

48-747 Shape Grammars

SHAPE GRAMMARS

computation

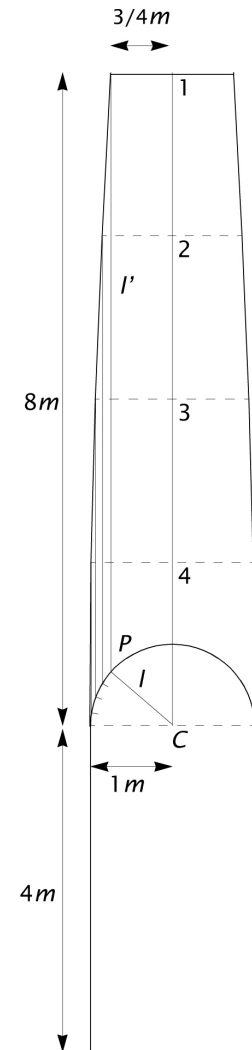
Giacomo Barozzi da Vignola (1507-73)

An elementary treatise on architecture comprising the complete study of five orders, with indication on their shadows and the first principles of construction

Constructing the profile of a classical tapered column

There are five steps.

1. Determine the height and largest diameter of the column, d . There are clear rules about preferred proportions between the height and diameter of various types of classical columns (*doric, ionic* etc.) These measures are normally related to each other as integral multiples of a common module, m . Figure shows the shaft of a column with diameter $2m$ and height $12m$. That is, the proportion between diameter and height is 2:12 or 1:6.
2. At the shaft's height, draw a straight line, l , across the shaft and draw a semi-circle, c , about the center point of l , C , with radius d ($1m$ in the figure). The shaft will have the uniform diameter d below line l .
3. Determine the smallest diameter at the top of the shaft ($1.5m$ in our case). Draw a perpendicular, l' , through an end-point of the diameter. l' intersects c at a point P . The line through P and C defines together with l a segment of c .
4. Divide the segment into segments of equal size and divide the shaft above l into the same number of sections of equal height.
5. Each of these segments intersects c at a point. Draw a perpendicular line through each of these points and find the intersection point with the corresponding shaft division as shown in the Figure. Each intersection point is a point of the profile.

