48-747 Shape Grammars

Forming New Languages from Old

Spatial Metathesis

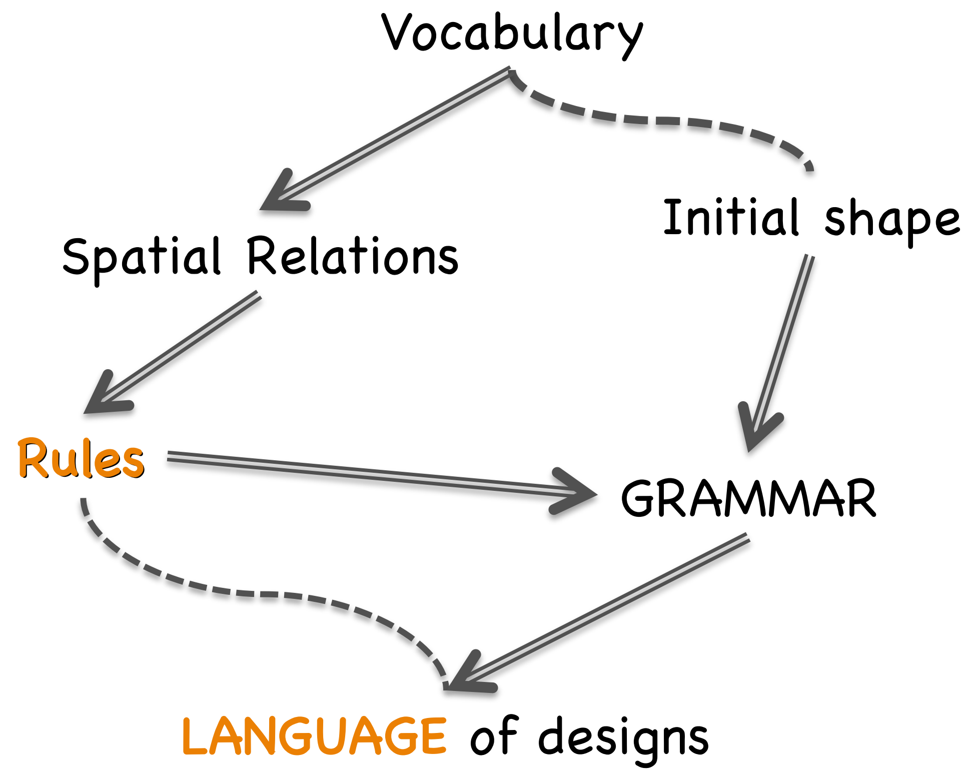
RECAP **grammar paradigm**

$x \rightarrow s + t$ (additive)

$s + t \rightarrow x$ (subtractive)

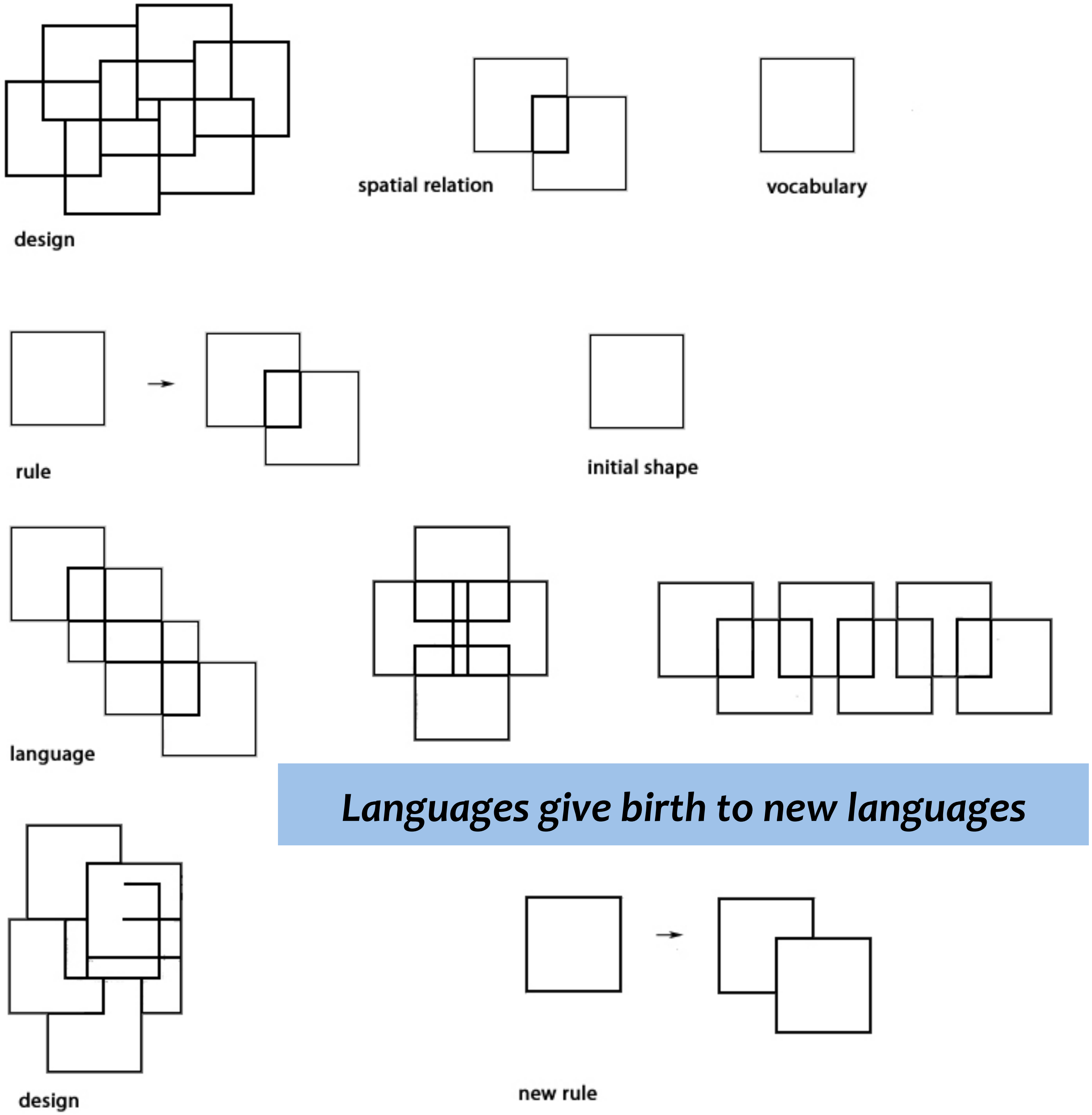
augmented by

the use of labels



Gothic
Classical
Islamic
Eastern styles

Encapsulates "theme"
Explored for design "styles"
Perhaps motivated by revival or restoration

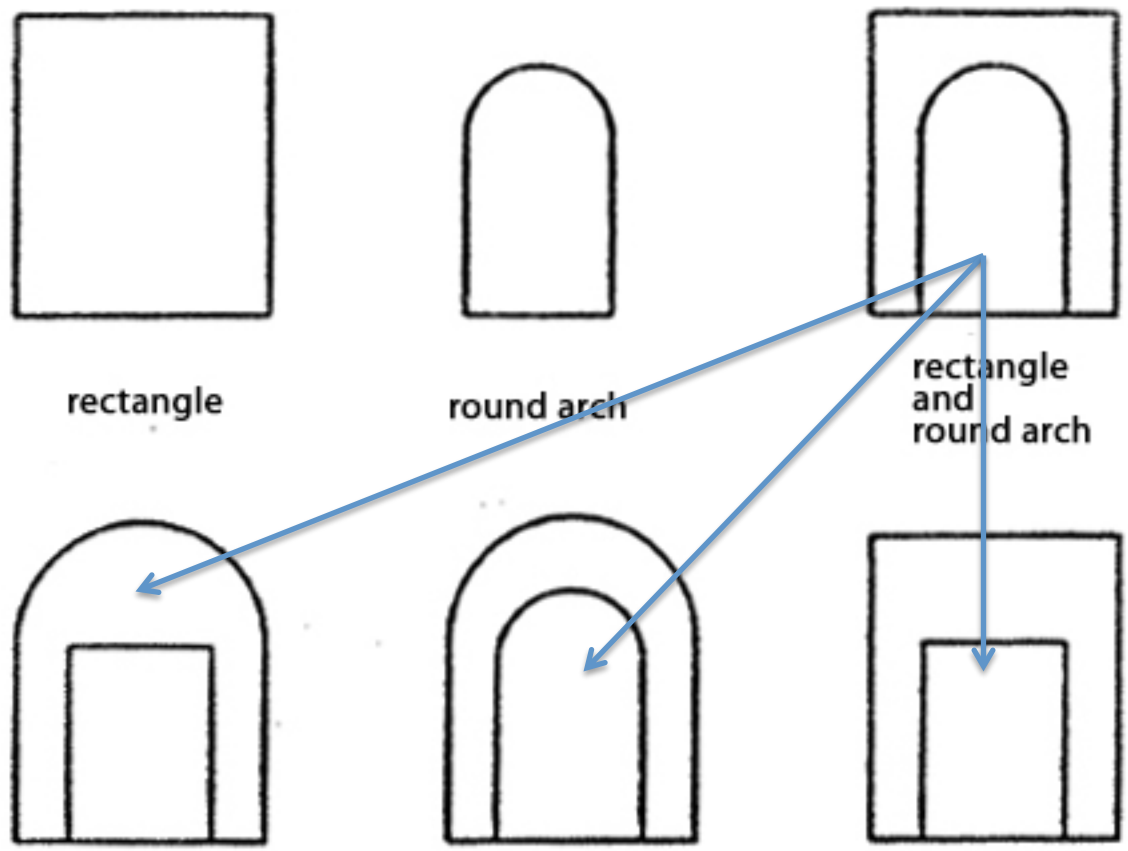

design
spatial relation
vocabulary
rule
initial shape
language
Languages give birth to new languages
design
new rule

transposition of letters, words, sounds, syllables

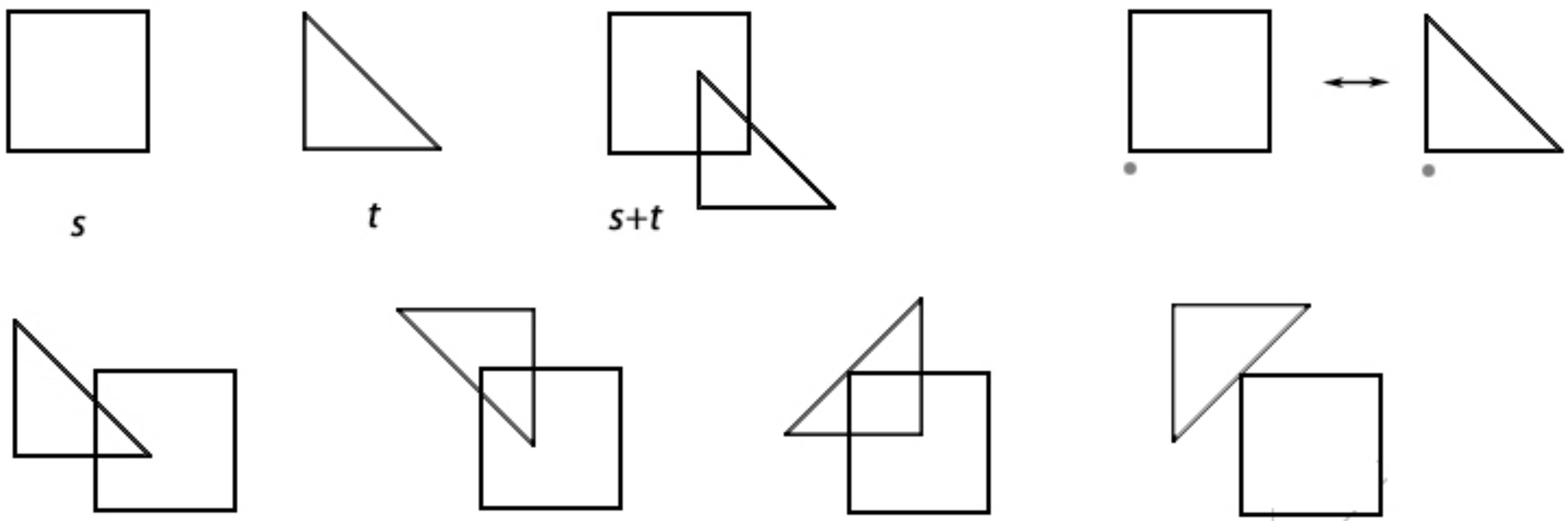
bird ← brid

evelate ← elevate (spoonerism)

why not transposition of shapes or images by shape replacement ?



metathesis



possible spatial metathesis

metathesis

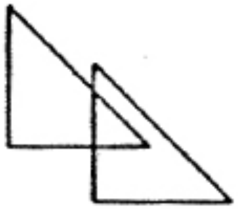
Is of the form $\boldsymbol{a} \leftrightarrow \boldsymbol{b}$ where neither $\boldsymbol{a}$ nor $\boldsymbol{b}$ is empty

Apply the rule to a spatial relation *R*, a set of shapes, to produce a new spatial relation *N provided R contains a shape s and there is a geometrical transformation f such that either* $s = f(\boldsymbol{a})$ *or* $s = f(\boldsymbol{b})$

$N = S - f(\boldsymbol{a}) + f(\boldsymbol{b})$ if $s = f(\boldsymbol{a})$

$N = S - f(\boldsymbol{b}) + f(\boldsymbol{a})$ if $s = f(\boldsymbol{b})$

shape equivalence rule



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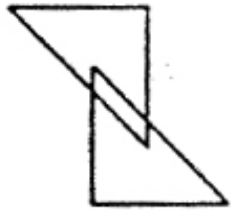

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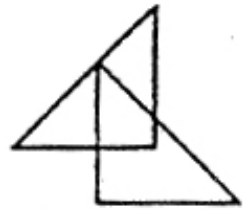

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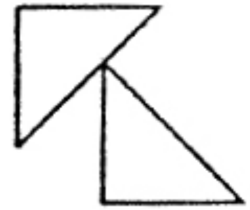

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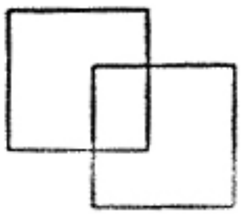
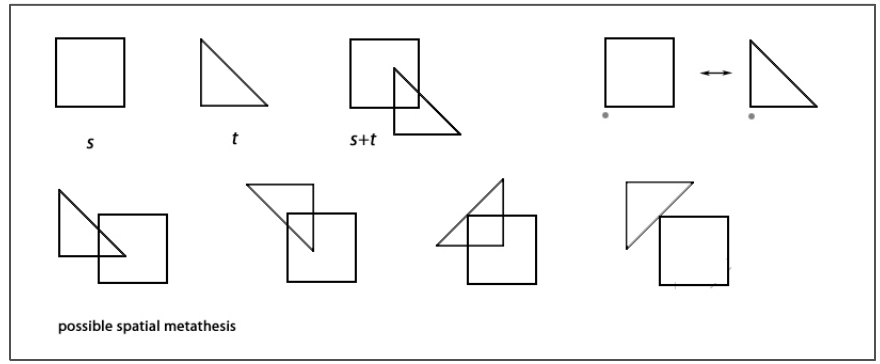
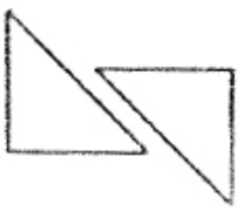
(→)



(←)





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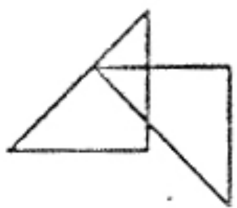

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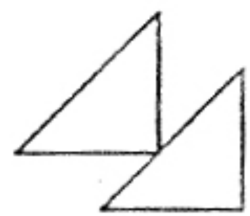

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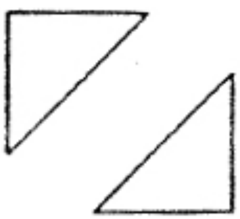

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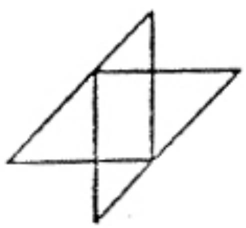

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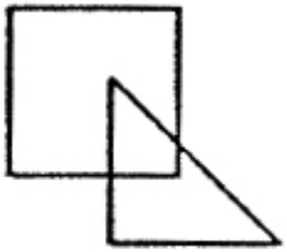

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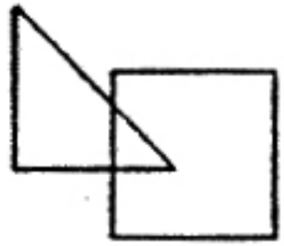

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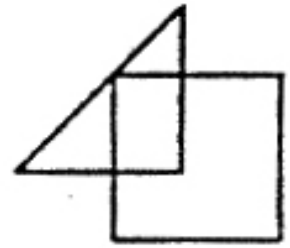

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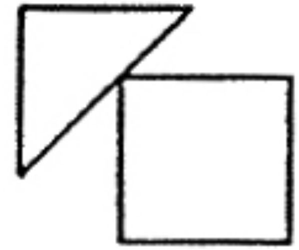

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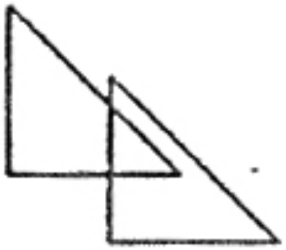

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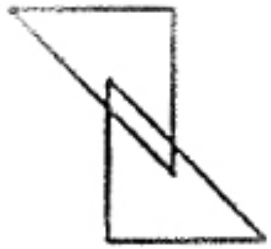

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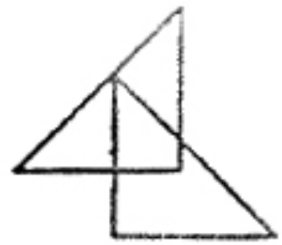

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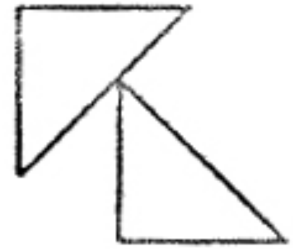

