48-747 Shape Grammars

PARAMETRIC SHAPE GRAMMARS

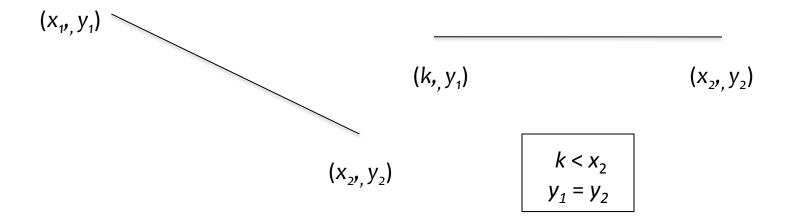A **parametric shape** is a shape with *open terms*

That is, certain points are specified in terms of *equations* or *constraints*

A parametric shape becomes a shape whenever we can find an assignment of real values to the points so that the constraints are satisfied
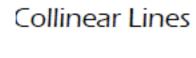
Example:

A square of given sides with one of its corners given fixed coordinates

parametric **shapes**

(x_1, y_1)

(k, y_1) (x_2, y_2)

(x_2, y_2)

$$k < x_2$$
$$y_1 = y_2$$

parametric shapes

Collinear Lines

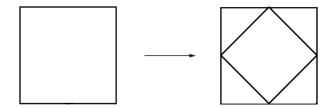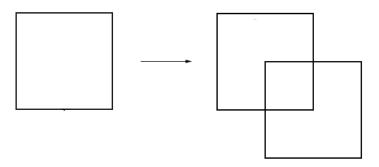(x_4, y_4)

(x_2, y_2)

(x_3, y_3)

(x_1, y_1)

(at a minimum)
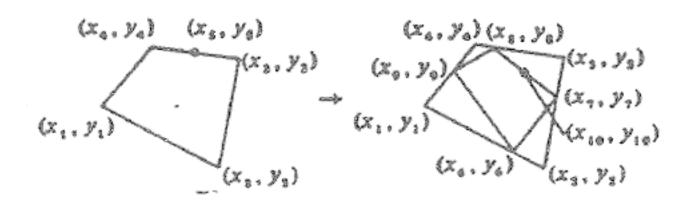is pictorially
equivalent
to the constraints

$$x_4 \geq x_3 \geq x_2 \geq x_1$$
$$y_4 > y_3 > y_2 > y_1$$
or
$$x_4 > x_3 > x_2 > x_1$$
$$y_4 \geq y_3 \geq y_2 \geq y_1$$
and
$$(y_2 - y_1)(x_4 - x_3) = (x_2 - x_1)(y_4 - y_3)$$
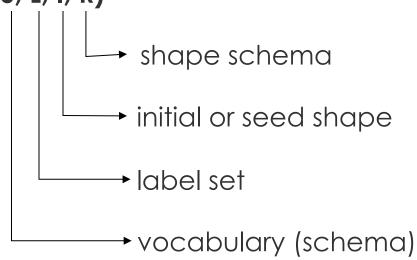and

x_i, y_i are real numbers.

shape rules

shape schema

parametric shape grammar G = (S, L, I, R)

shape schema

initial or seed shape

label set

vocabulary (schema)

initial (seed) shape
belongs to the *universe*
of labeled shapes made
up of schemas in S and
labels in L

R contain schemas of the form a → b where a and b
belong to the universe of labeled shape schemas made of
schemas in S and labels in L except **a** cannot be empty

formally: a parametric shape grammar is

A shape schema is *applicable* to the *current shape*
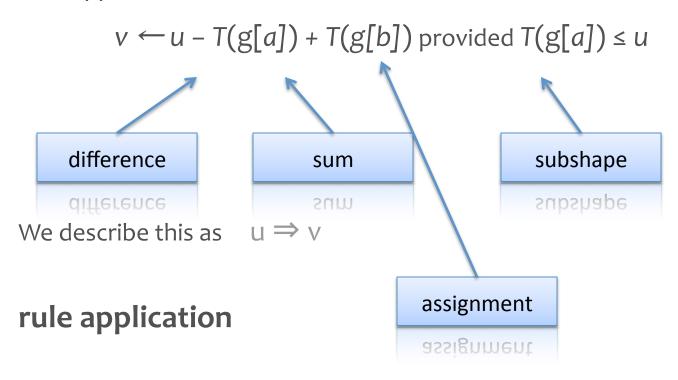
which is either the initial shape

       or a shape produced from the initial shape

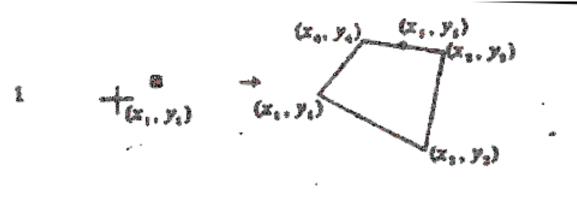*whenever* the left hand side of the rule '*occurs*' in the object

in which case

it is *replaced* by the right hand side of the rule under rule

application

shape rule application

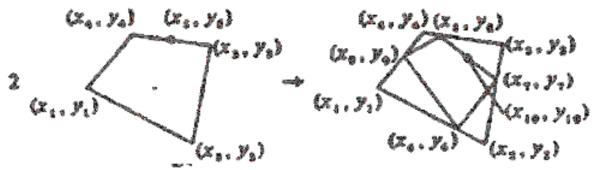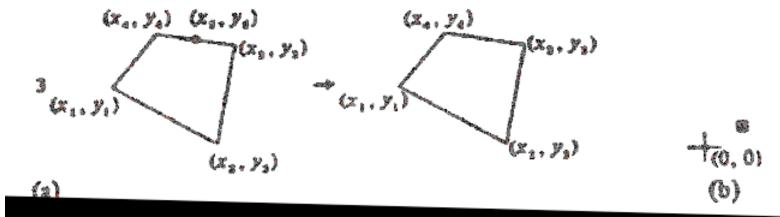A schema $a \rightarrow b$ is applies only if a 'occurs' in the given shape u under some 'transformation' T I and an assignment g in which case $T(g[a])$ is substituted by $T(g[b])$ in the current shape

Rule application

$$v \leftarrow u - T(g[a]) + T(g[b]) \text{ provided } T(g[a]) \leq u$$

difference

sum

subshape

We describe this as $u \Rightarrow v$

rule application

assignment

1

(x_1, y_1) → (x_4, y_4) (x_5, y_5) (x_6, y_6)

(x_8, y_8)

(x_3, y_3)

2

(x_4, y_4) (x_5, y_5) (x_6, y_6)

(x_1, y_1)

(x_2, y_2)

→

(x_4, y_4) (x_5, y_5)

(x_8, y_8) (x_9, y_9)

(x_7, y_7)

(x_1, y_1) (x_{10}, y_{10})

(x_6, y_6) (x_3, y_3)

3

(x_4, y_4) (x_5, y_5)

(x_6, y_6)

(x_1, y_1)

(x_2, y_2)

→

(x_4, y_4)

(x_5, y_5)

(x_1, y_1)

(x_2, y_2)

$(0, 0)$

(a)   (b)

go to some grammar examples