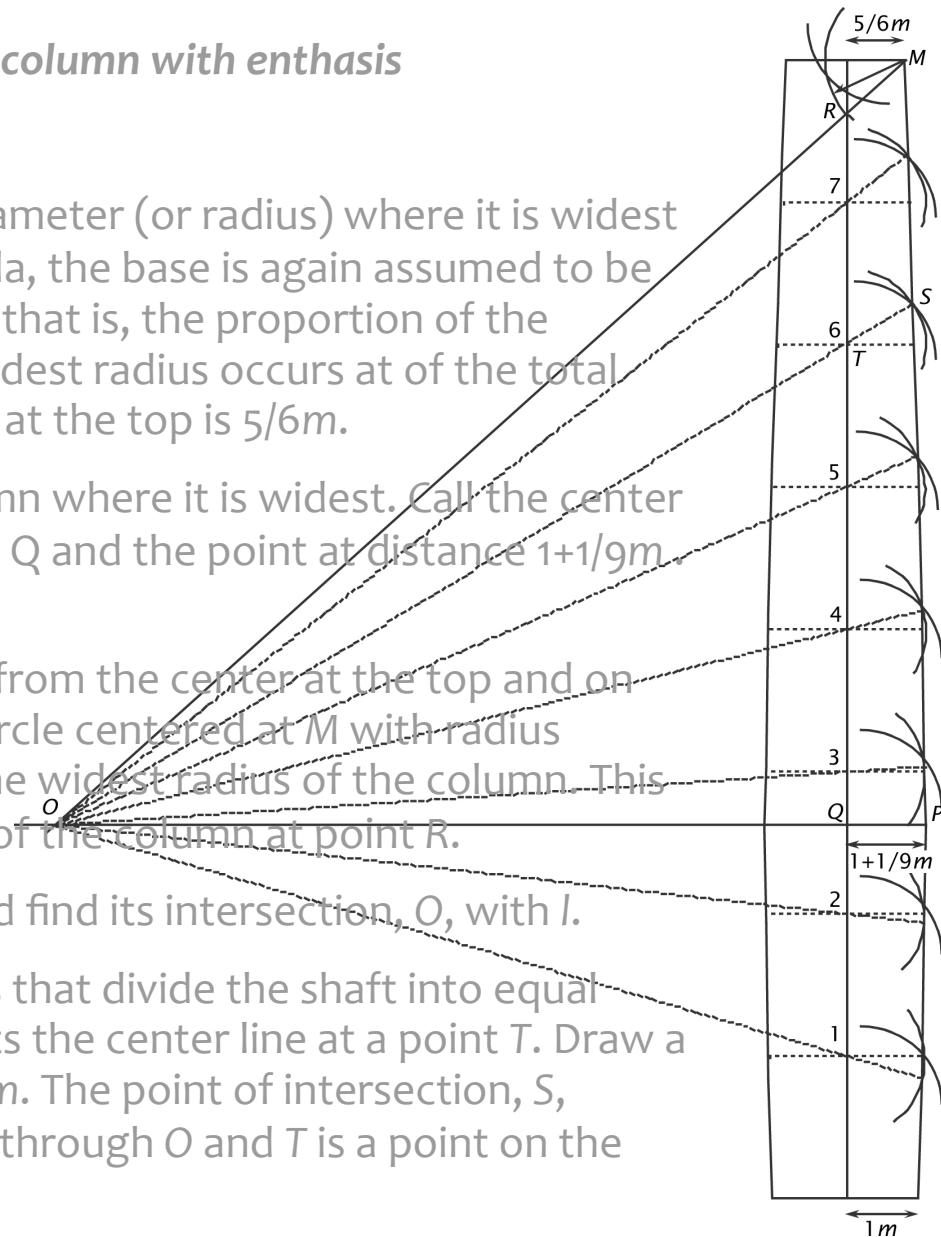


shape grammars have something to do with **computation**

Constructing the profile of a classical column with entasis

Here, too, there are five steps.

1. Determine the height and its diameter (or radius) where it is widest and at the top. Following Vignola, the base is again assumed to be $2m$ wide, and the height is $16m$; that is, the proportion of the diameter to height is $1:8$. The widest radius occurs at of the total height and is $1+1/9m$. The radius at the top is $5/6m$.
2. Draw a line, l , through the column where it is widest. Call the center point of the column on that line Q and the point at distance $1+1/9m$ from Q on l , P .
3. Call the point at distance $5/6m$ from the center at the top and on the same side as P , M . Draw a circle centered at M with radius $1+1/9m$, that is, the circle with the widest radius of the column. This circle intersects the center line of the column at point R .
4. Draw a line through M and R and find its intersection, O , with l .
5. Draw a series of horizontal lines that divide the shaft into equal sections. Any such line intersects the center line at a point T . Draw a circle about each T with radius m . The point of intersection, S , between this circle and the line through O and T is a point on the profile.