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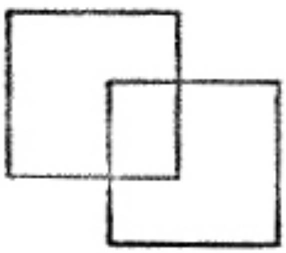

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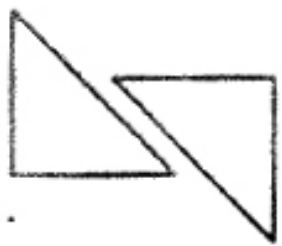

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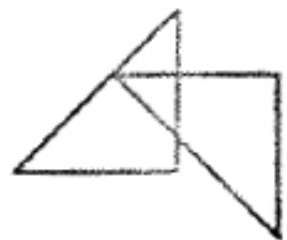

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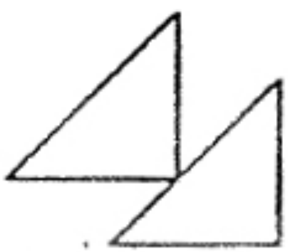

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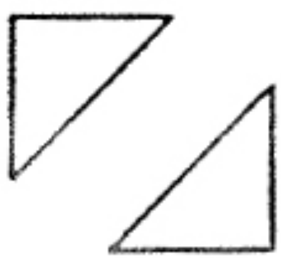

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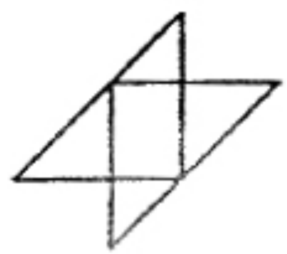

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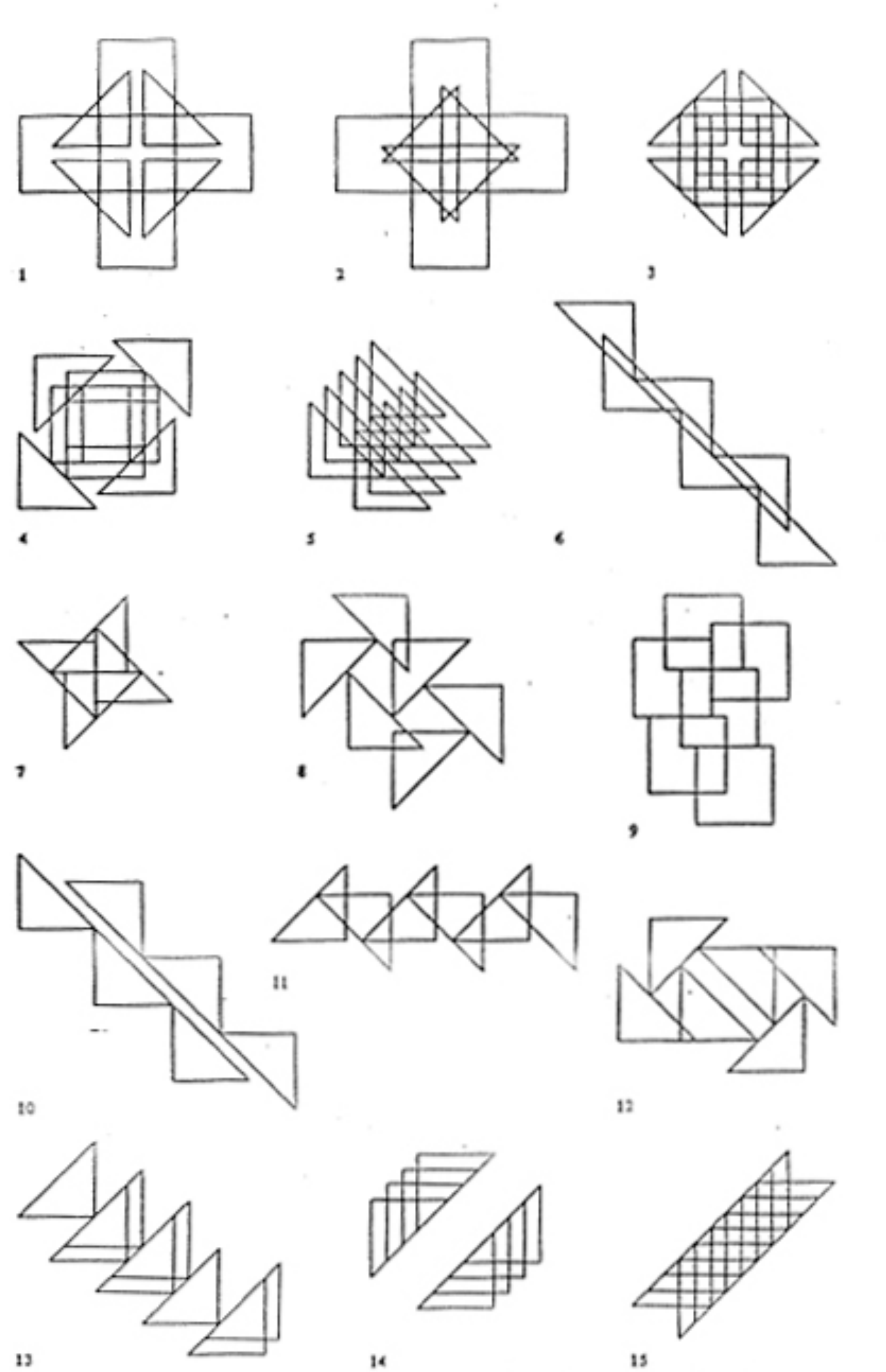
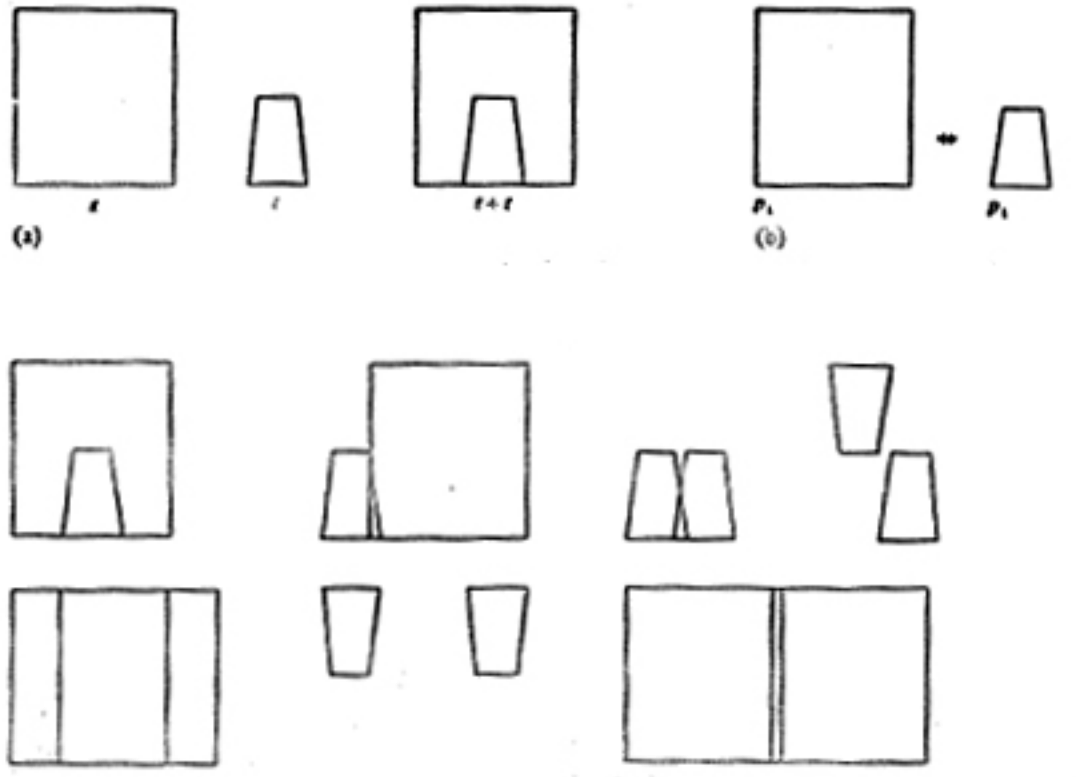
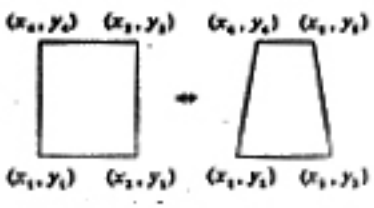
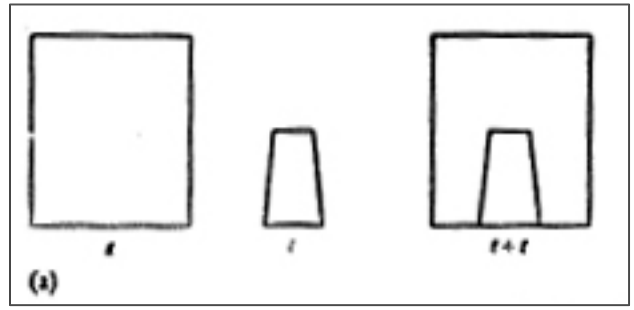
example designs

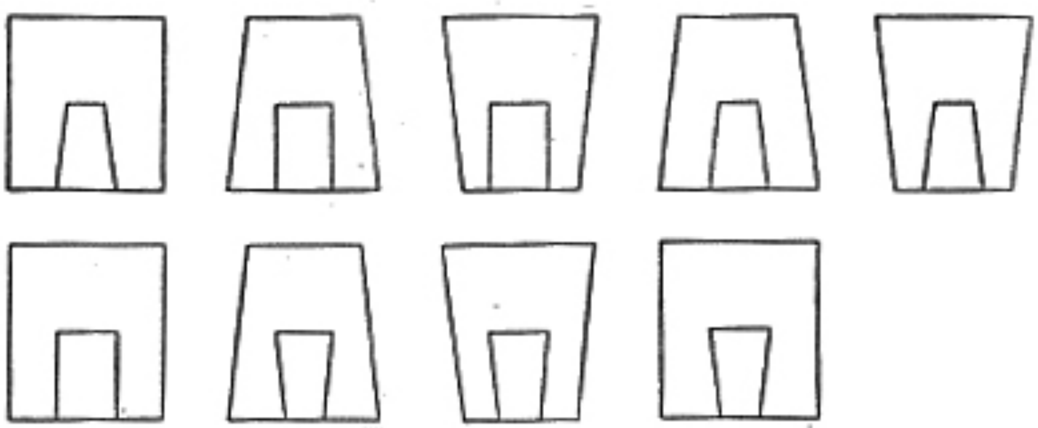




if we can have shape equivalence rules why not **shape equivalence schemas**?







shape equivalence schema

Is a schema of the form $\boldsymbol{a} \leftrightarrow \boldsymbol{b}$ where neither $\boldsymbol{a}$ nor $\boldsymbol{b}$ is empty, $\boldsymbol{a}$ and $\boldsymbol{b}$ have open terms

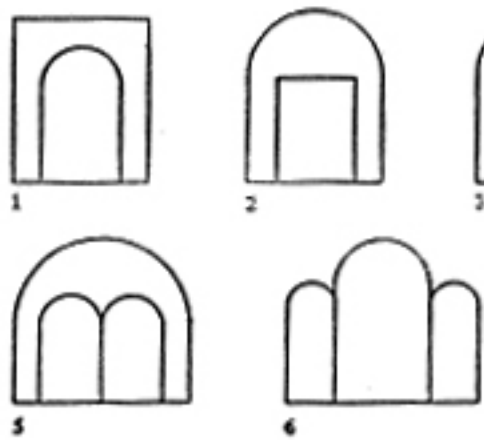
Apply the schema to a spatial relation *R*, a set of shapes, to produce a new spatial relation *N provided R contains a shape s, there is an assignment g to all open variables in* $\boldsymbol{a}$ and $\boldsymbol{b}$, *and there is a geometrical transformation f such that either s= f(*$\boldsymbol{a}$*) or s=f(*$\boldsymbol{b}$*)*

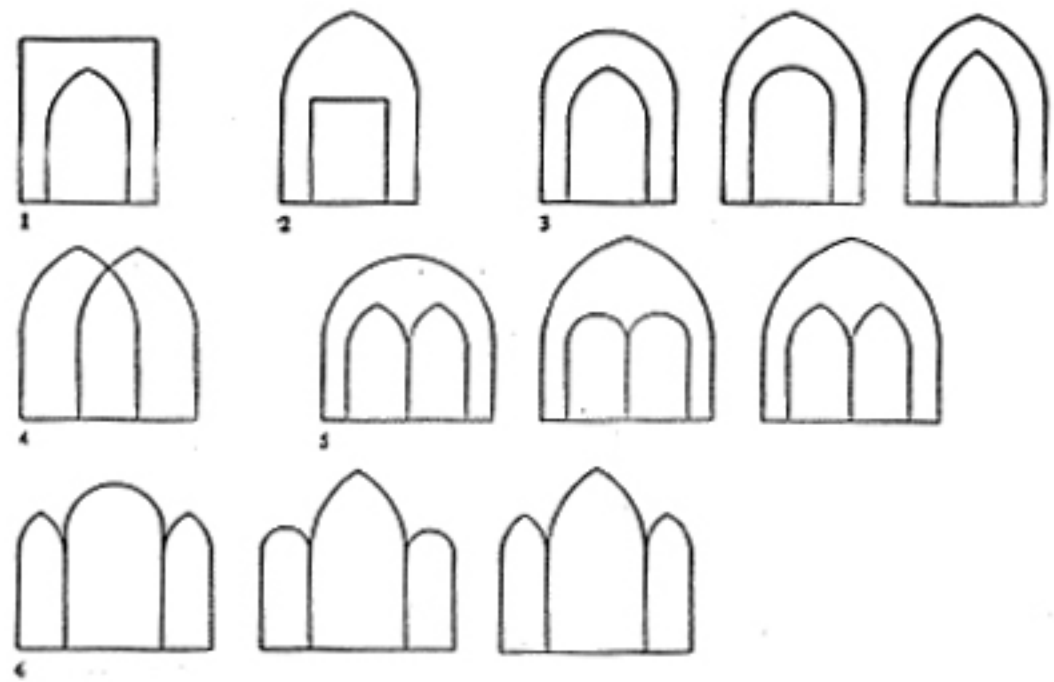
$N = S - f(g[\boldsymbol{a}]) + f(\boldsymbol{b})$ if $s = f(g[\boldsymbol{a}])$

$N = S - f(\boldsymbol{b}) + f(\boldsymbol{a})$ if $s = f(g[\boldsymbol{b}])$

shape equivalence schema

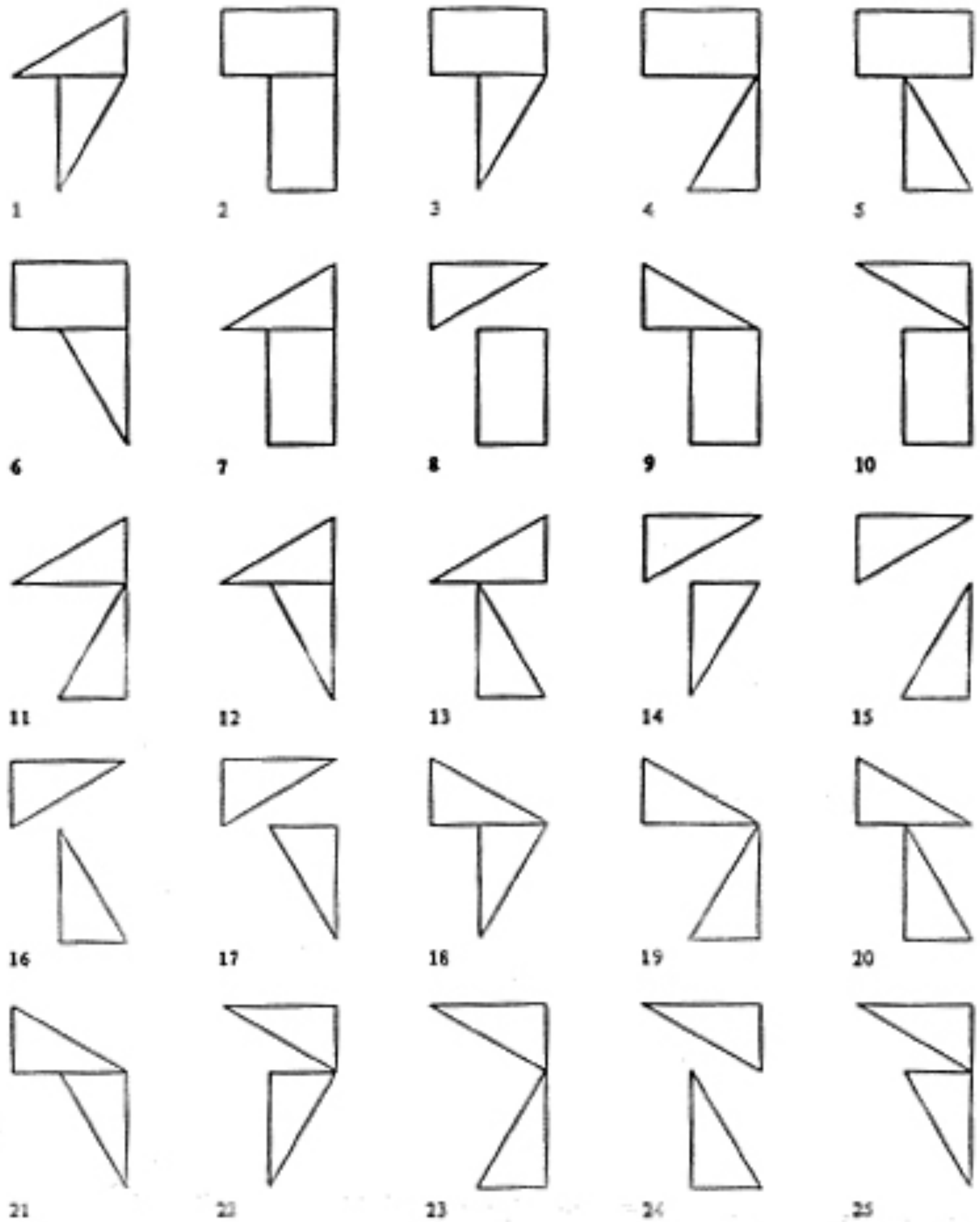
What we have seen so far is a FLIP-FLOP between shapes/schemas with the implicit PROVISO that no new shapes are introduced into the relation





transition from Romanesque to Gothic arches?

