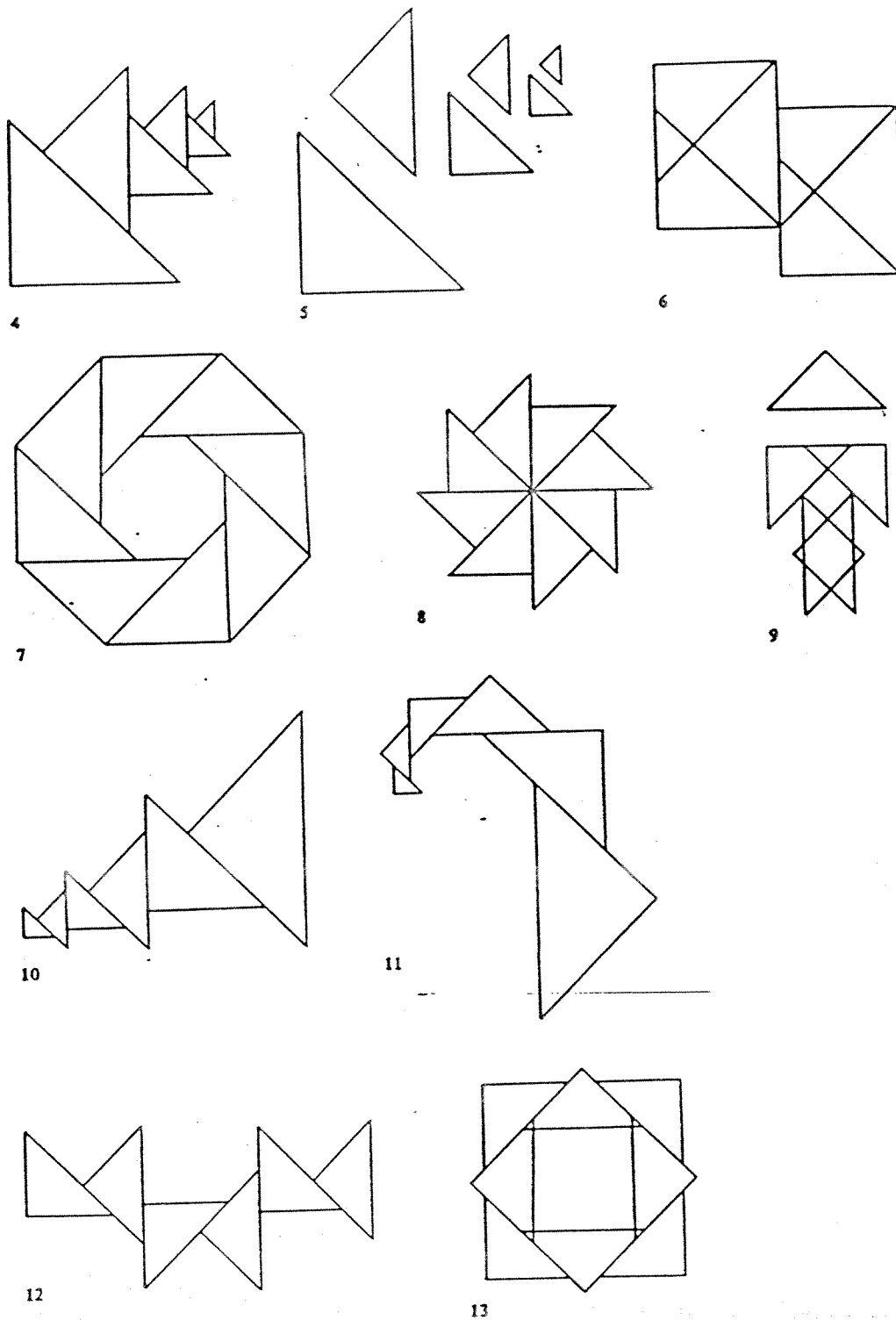
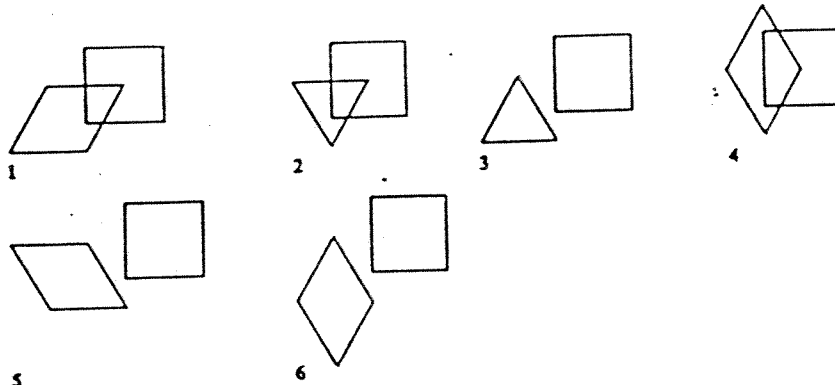
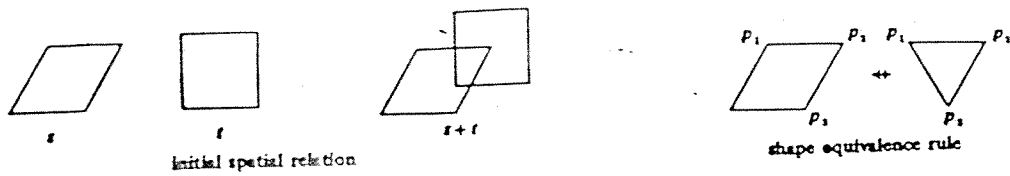