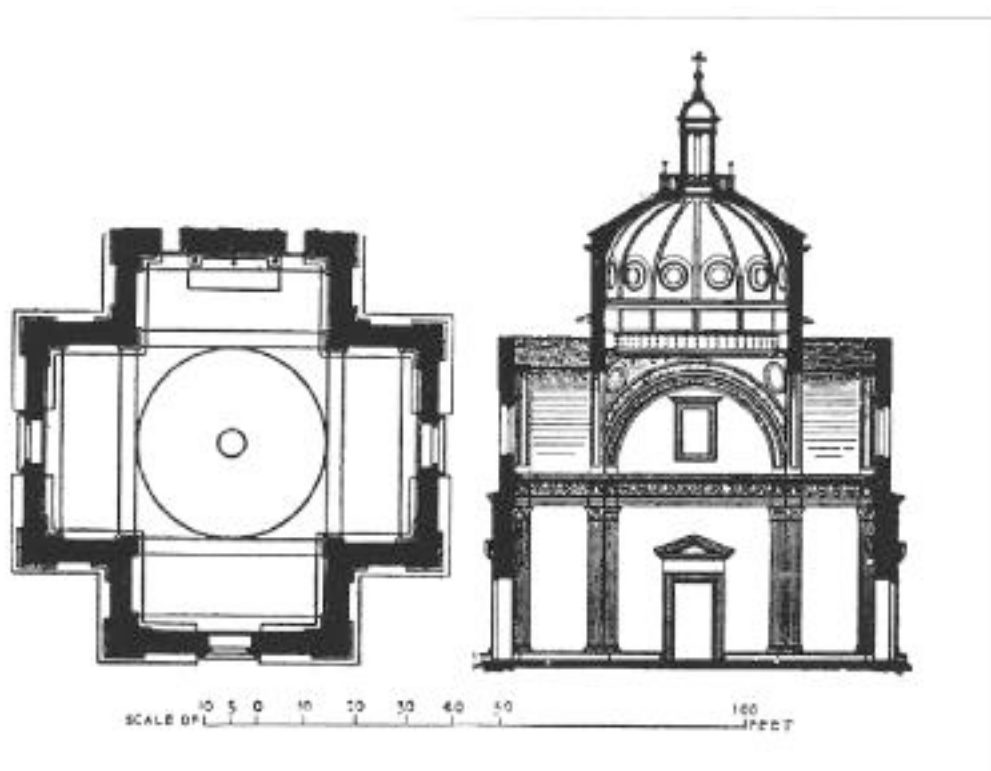
creative

descriptive

‘mechanical’

computation is ...

what is a shape grammar?



Shape



Shape relation

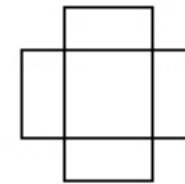
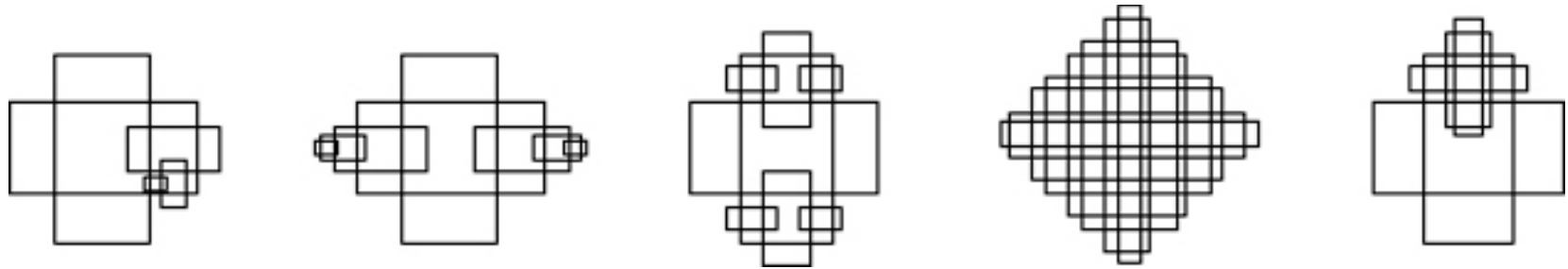
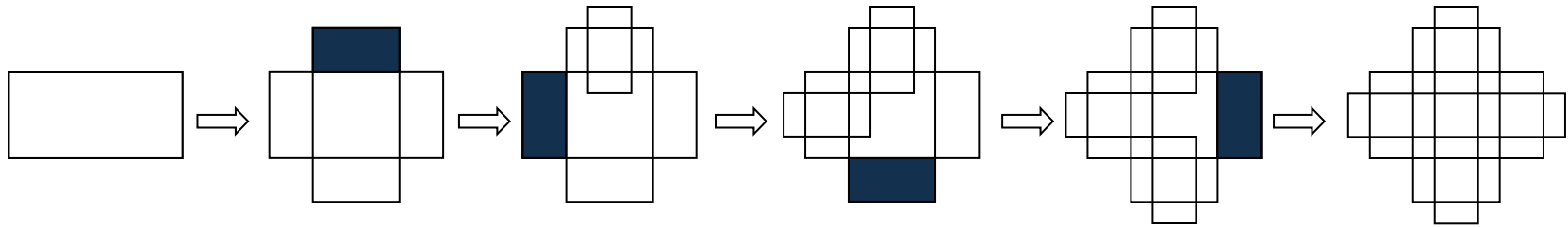
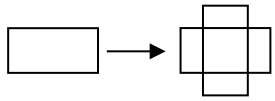


Illustration by Peter Murray, "the Architecture of the Italian Renaissance", Shoken Books Inc. 1963, Pp.96