what about **introducing new shapes** into the **equivalence rule**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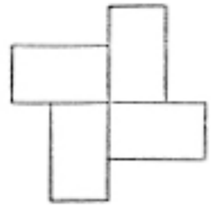

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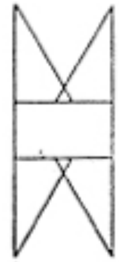

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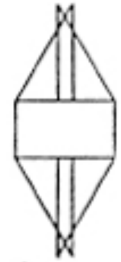

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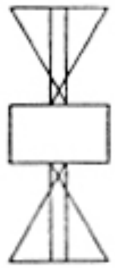

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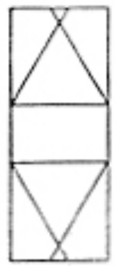

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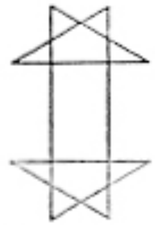

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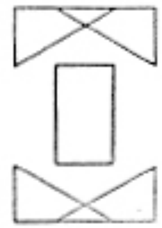

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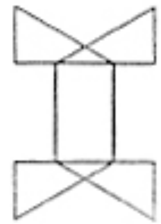

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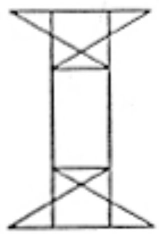

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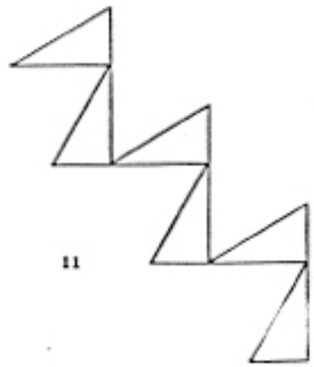

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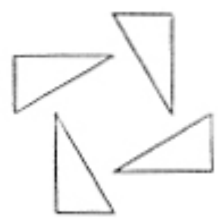

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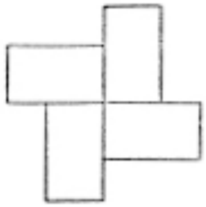
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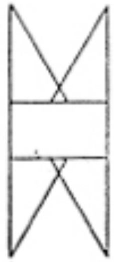
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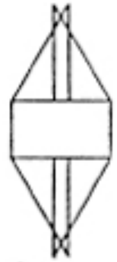
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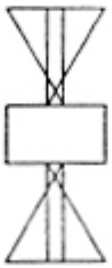
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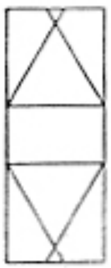
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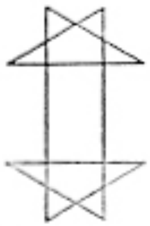
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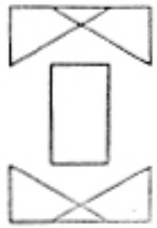
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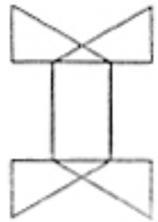
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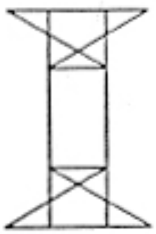
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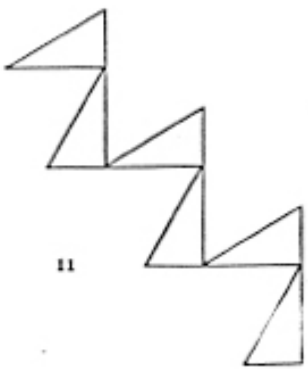
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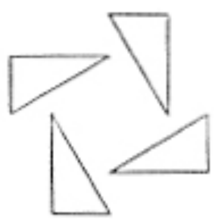
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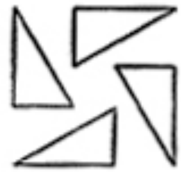
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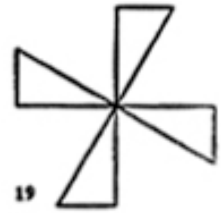
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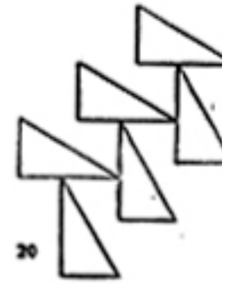
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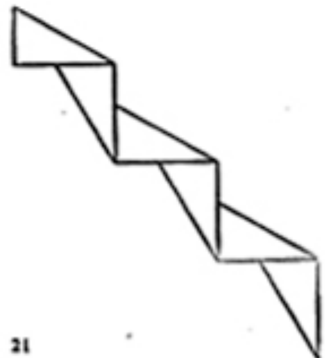
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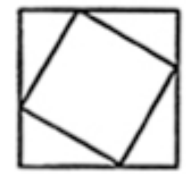
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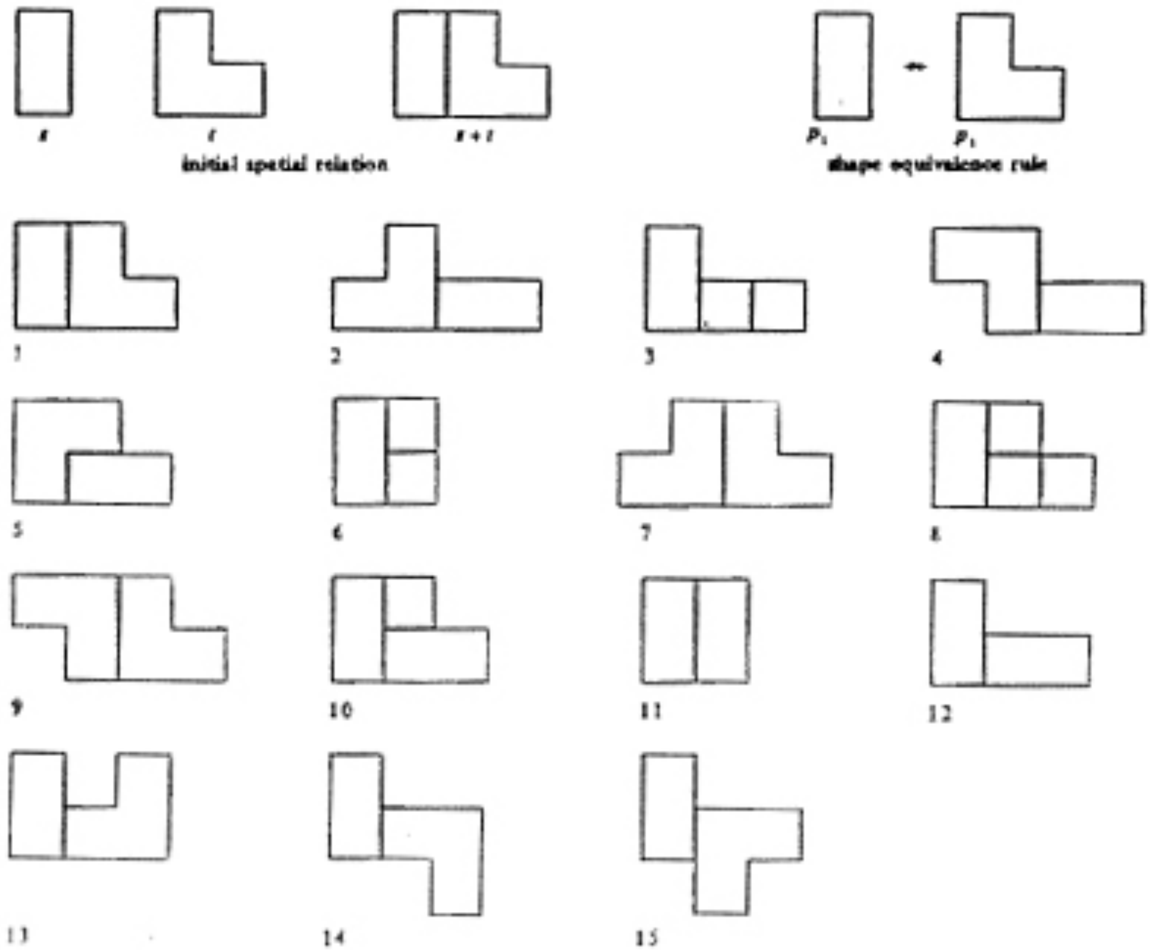
We have $\boldsymbol{a} \leftrightarrow \boldsymbol{b}$

We can construct classes of spatial relations by looking at $f[h(\boldsymbol{a})]$ and $g[j(b)]$ so that

$N = S - f(\boldsymbol{a}) + g(\boldsymbol{b})$

$N = S - f(h(\boldsymbol{b})) + g(j(\boldsymbol{a}))$

any more variations?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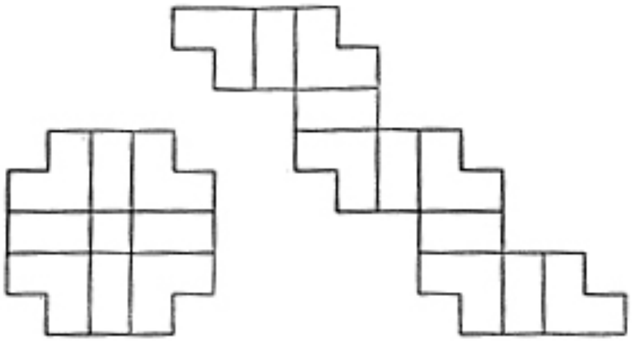

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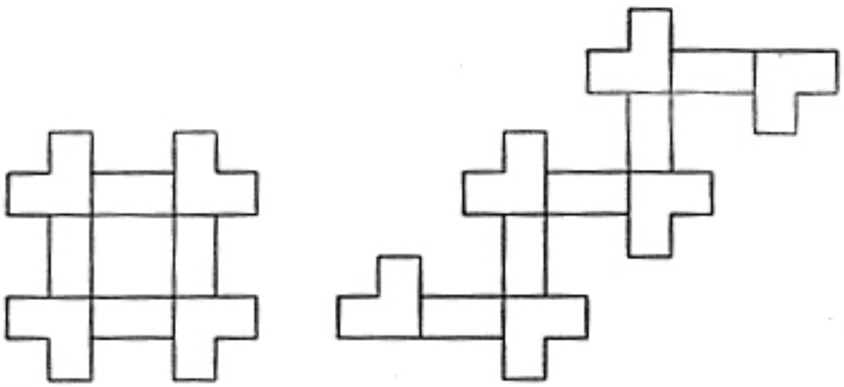

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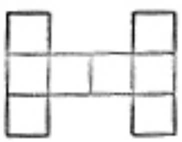

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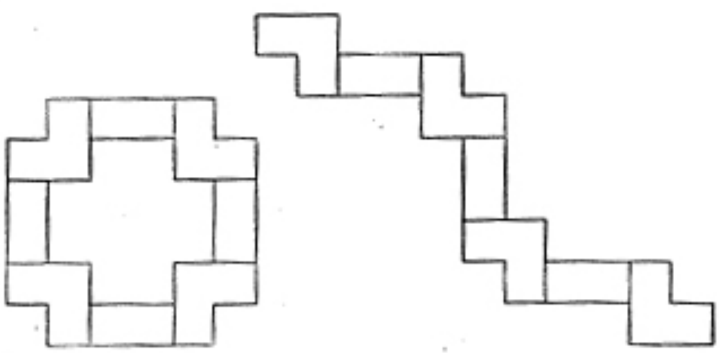

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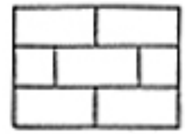

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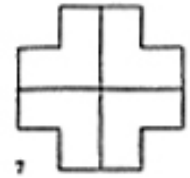
6



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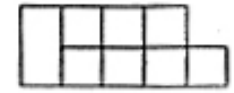

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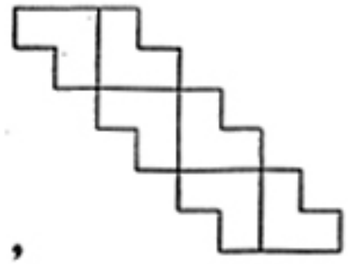

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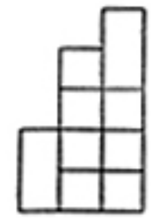

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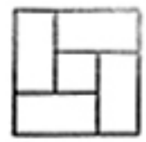
11



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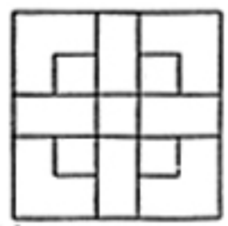
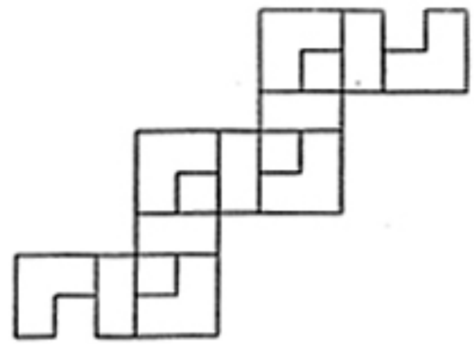

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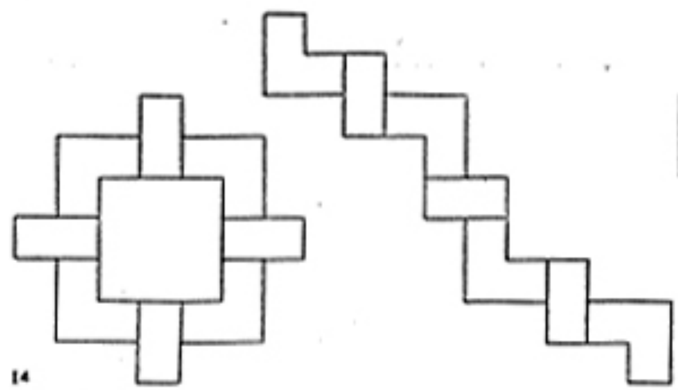

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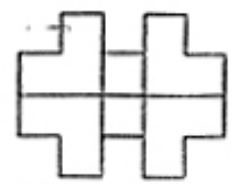