Figure 24. An example of a shape equivalence rule which generates an infinite class of spatial relations when applied to the initial spatial relation.

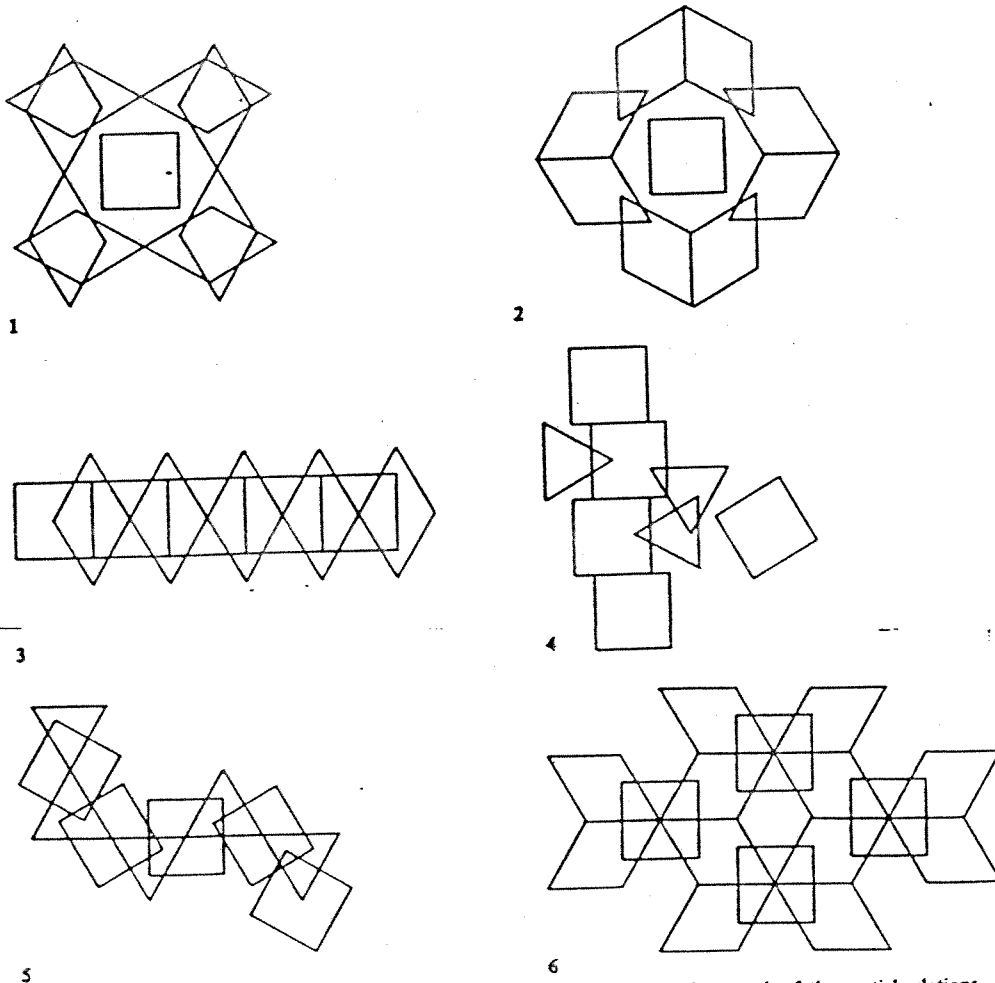


a design in each of the languages determined by type 1 shape rules defined from each of the spatial relations above—the number given beneath each design corresponds to the number of the spatial relation used to construct it

Figure 24 (continued)



some members of the infinite class of spatial relations generated by the shape equivalence rule



a design in each of the languages determined by type 1 shape rules defined from each of the spatial relations above—the number given beneath each design corresponds to the number of the spatial relation used to construct it

Figure 25. An example of a shape equivalence rule which generates an infinite class of spatial relations when applied to the initial spatial relation.