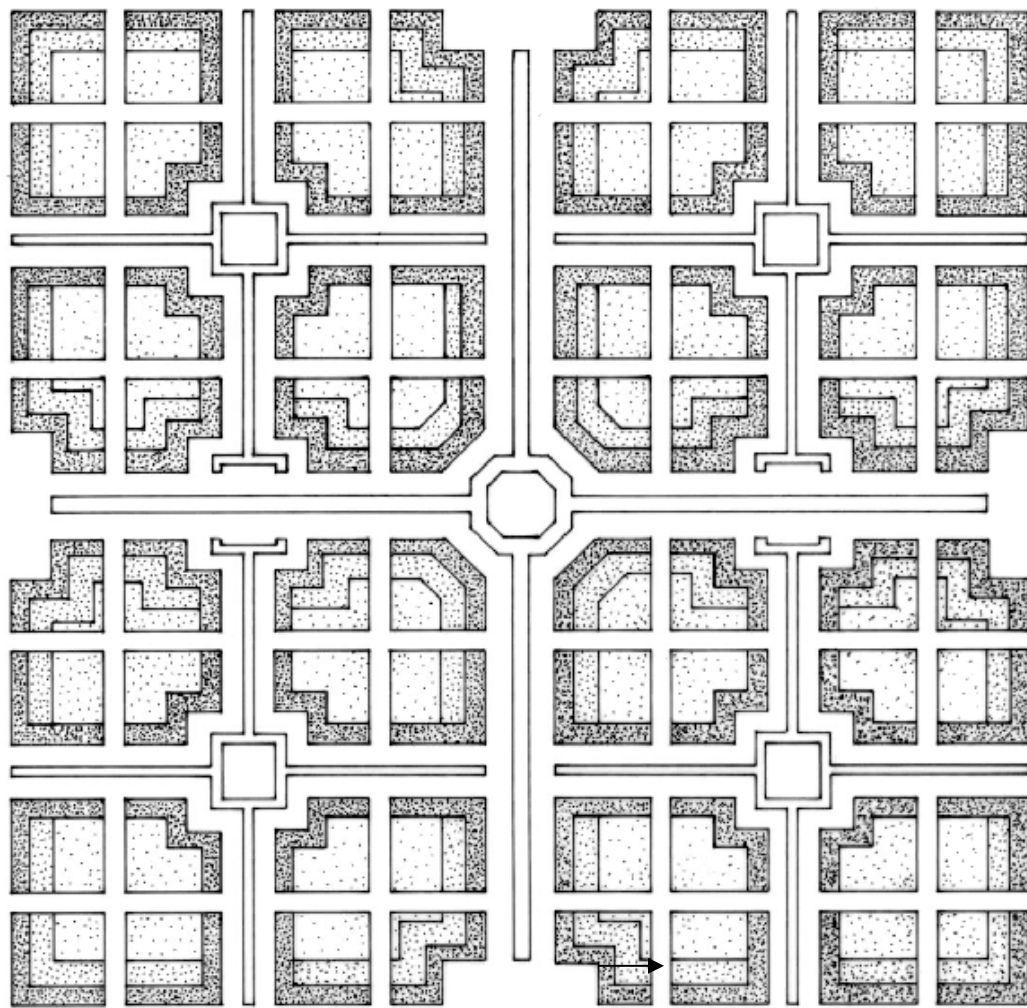
derivation

analyses

original designs

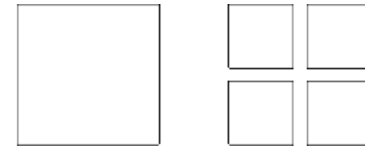
as explanatory devices

how are they used?