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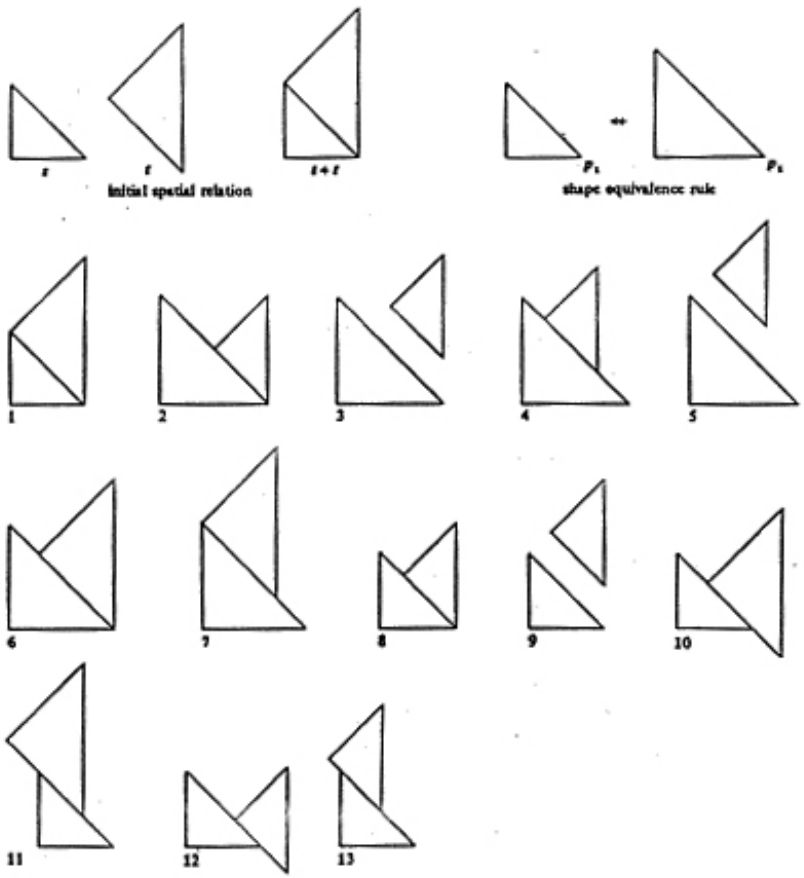
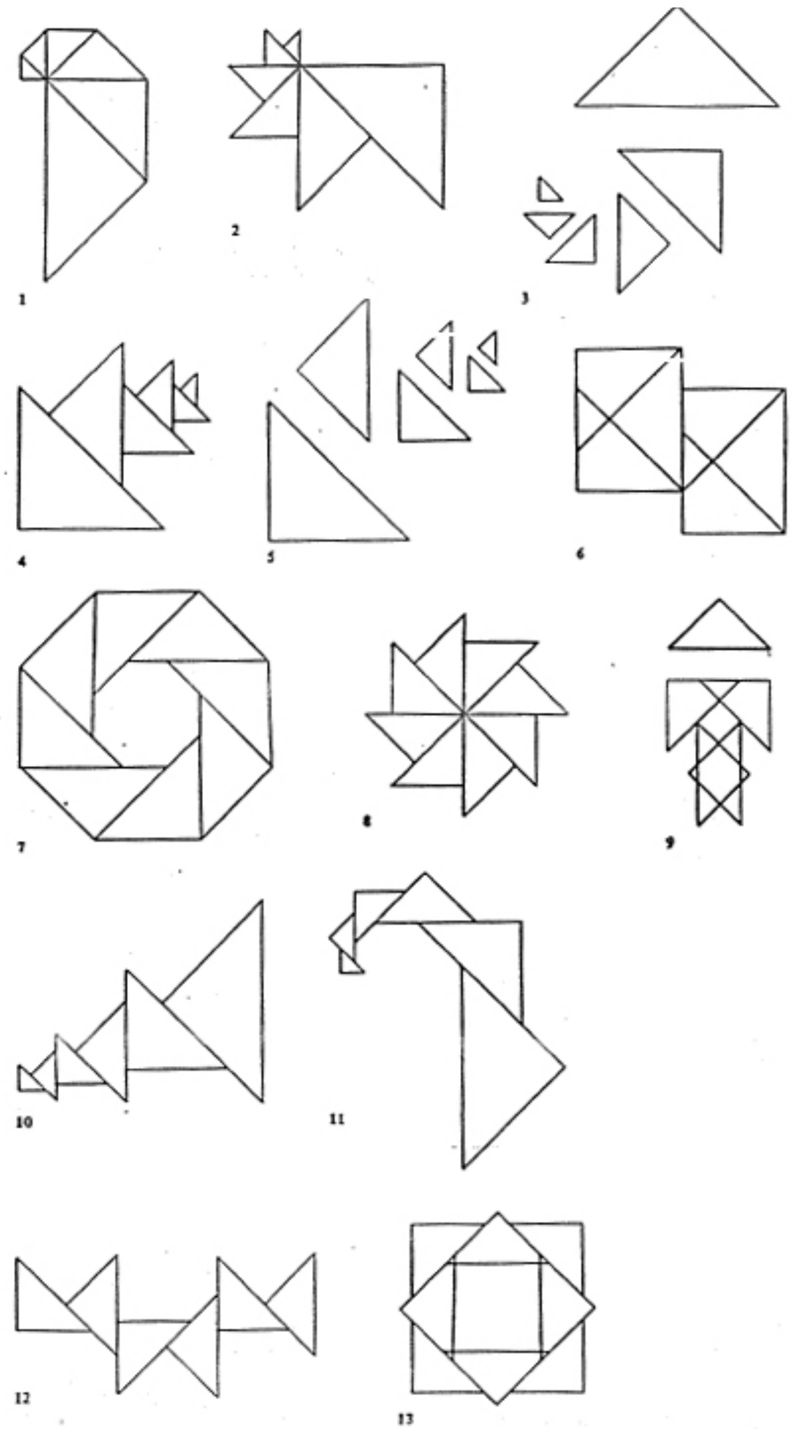
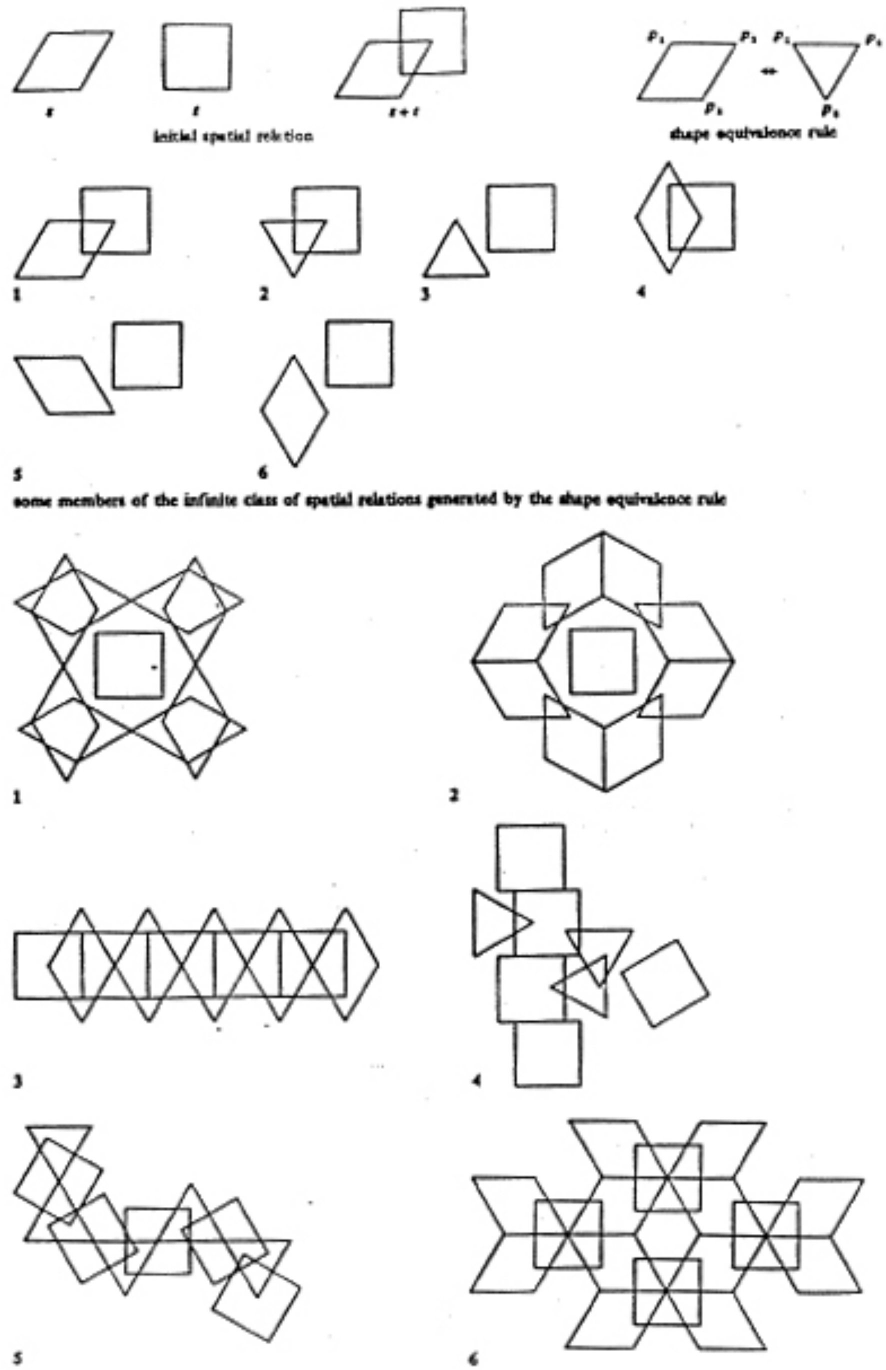
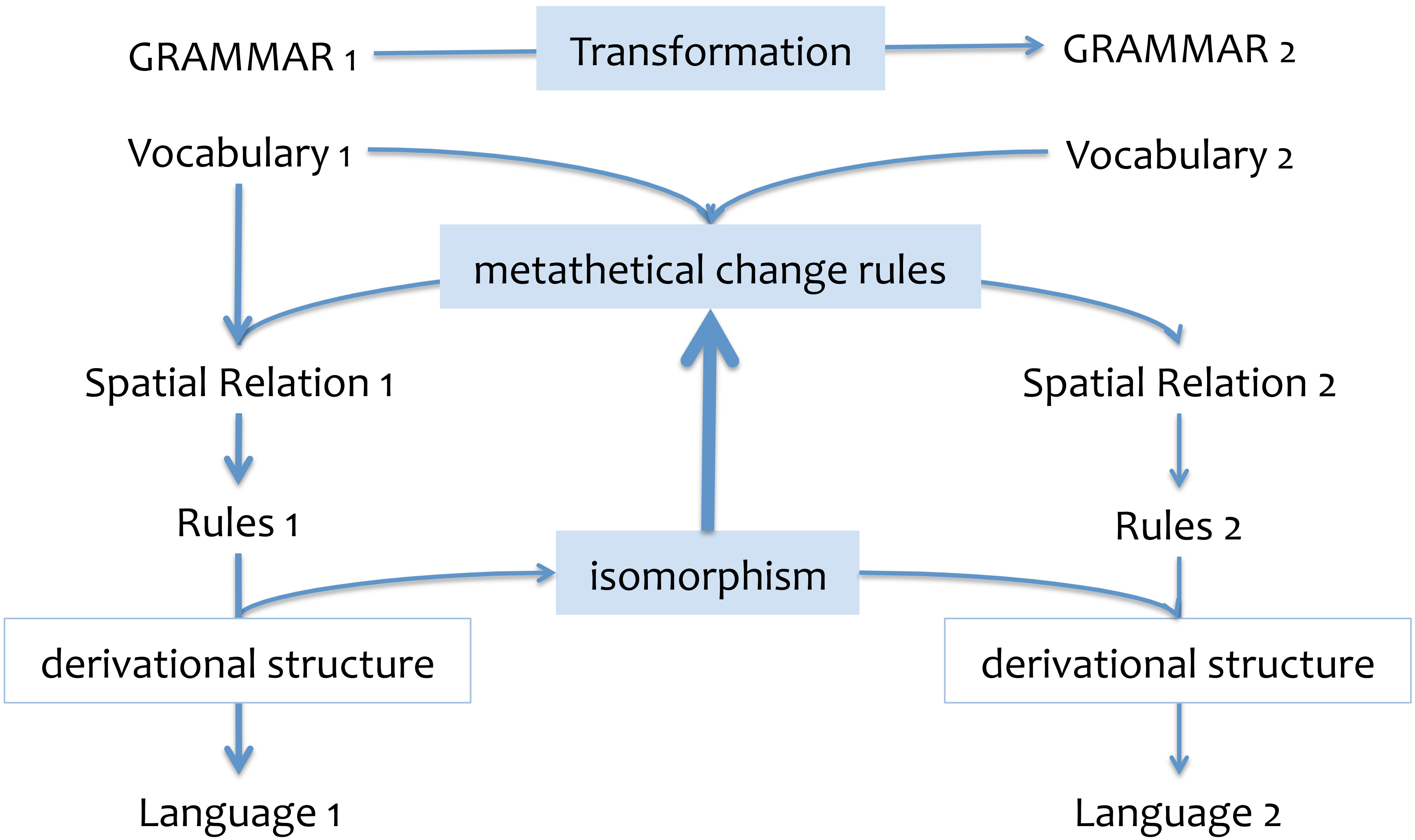

t
t
t + t
initial spatial relation
shape equivalence rule
1
2
3
4
5
6
7
8
9
10
11
12
13


1
2
3
4
5
6
7
8
9
10
11
12
13

to make this transition idea work one must consider heuristics in how the shape equivalence rules are applied.



Transformation of Grammars



the basic idea

we need to ensure that grammars are specified in an normalized fashion –
i.e., *in the same sort of way every time*