Supplementary remarks and conclusions

In the preceding sections, a simple method for defining new spatial relations from a given one was presented. It consists, basically, of replacing one or more shapes in the given spatial relation with another shape which may or may not be in the given spatial relation. This shape replacement operation can be specified precisely by a shape equivalence rule or rule schema. Heuristics were suggested which enable one to easily define these rules or rule schemata so that they are simple to apply and produce interesting results.

A shape equivalence rule or rule schema recursively defines a class of spatial relations. For the sake of simplicity, each of the classes of spatial relations given in the examples were determined by one shape equivalence rule or rule schema; more than one shape equivalence rule or rule schema can be applied to a given spatial relation to determine a larger, more diverse class of spatial relations. The spatial relations in this class can be used to define a wide variety of new languages of designs.

Although all of the examples of spatial relations were specified by pairs of two-dimensional shapes, the methods and results given here apply to spatial relations containing any number of shapes defined in two, three, or more dimensions. When a given spatial relation consists of more than two shapes, a shape equivalence rule or rule schema can be specified in terms of any two shapes in the spatial relation or in terms of any one shape in the relation and a new shape. This rule or rule schema is applied to the given spatial relation in the usual way. If the shapes in the given spatial relation are defined in more than two dimensions, shape equivalence rules or rule schemata are easily specified by means of the same heuristics used for two-dimensional shapes. These rules or rule schemata are applied in the same way that rules or rule schemata consisting of two-dimensional shapes are applied.

The methods proposed here are also easily extended to apply to spatial relations between parameterized shapes. These more general types of spatial relations are particularly useful in the description of existing design languages in which the proportions of different manifestations of a recurring spatial relation are often not geometrically similar. For example, the spatial relation between a rectangle and arch (figure 5) occurs throughout Western architecture; the relative dimensions of the shapes in this spatial relation, however, vary in separate instances of it. If a spatial relation is specified by a set of parameterized shapes, each assignment of values to the variables associated with these shapes defines a spatial relation between non-parameterized shapes. A shape equivalence rule or rule schema may be applied in the usual way to each of the spatial relations so defined.

It should be clear then that shape equivalence rules or rule schemata are a powerful tool for creating new and original languages of designs. Instead of attempting to create new designs in a random or ad hoc manner which may be time-consuming and unproductive, a designer can employ these rules or rule schemata to construct quickly and effortlessly not just a single design or language of designs, but a multiplicity of new languages of designs. Because shape equivalence rules or rule schemata define new languages of designs by changing existing ones in terms of the most basic elements used to construct and understand them—spatial relations—and because they make explicit these changes, they enable the designer to understand and control the languages of designs produced by them. In this context, therefore, creativity or inventiveness is not an abstruse, unconscious process, but an informed and deliberate manipulation of known or given information.

Shape equivalence rules or rule schemata and the classes they define are not only a means for producing new languages of designs; they are also the basis for a potentially rigorous and substantial analysis of the formal composition of historic styles or

languages of designs in both diachronic and synchronic terms. In the former, the spatial elements of styles as they change over a period of time are characterized. In the latter, the various spatial elements of a single style, irrespective of historical antecedents, are characterized. The feasibility of using shape equivalence rules or rule schemata for modelling the relationships between spatial elements of a single style as well as the transition between spatial elements of different styles was illustrated in a very rudimentary way for Romanesque and Gothic design.

Devices which delimit a class of spatial relations determined by a shape equivalence rule or rule schema may be of significant value in any such analysis or description. One way of regulating the application of shape equivalence rules or rule schemata and the spatial relations defined by them was suggested previously. It entails restricting the transformations under which shape equivalence rules or rule schemata are applied to spatial relations. A class of spatial relations can also be restricted by associating labels with shapes in the initial spatial relation and shape equivalence rule or rule schema. Shape equivalence rules or rule schemata containing labelled shapes are applied to a spatial relation containing labelled shapes according to the formal definitions given in preceding sections. Some shapes in the spatial relations so defined may have labels associated with them; these labels can be erased by specifying rules which do so. Further research into the possible applications of shape equivalence rules and rule schemata as a descriptive and explanatory tool in stylistic analysis should prove fruitful.

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