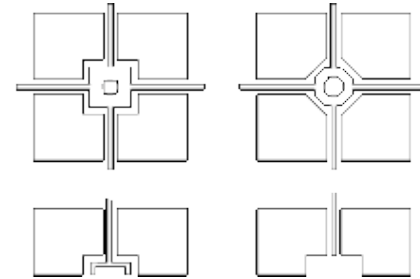


mughal gardens

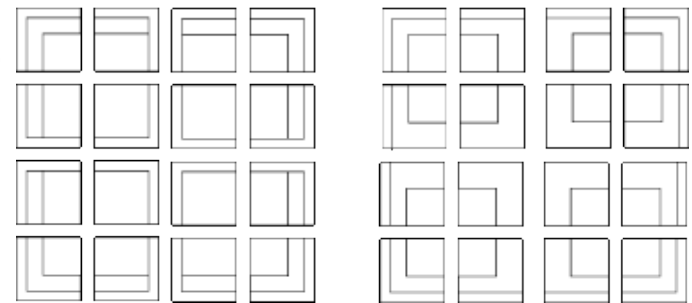
parti subdivision



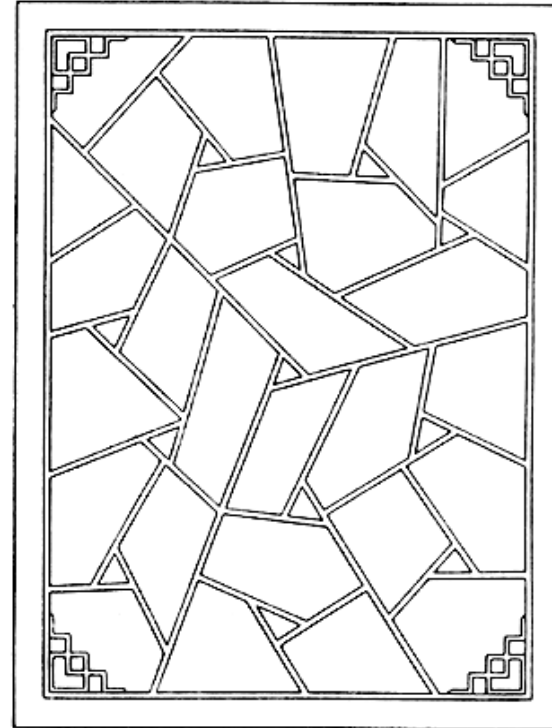
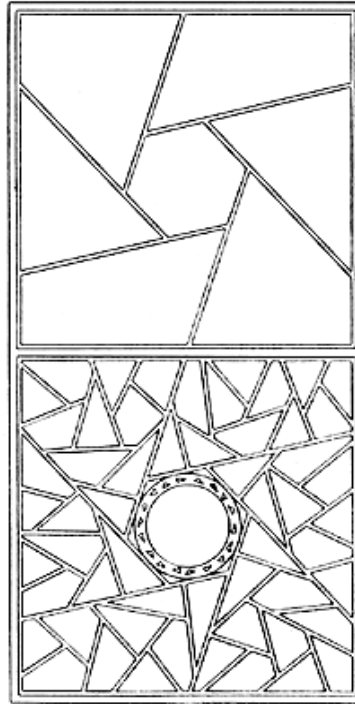
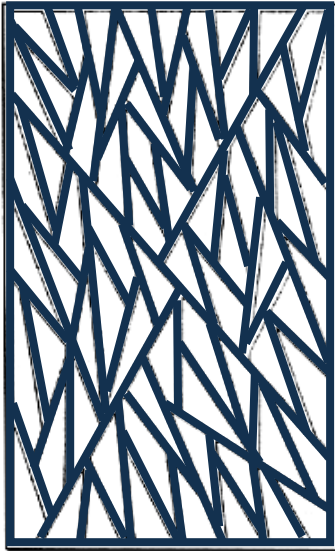
dealing with canals