hence, ***grammars*** in ***normal form***

Vocabulary

Purely Additive rules

Purely Subtractive rules

Labels are spatial

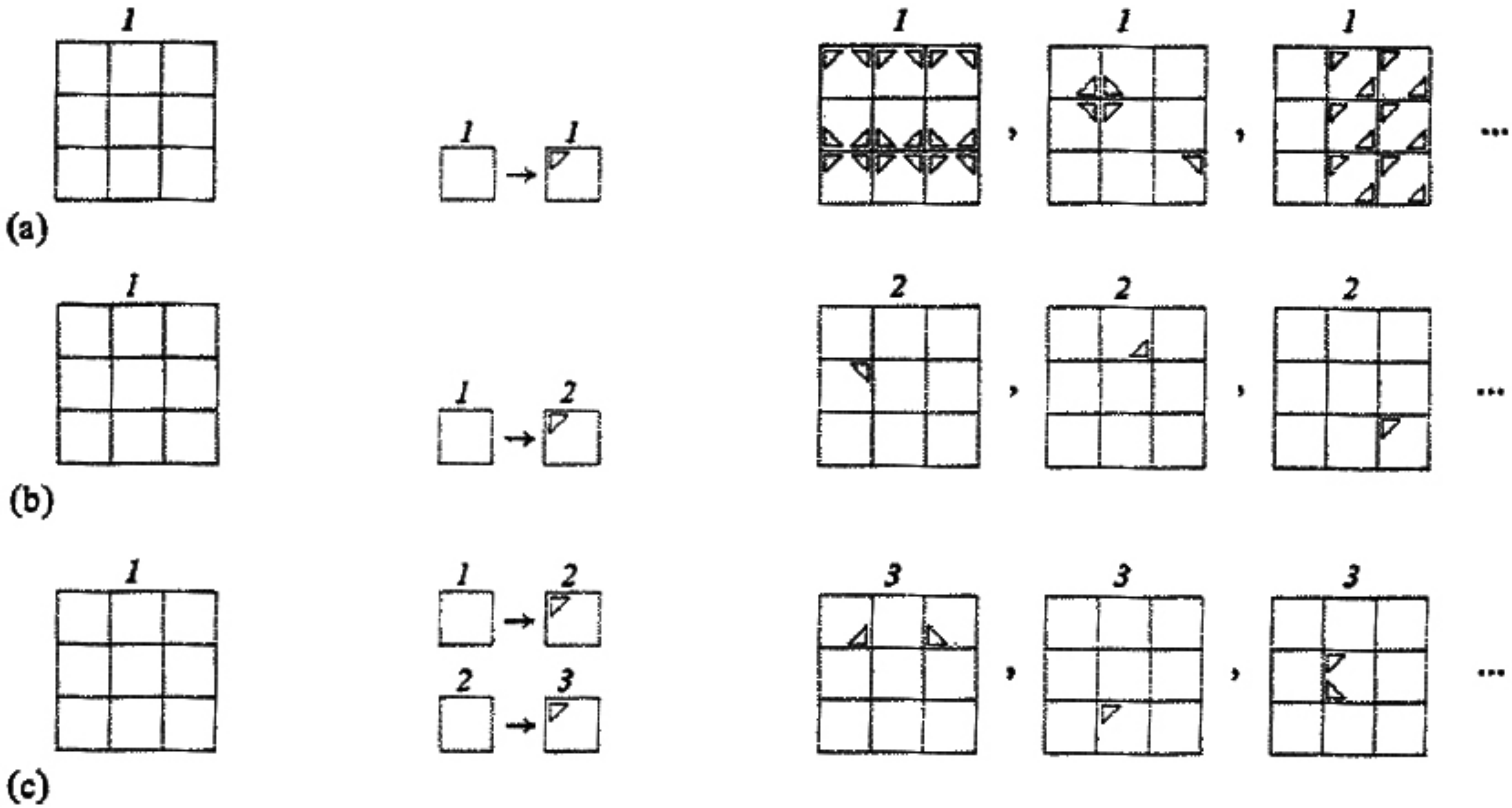
– *how*

– *where*

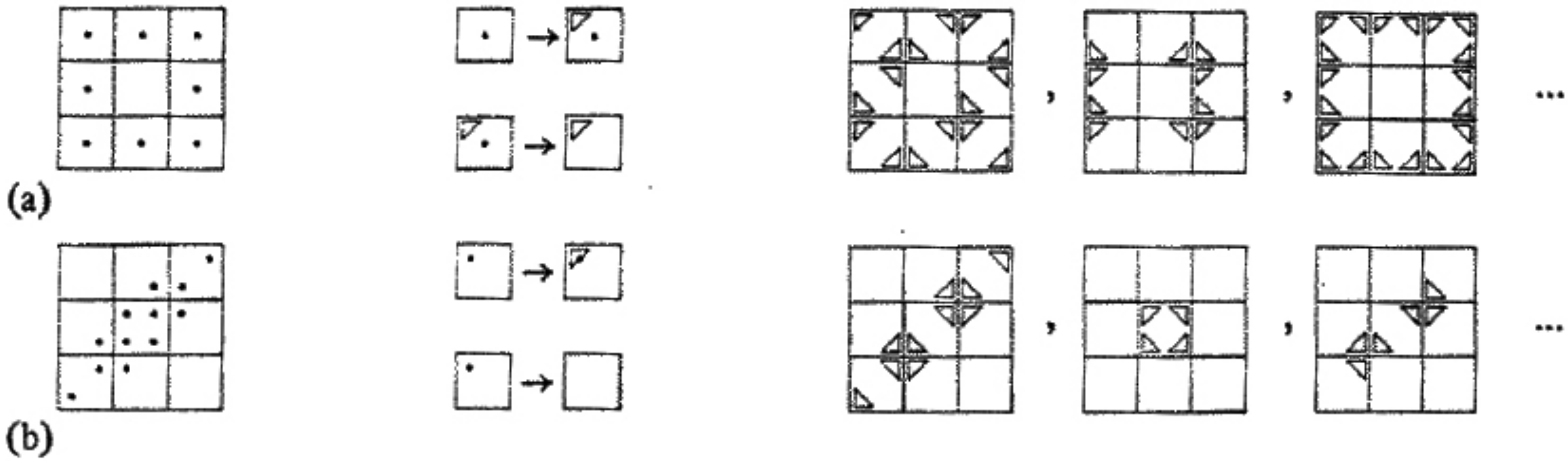
States are nonspatial

– *when*

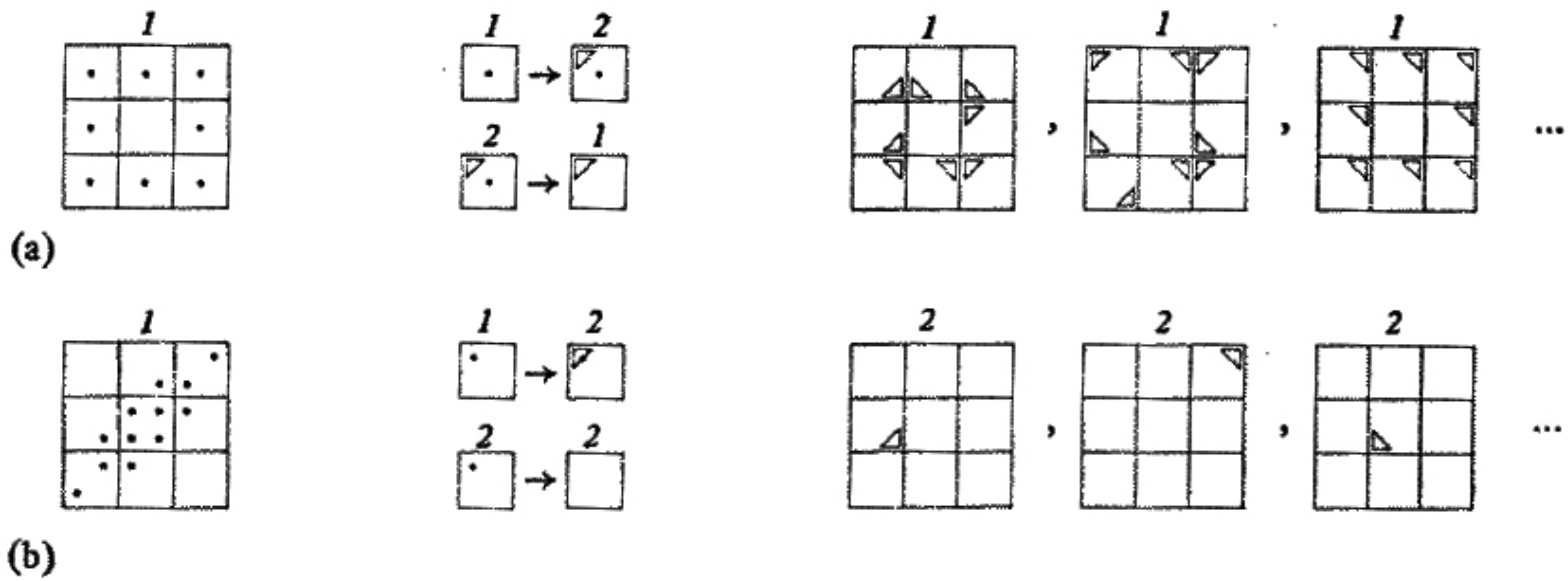
to compare languages



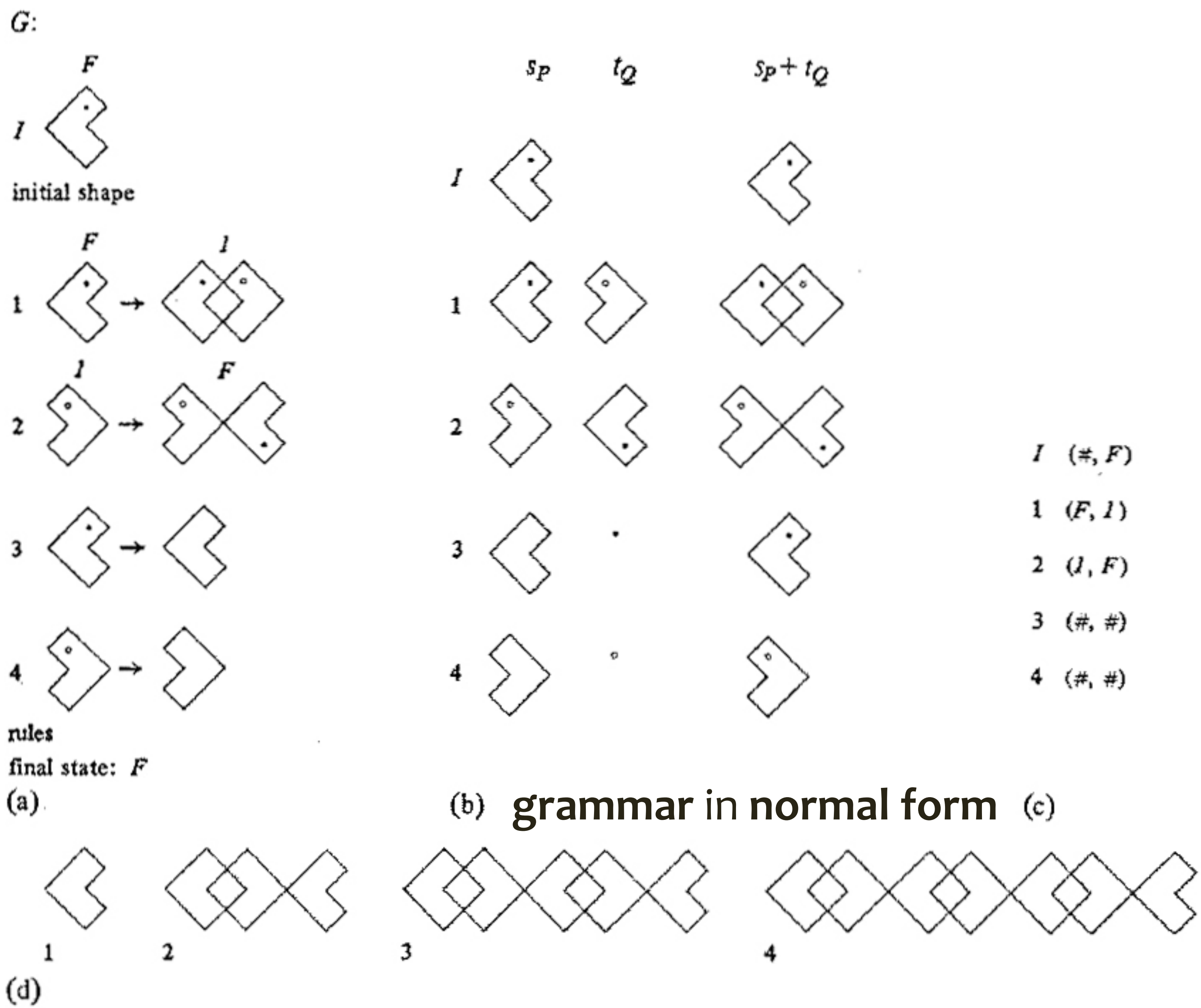
nonspatial or state labels



spatial – *where* and *how* labels



state and **spatial labels**


G:
F
I
initial shape
F
I
1
I
F
2
3
4
rules
final state: F
(a)
s_P
t_Q
$s_P + t_Q$
I
1
2
3
4
(b)
grammar in normal form
I (#, F)
1 (F, I)
2 (I, F)
3 (#, #)
4 (#, #)
(c)
1
2
3
4
(d)

Is a basic property of grammars

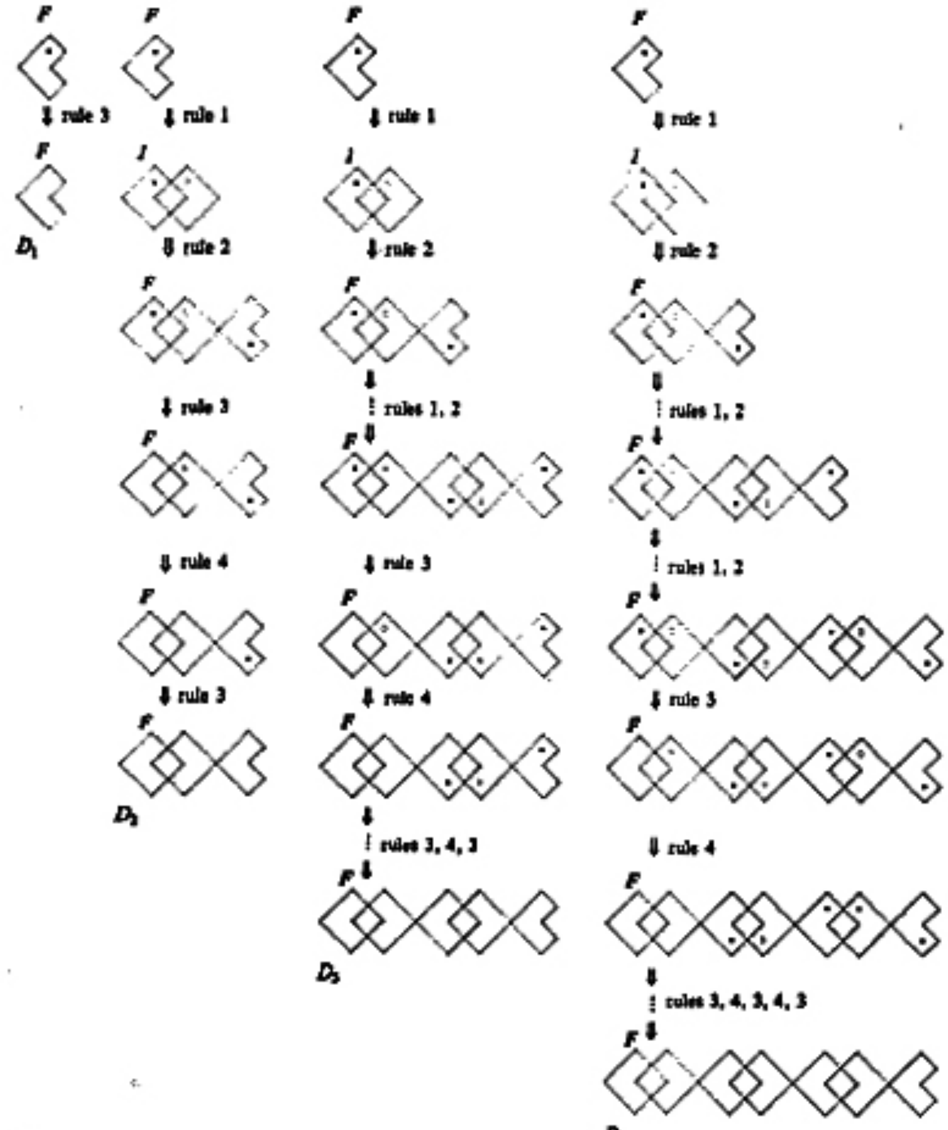
Expresses a relationship on rules, the initial shape and selected typical derivations of designs in the grammar

R(G) = {(rule x, rule y) … }where (rule x, rule y) is a member of R(G) whenever

- Rule x is additive or is the initial shape
- Rule y is purely additive or purely subtractive and rule y is applied to that part of the design that includes a subshape of a labeled shape which was added by a previous application of rule x

i.e., rule x makes rule y possible

recursive structure R(G)



(a)

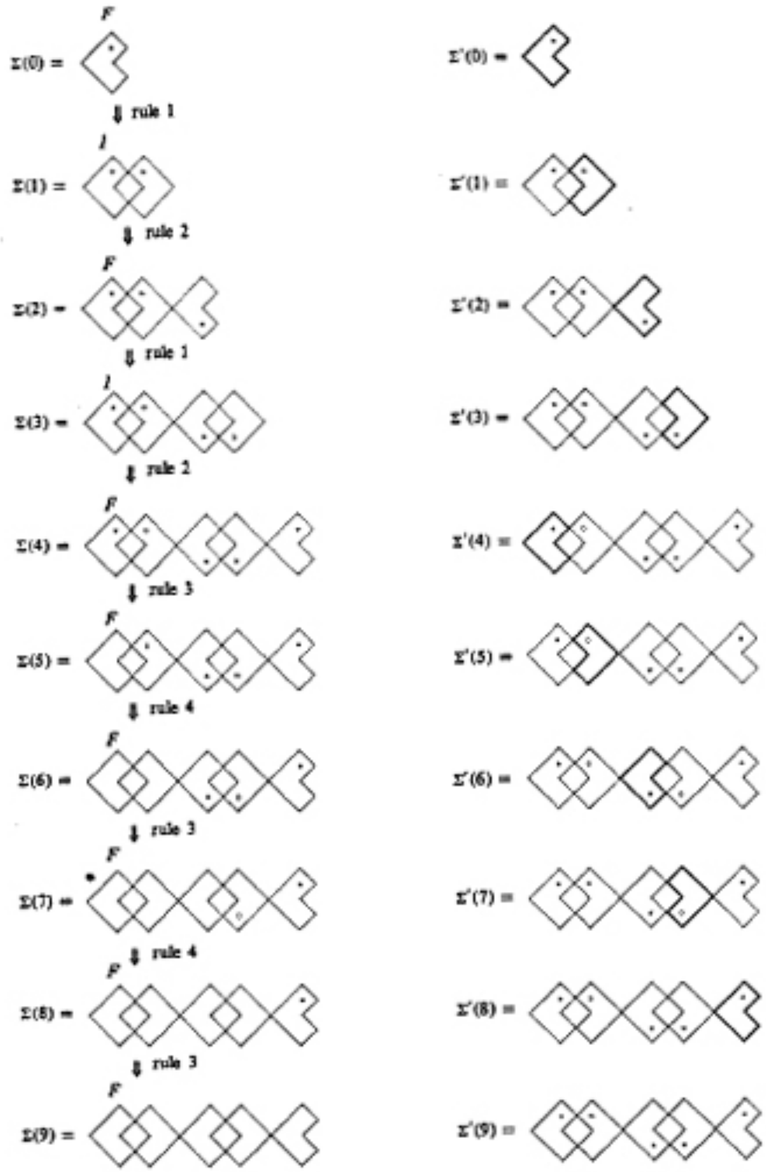
$R_1(G) = \{(I, \text{rule } 3)\}$

$R_2(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$

$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$

$R_4(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$

(b)



(c)

$$R(G) = R_1(G) \cup R_2(G) \cup R_3(G) \cup R_4(G)$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$$

(d)

$$R_3(G) = \emptyset$$

$$R_3(G) = \emptyset \cup \{(I, \text{rule } 1)\}$$
$$= \{(I, \text{rule } 1)\}$$

$$R_3(G) = \{(I, \text{rule } 1)\} \cup \{(\text{rule } 1, \text{rule } 2)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2)\} \cup \{(\text{rule } 2, \text{rule } 1)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1)\} \cup \{(\text{rule } 1, \text{rule } 2)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1)\} \cup \{(I, \text{rule } 3)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3)\} \cup \{(\text{rule } 1, \text{rule } 4)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4)\} \cup \{(\text{rule } 2, \text{rule } 3)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\} \cup \{(\text{rule } 1, \text{rule } 4)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$$

$$R_3(G) = \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\} \cup \{(\text{rule } 2, \text{rule } 3)\}$$
$$= \{(I, \text{rule } 1), (\text{rule } 1, \text{rule } 2), (\text{rule } 2, \text{rule } 1), (I, \text{rule } 3), (\text{rule } 1, \text{rule } 4), (\text{rule } 2, \text{rule } 3)\}$$

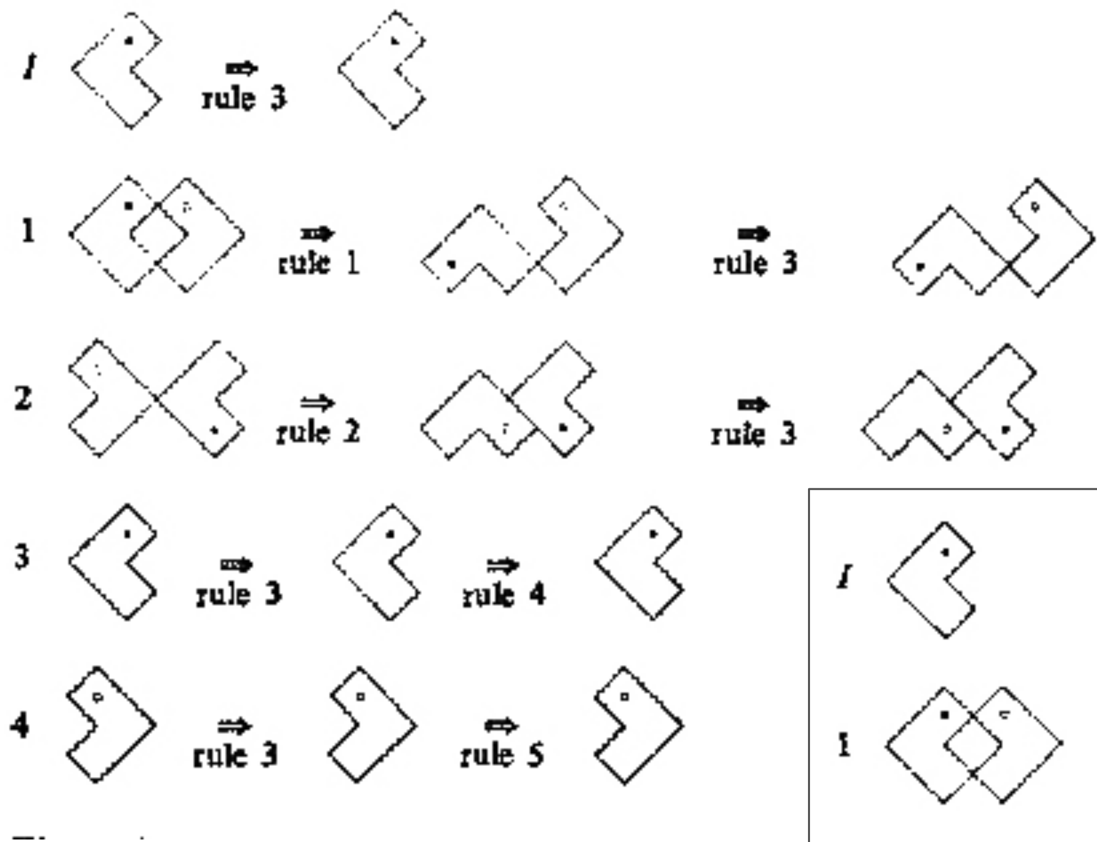
Comprises two independent stages

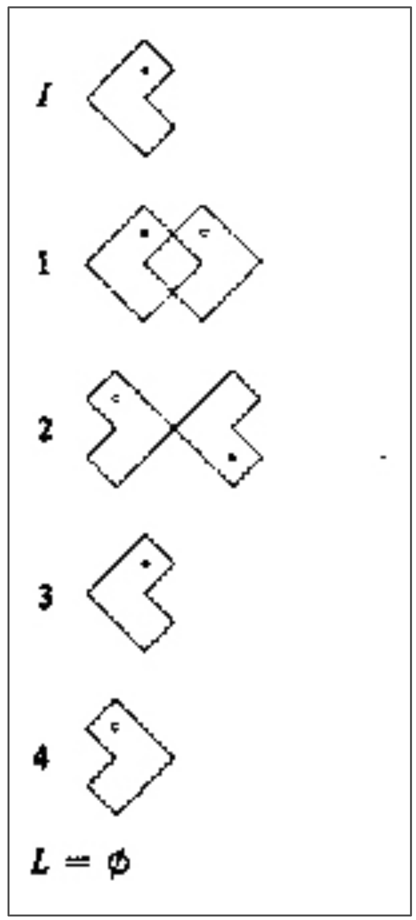
Defining shape change rules specifying transformation T_A between an initial and final set of relations

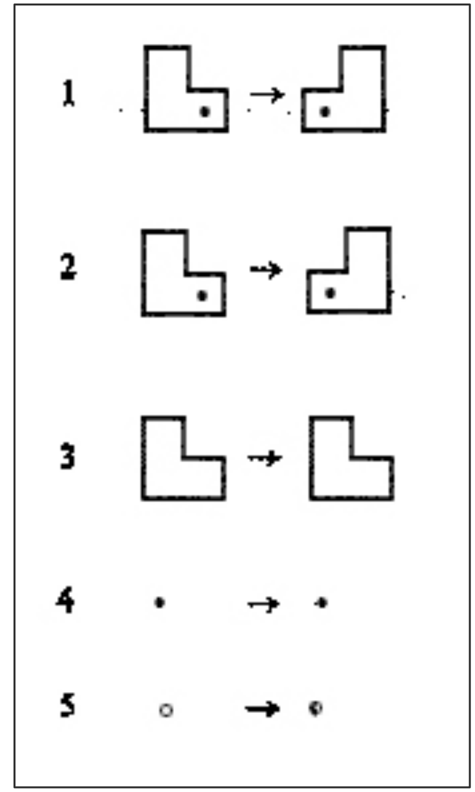
Defining state change rules specifying transformation T_B

T_A and T_B are combined to produce a complete transformation T of G.