dealing with borders



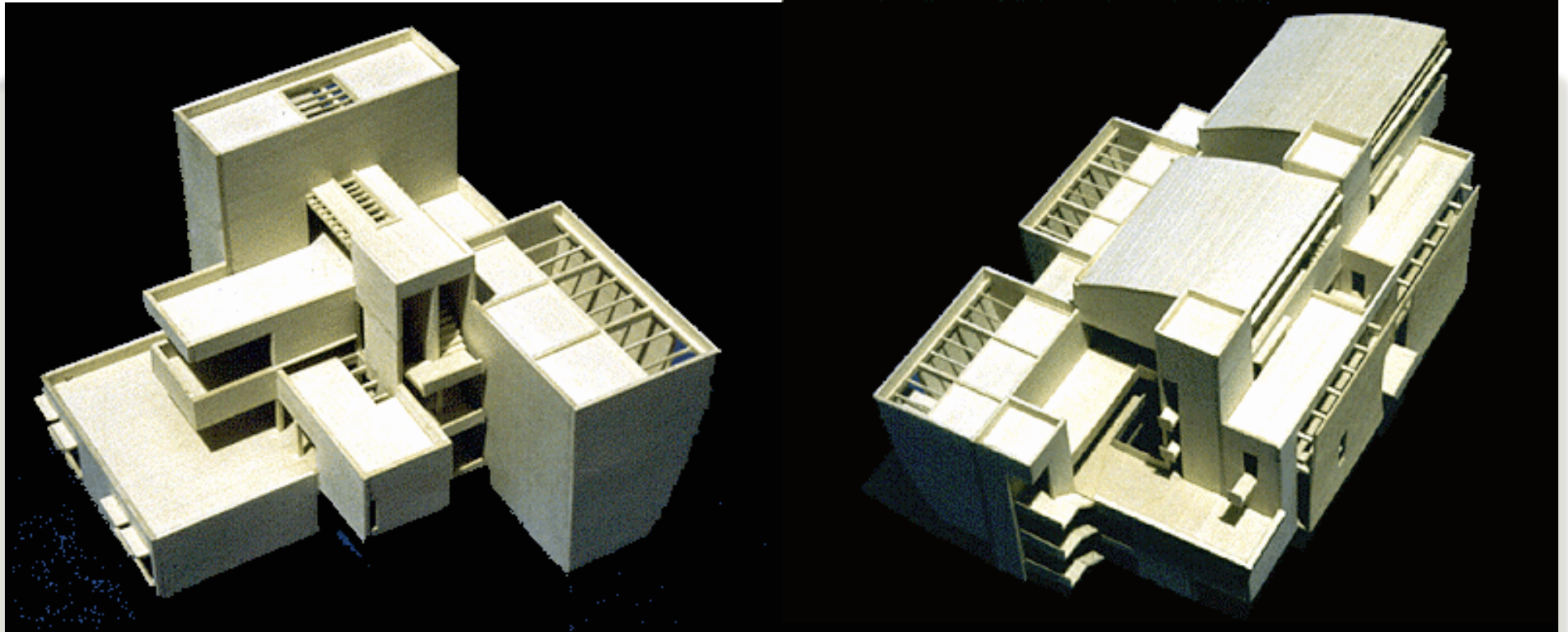
[demo](#)