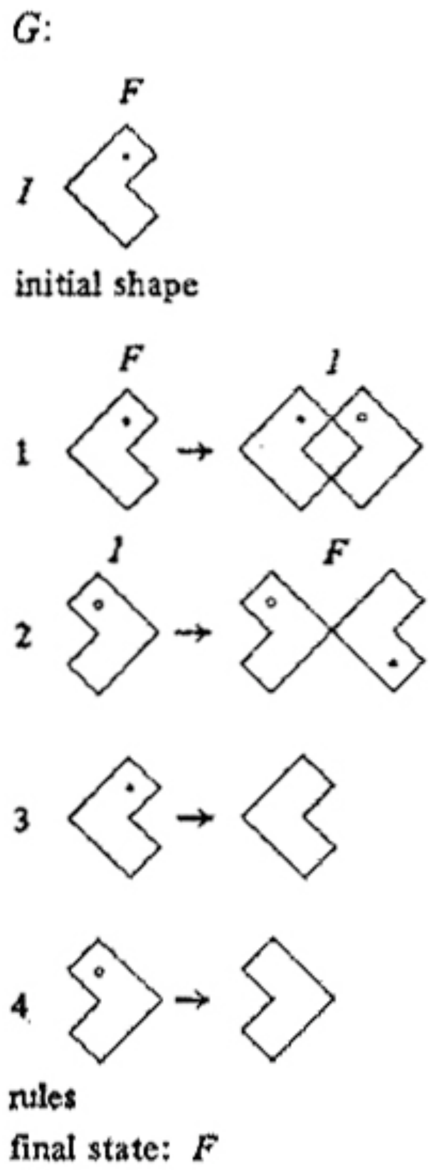
transformation of grammars

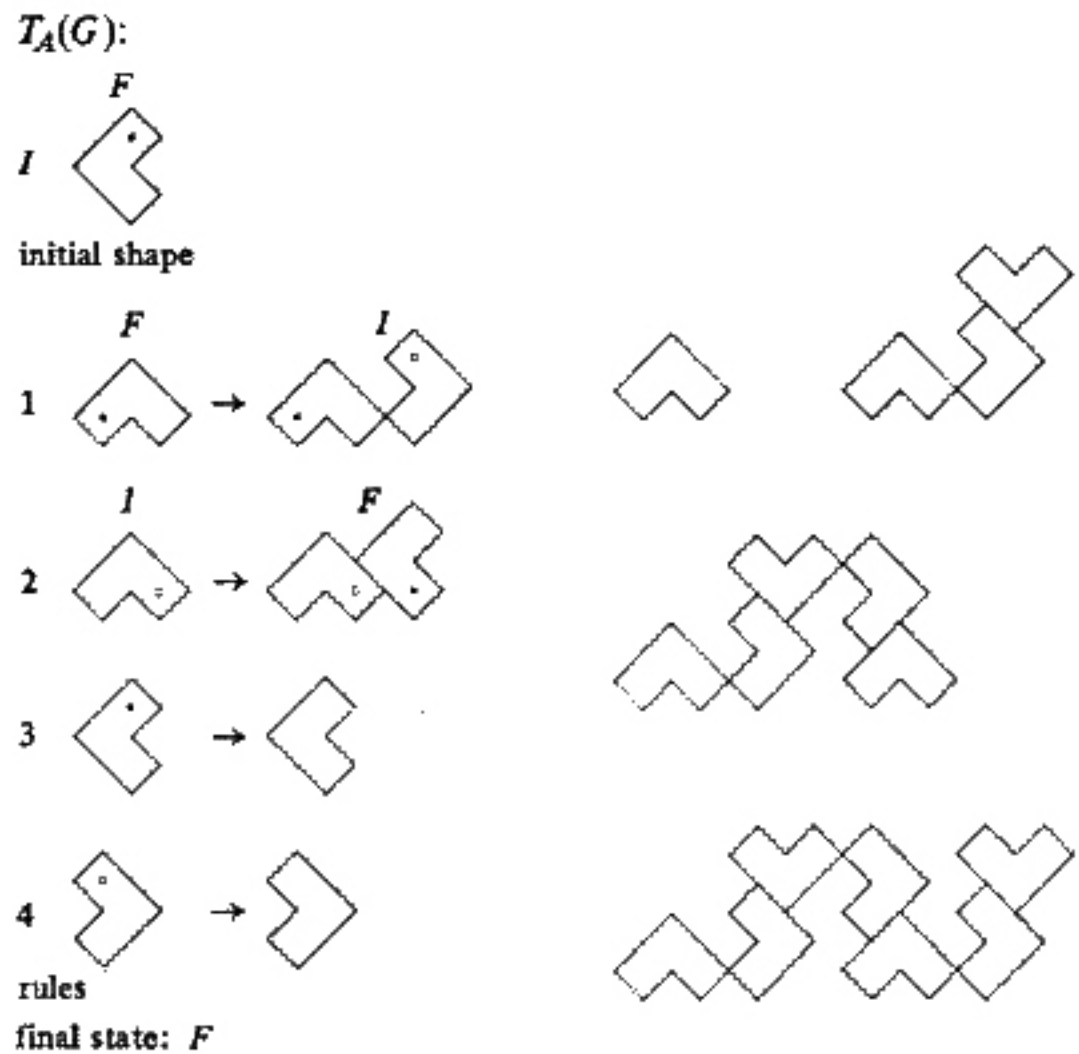


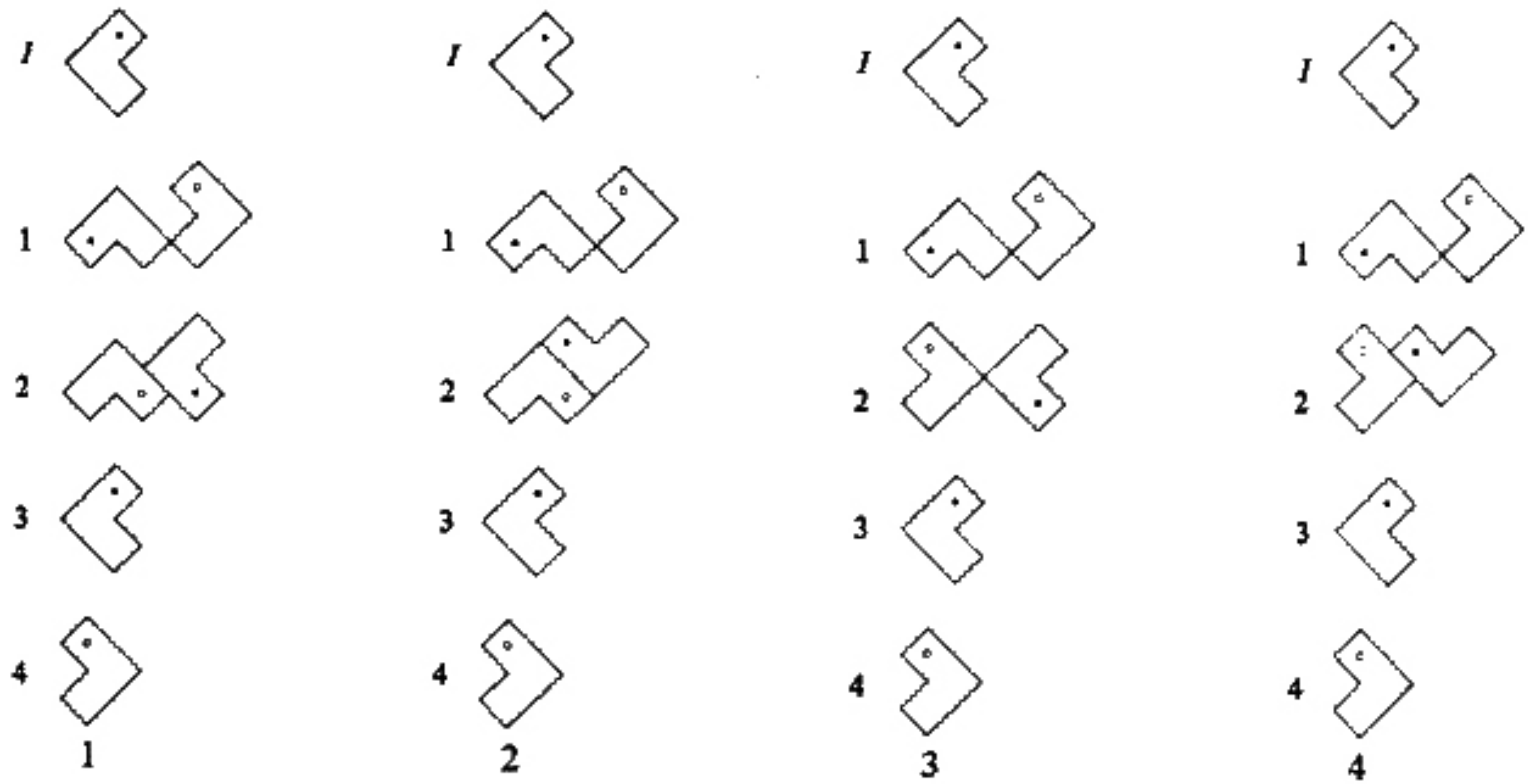


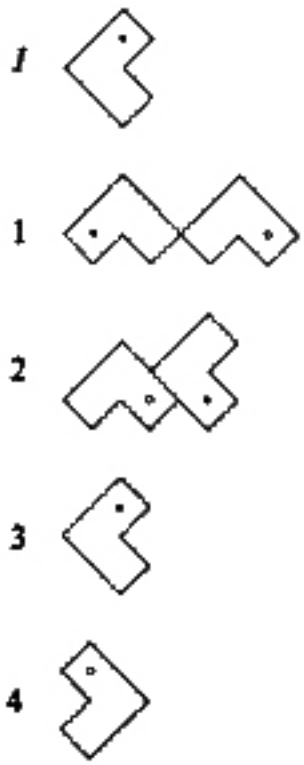


deriving a final set of relations

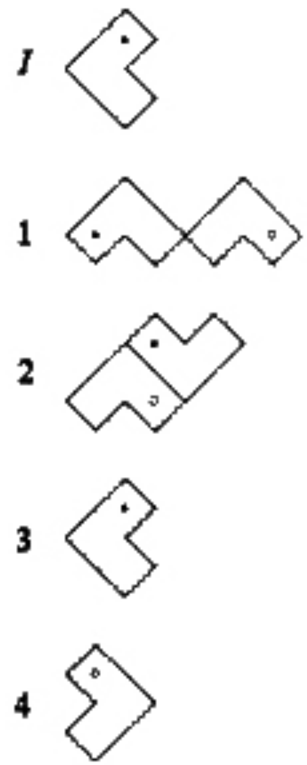

G:
F
I
initial shape
F
I
1
I
F
2
3
4
rules
final state: F
(a)


T_A(G):
F
I
initial shape
F
I
1
I
F
2
3
4
rules
final state: F


I
1
2
3
4
1
I
1
2
3
4
2
I
1
2
3
4
3
I
1
2
3
4
4



5



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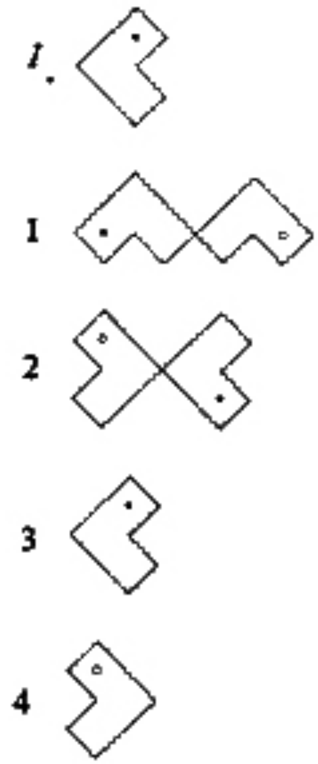
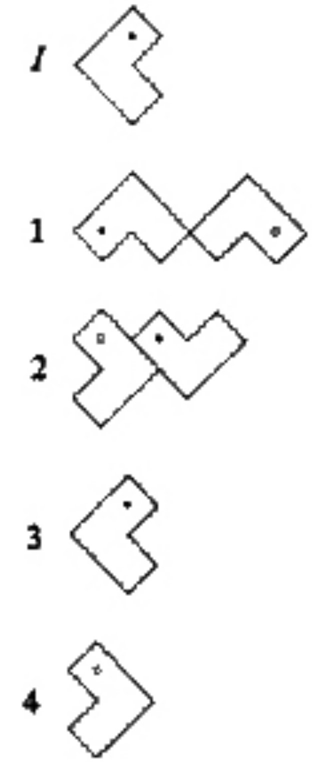


7



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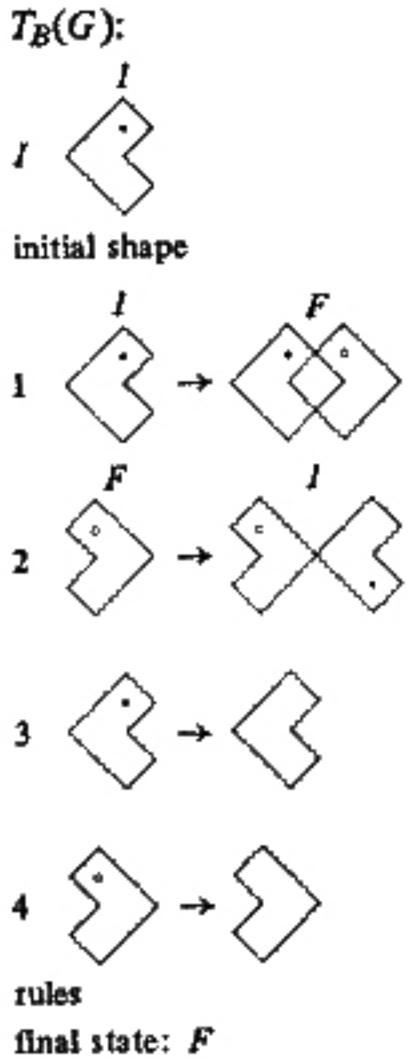
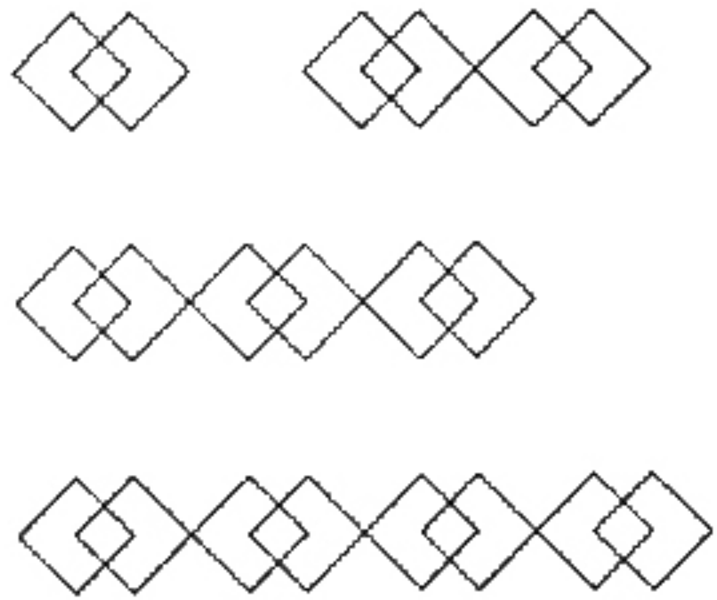
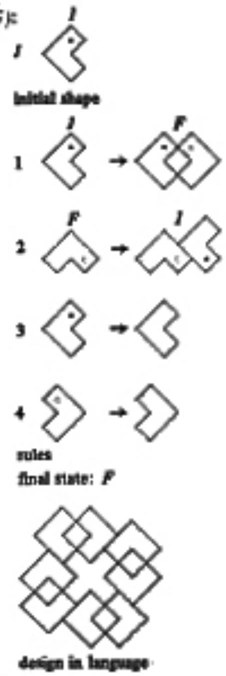


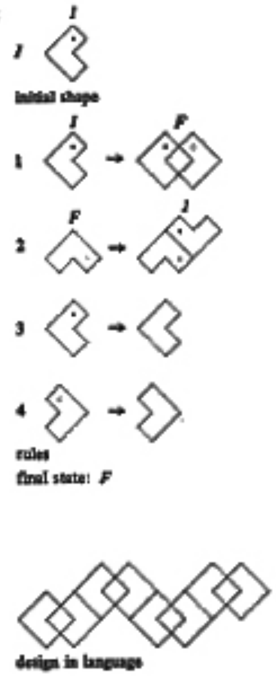
(a)


(b)

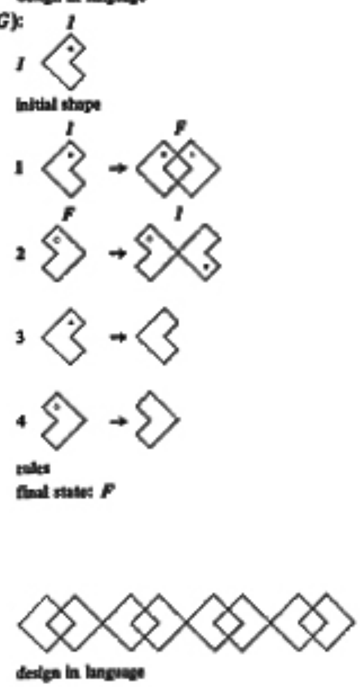
$T_9(G)$:



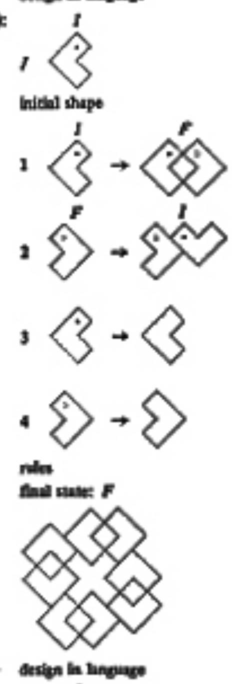
$T_{10}(G)$:



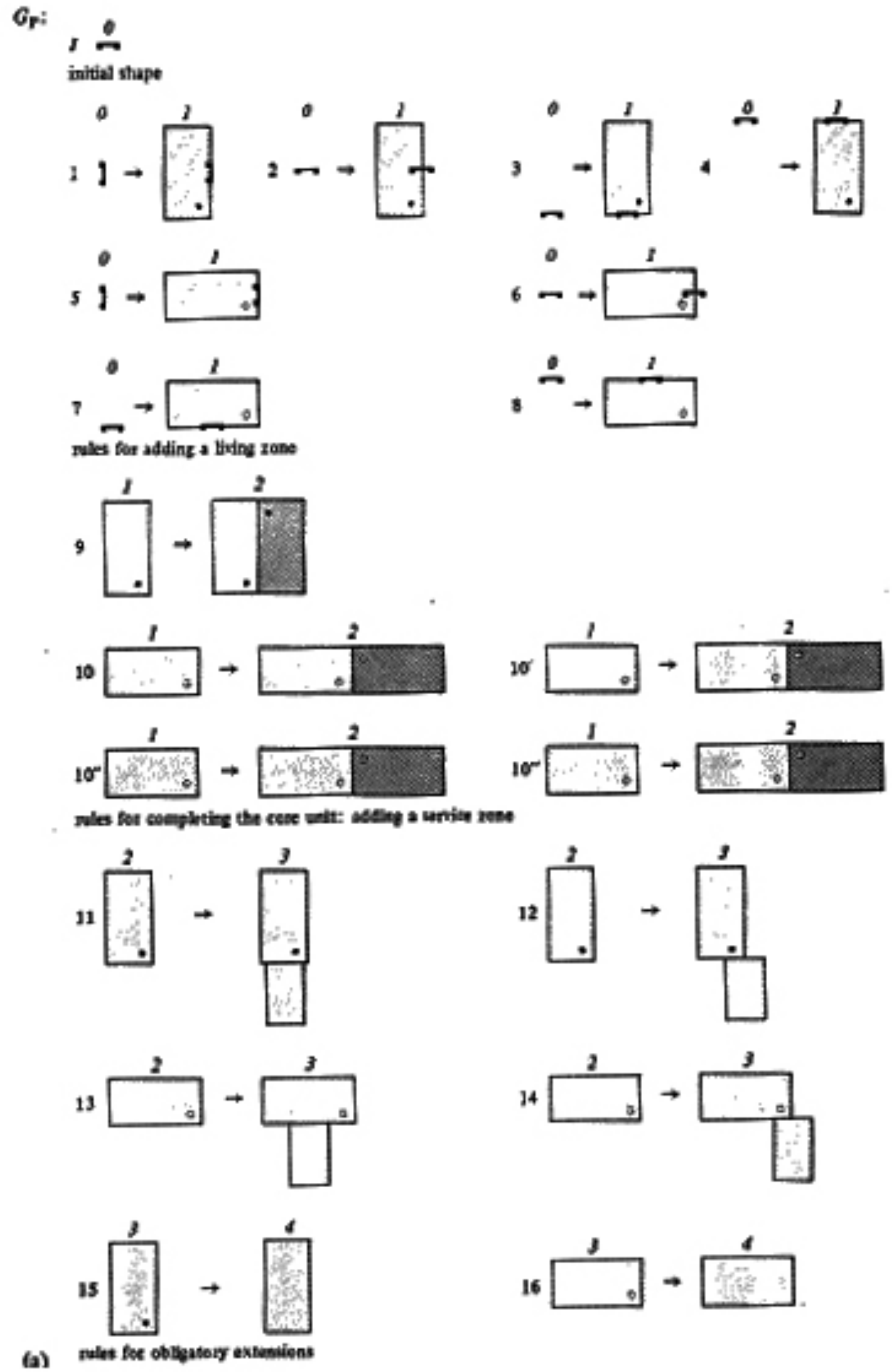
$T_{11}(G)$:



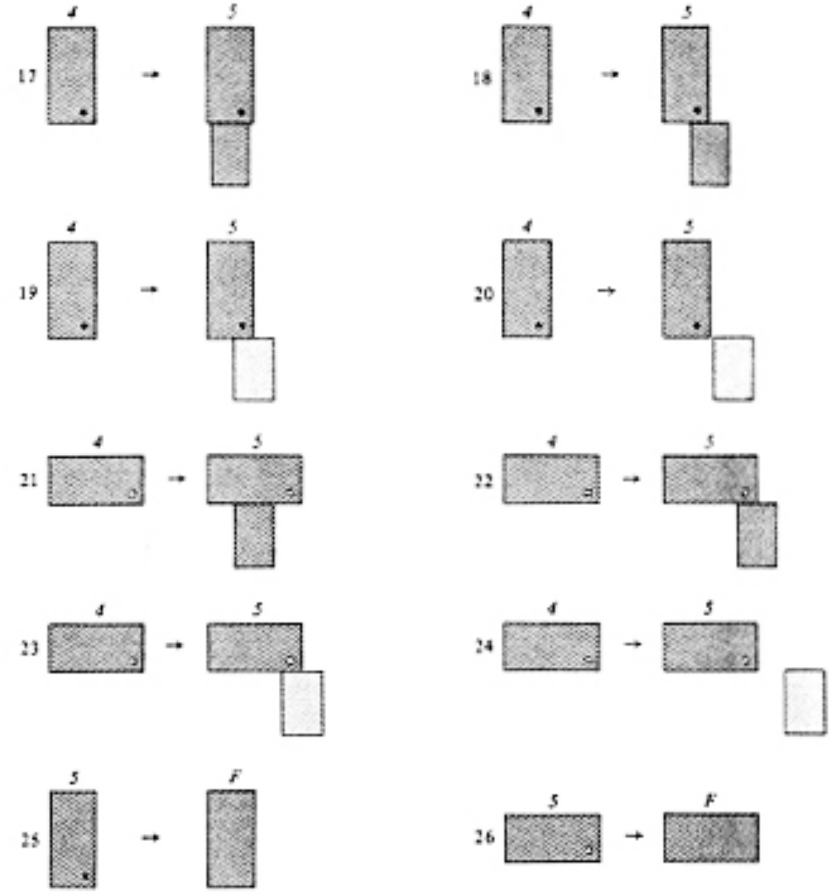
$T_{12}(G)$:



Prairie to Usonian

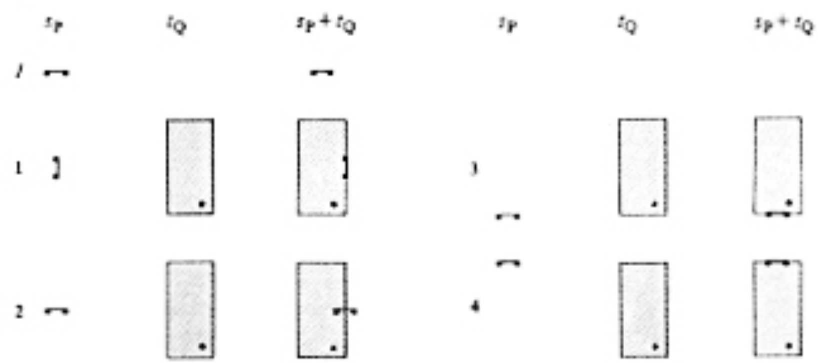


the prairie house grammar in formal form

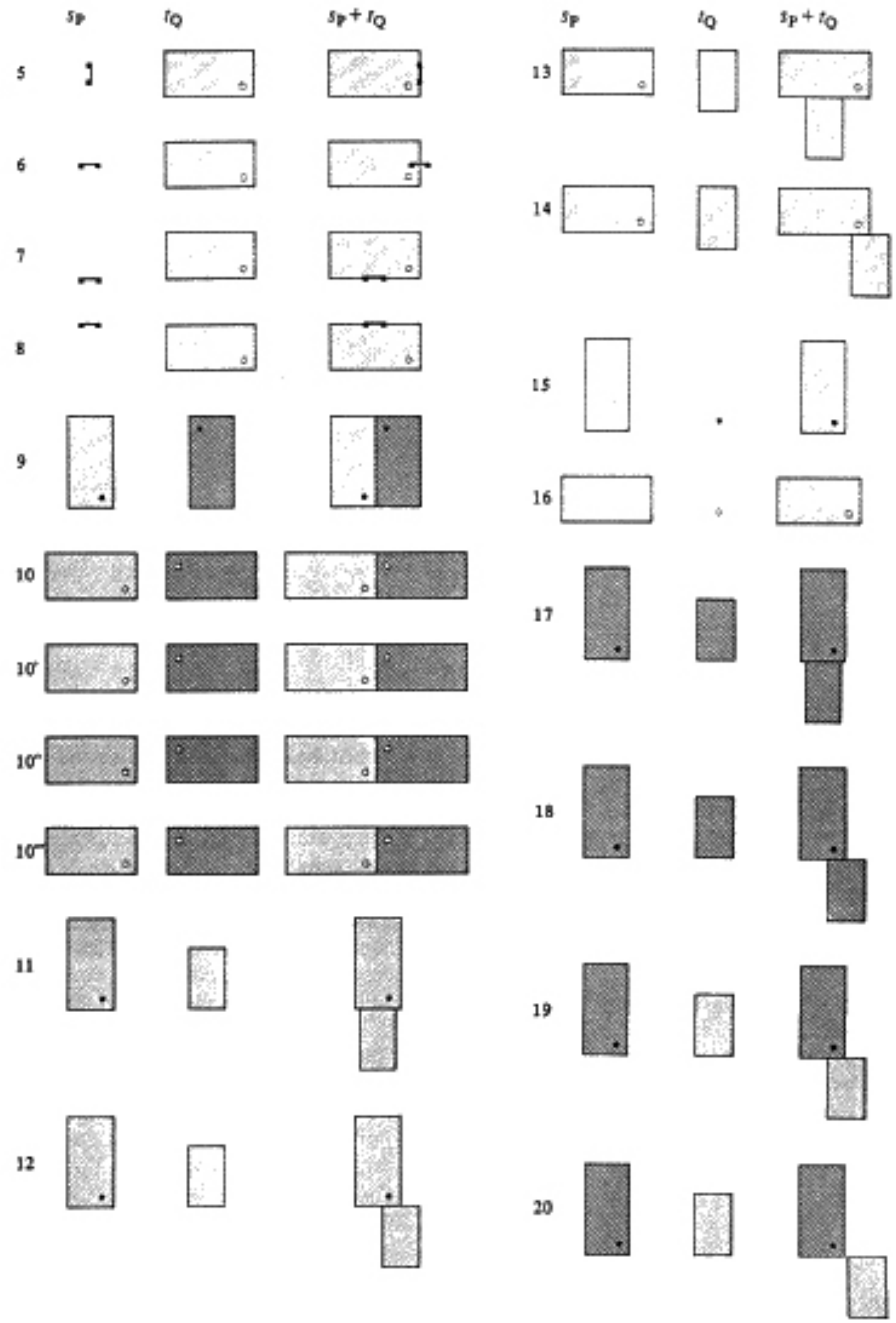


rules for obligatory extensions (continued)
final state: *F*

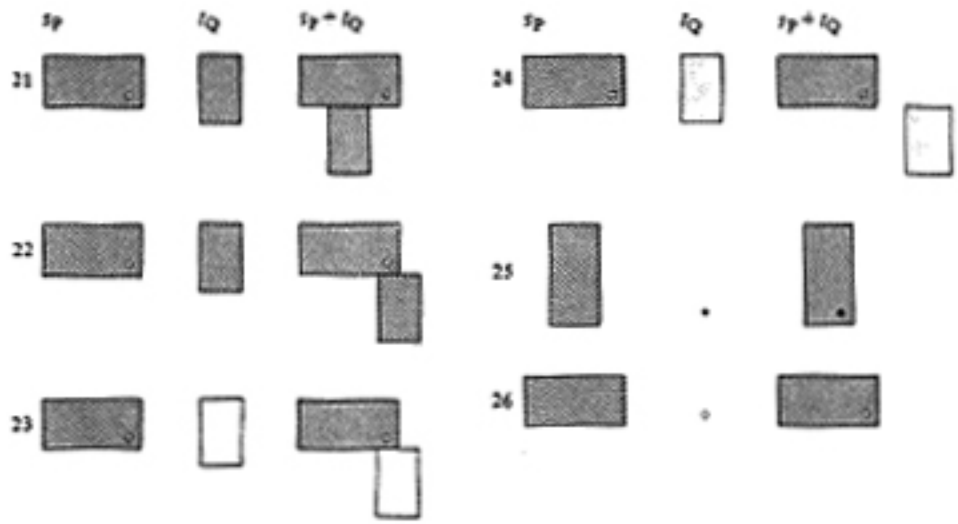
(a) continued



(b)



(b) continued

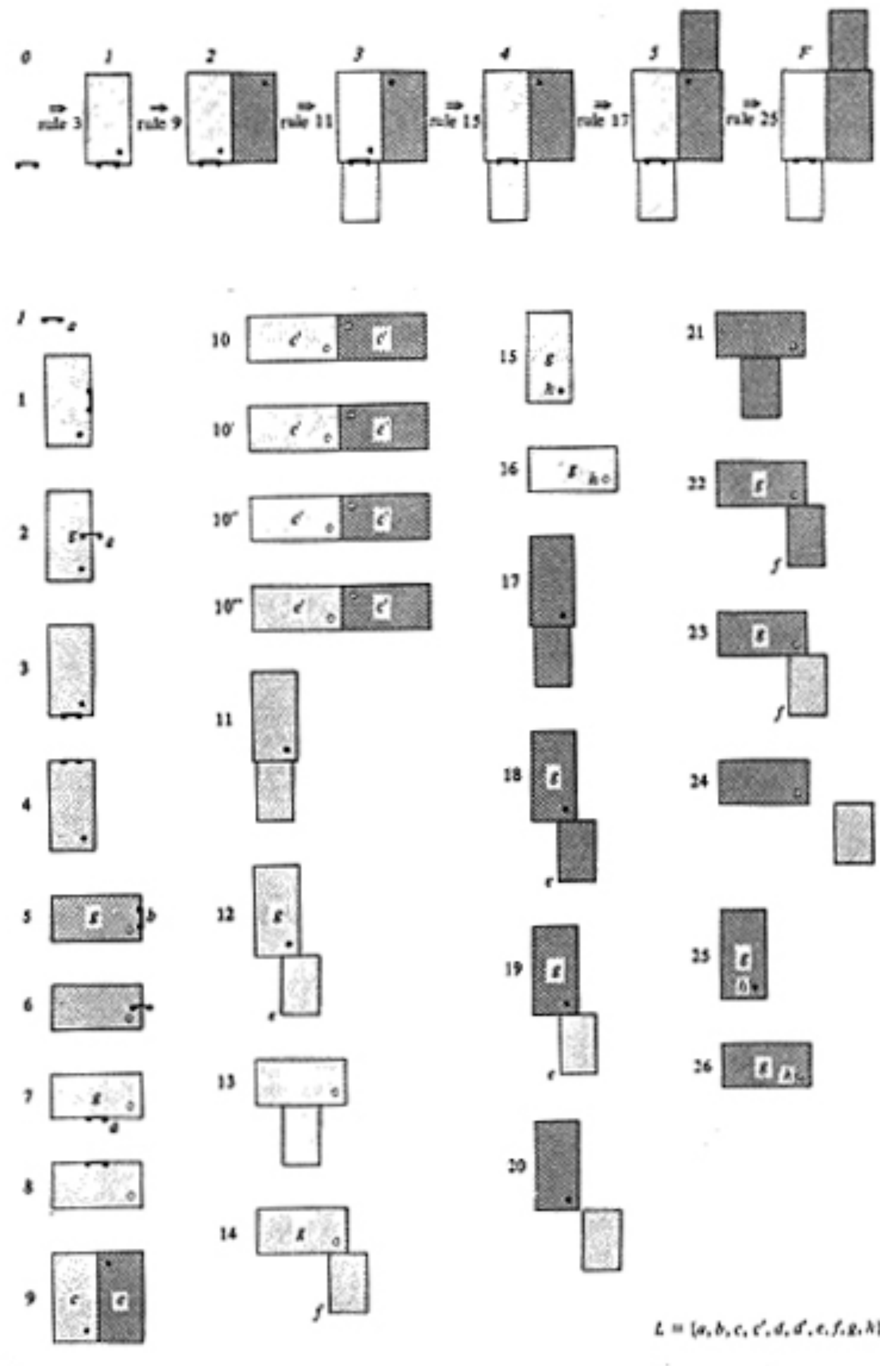


(b) continued

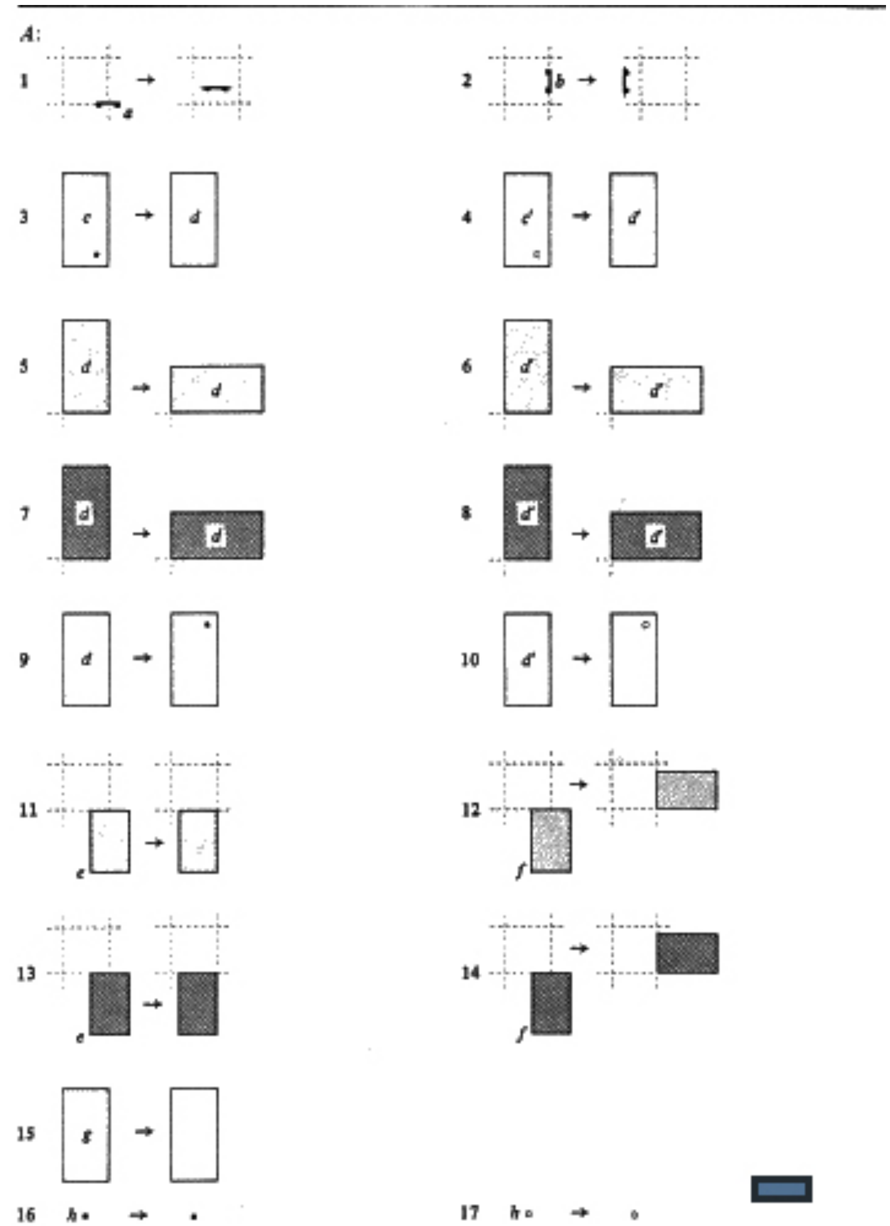
I	(*a*, *0*)	6	(*0*, *1*)	10″	(*1*, *2*)	15	(*3*, *4*)	21	(*4*, *5*)
1	(*0*, *1*)	7	(*0*, *1*)	10‴	(*1*, *2*)	16	(*3*, *4*)	22	(*4*, *5*)
2	(*0*, *1*)	8	(*0*, *1*)	11	(*2*, *3*)	17	(*4*, *5*)	23	(*4*, *5*)
3	(*0*, *1*)	9	(*1*, *2*)	12	(*2*, *3*)	18	(*4*, *5*)	24	(*4*, *5*)
4	(*0*, *1*)	10	(*1*, *2*)	13	(*2*, *3*)	19	(*4*, *5*)	25	(*5*, *F*)
5	(*0*, *1*)	10′	(*1*, *2*)	14	(*2*, *3*)	20	(*4*, *5*)	26	(*5*, *F*)

(c)

initial set of spatial relations



change rules



(a)

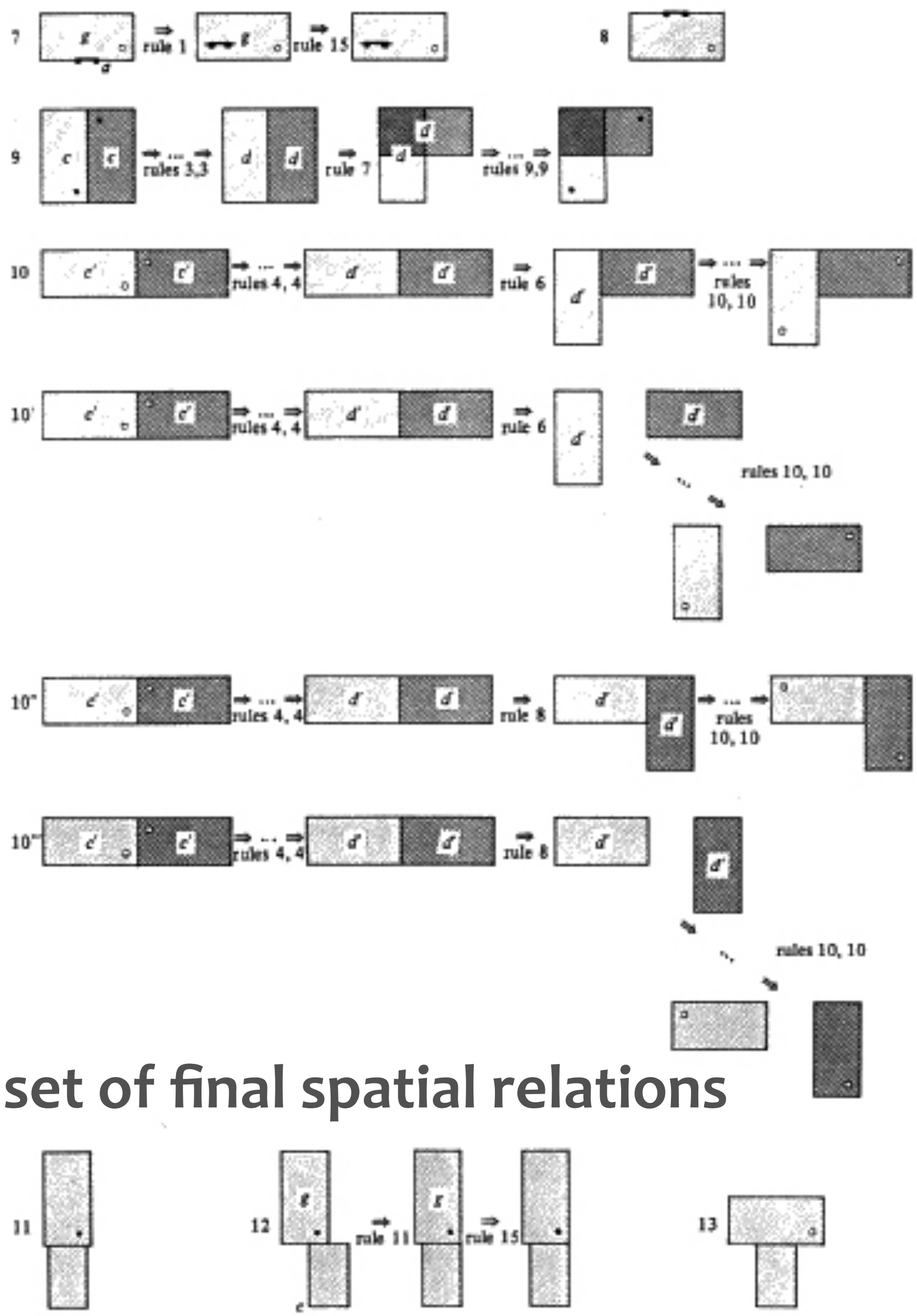
B:

$r_1, r_2, r_5, r_7, r_9, r_{10}, r_{10'}, r_{10''}, r_{10'''}$: identity state change rules

r_{13}, r_{14}:	$(2, 3) \rightarrow (2, 4)$
r_{15}, r_{16}:	$(3, 4) \rightarrow (\#, 3)$
$r_{18}, r_{19}, r_{22}, r_{23}$:	$(4, 5) \rightarrow (3, F)$
r_{20}, r_{24}:	$(5, F) \rightarrow (\#, F)$

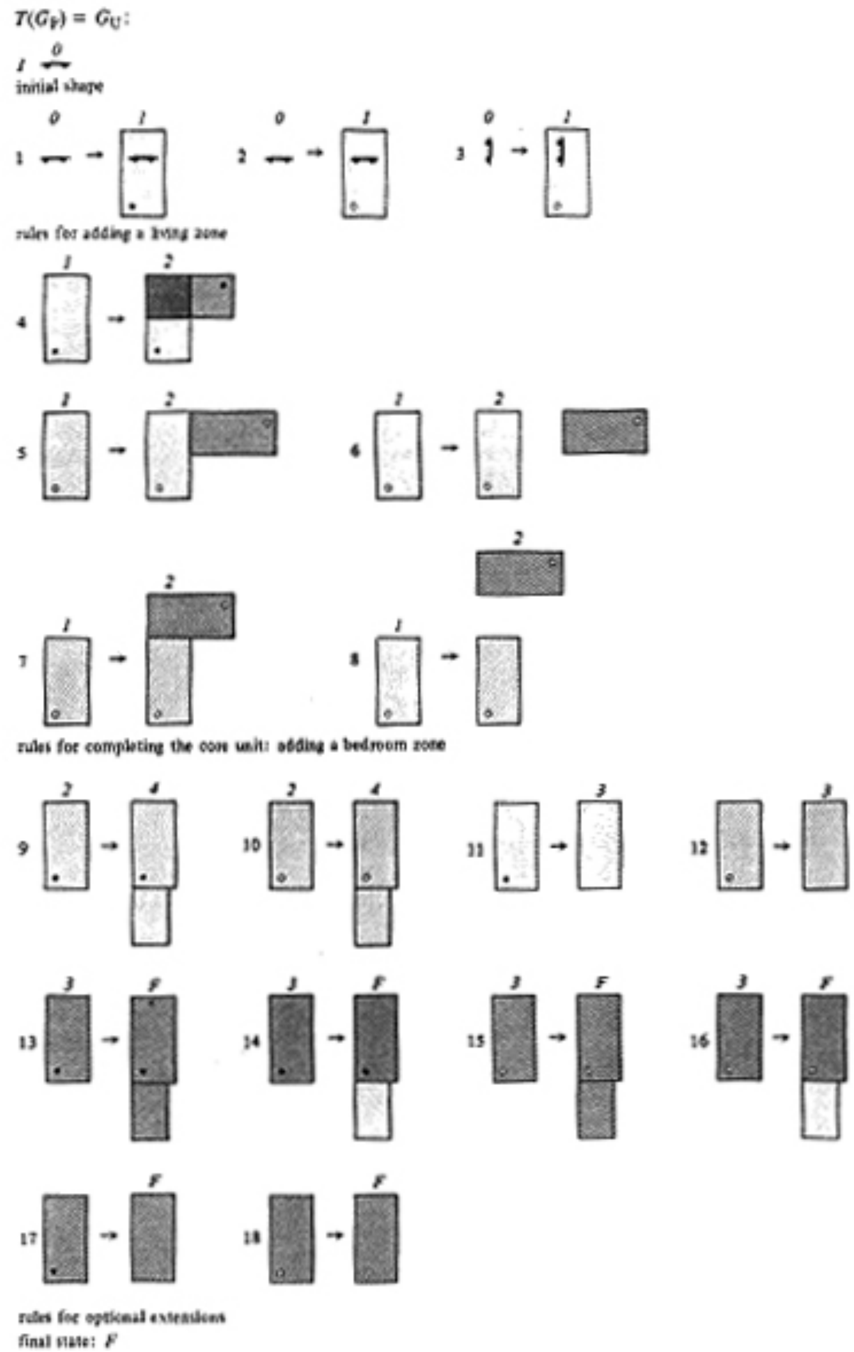
(b)

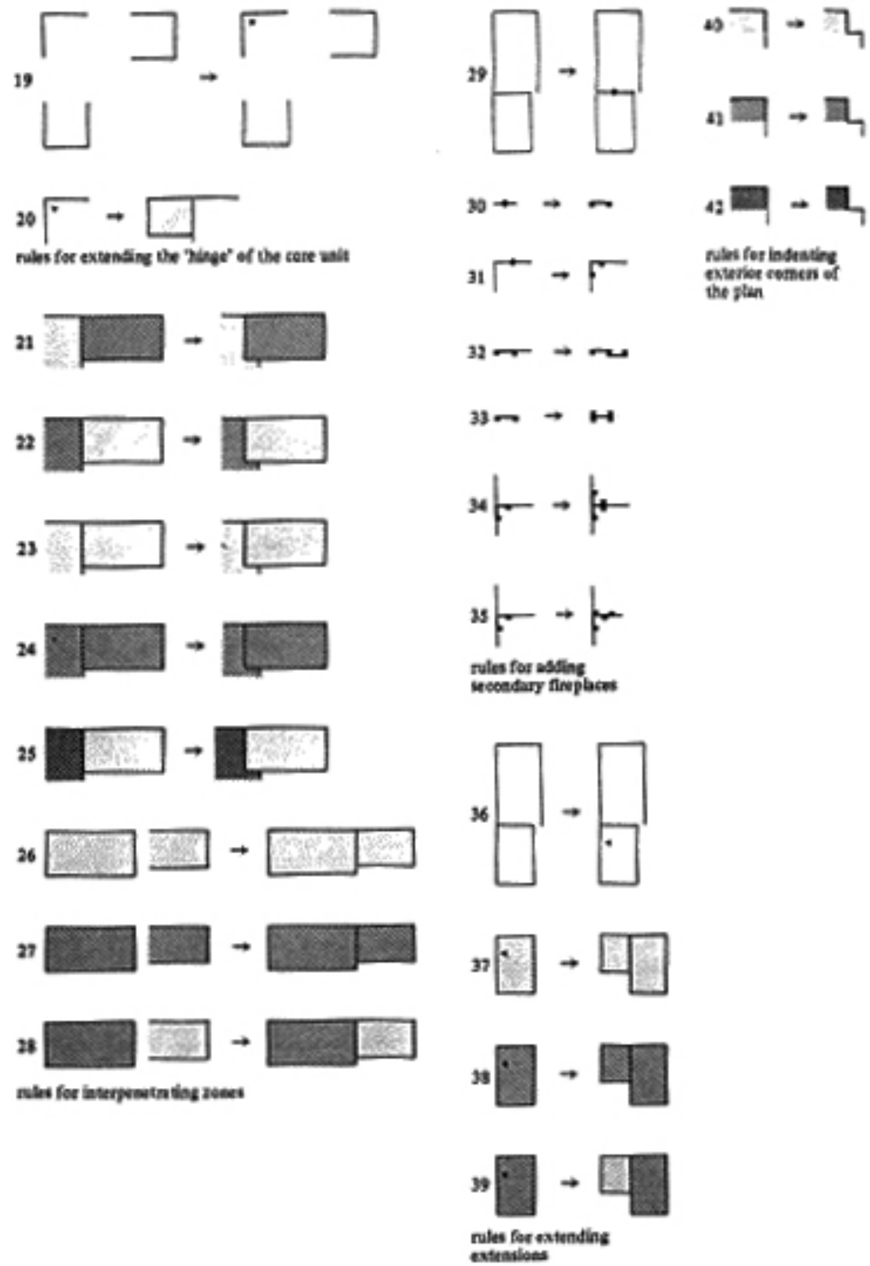
deriving a set of final spatial relations



(a) continued

transformation !




19
20
rules for extending the 'hinge' of the core unit
21
22
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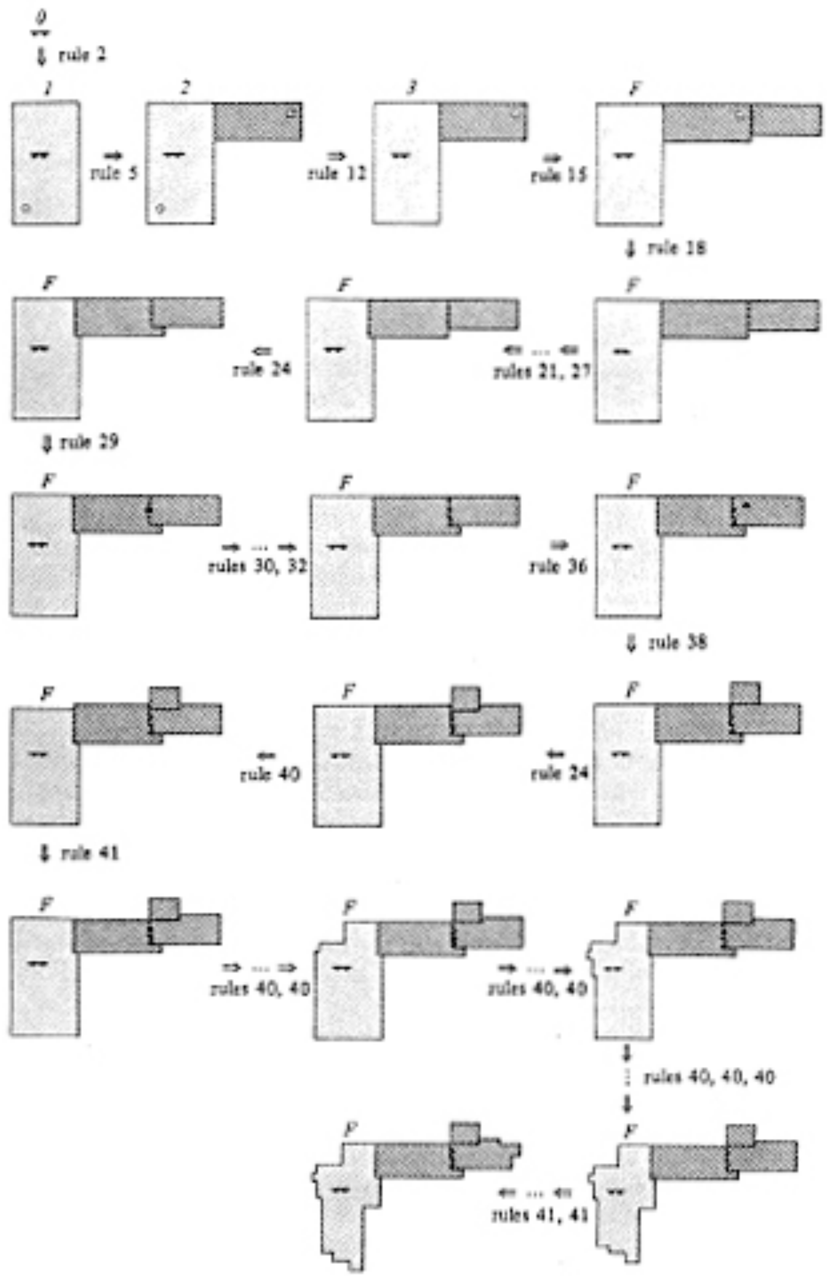
27
28
rules for interpenetrating zones
29
30
31
32
33
34
35
rules for adding secondary fireplaces
36
37
38
39
rules for extending extensions
40
41
42
rules for indenting exterior corners of the plan



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rule 2
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rule 5
rule 12
rule 15
rule 18
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