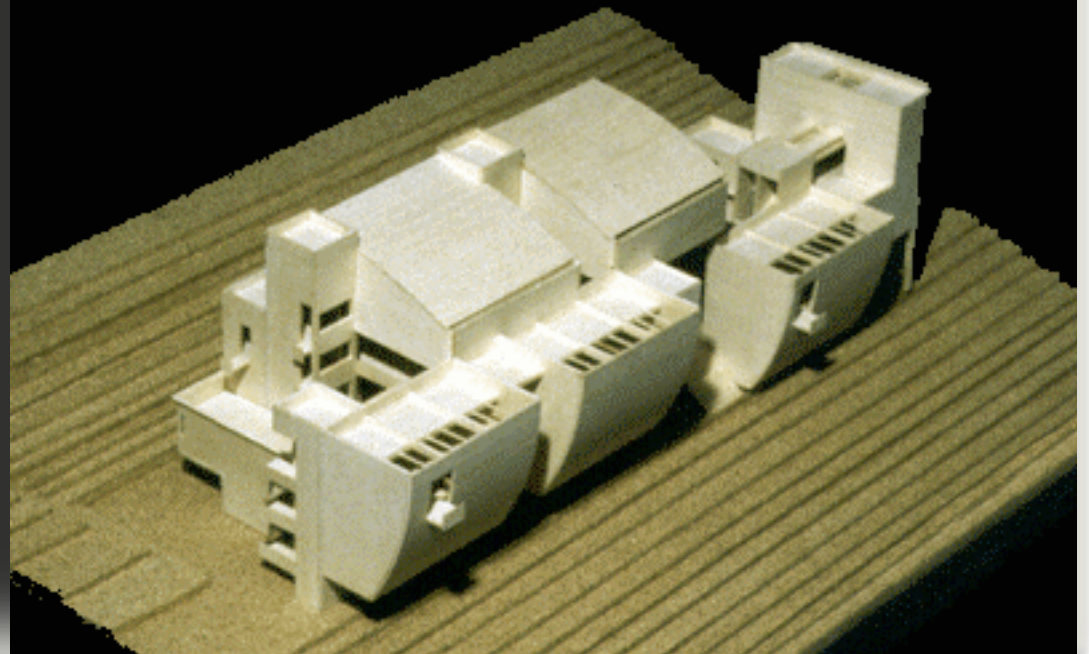
another - **ice-ray designs**



apartment building in manhattan



cultural museum in LA



developing a shape grammar

shapes

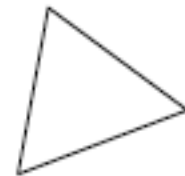
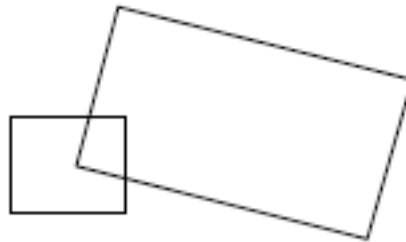
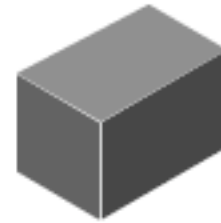
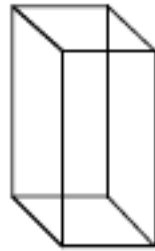
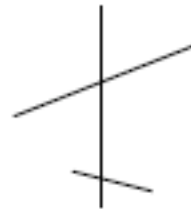
spatial relations

rules

shape grammar

designs

stages in a shape grammar **development**



basic components of grammars and design

shapes

Shape is a finite arrangement of lines of non-zero length with respect to a coordinate system

Shapes can be formed by *addition* of shapes which consists of lines in both shapes.

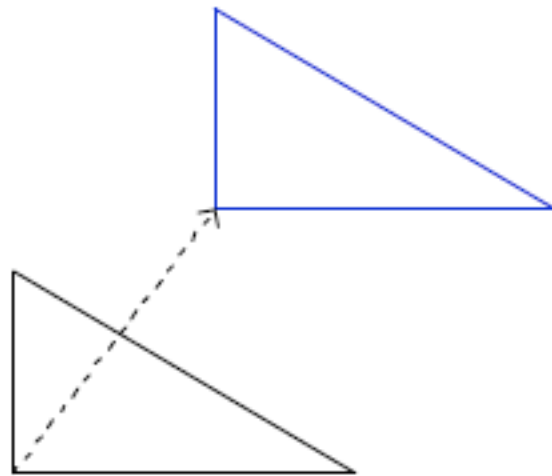
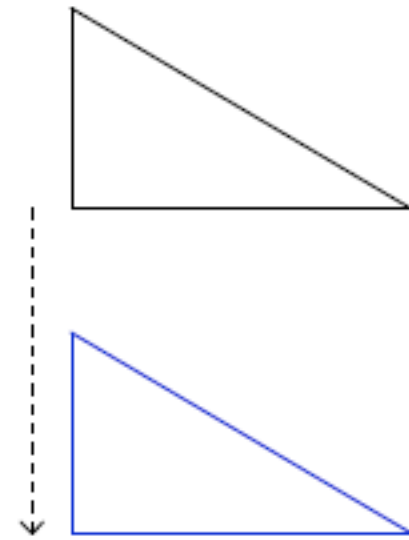
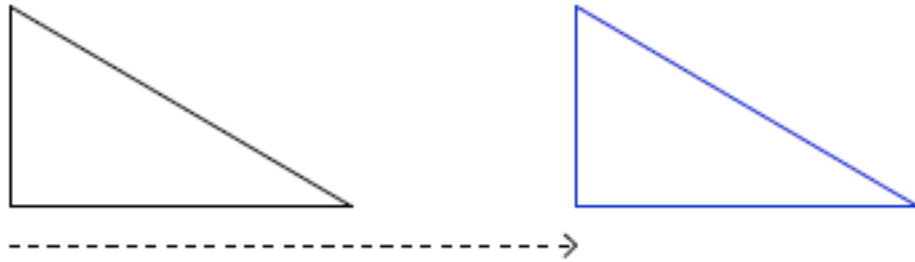
Shapes can be formed by *subtraction* of shapes which consists of the lines in the first shape that are not in the second.

Shapes can be formed by combinations of the two under an ***Euclidean transformation***

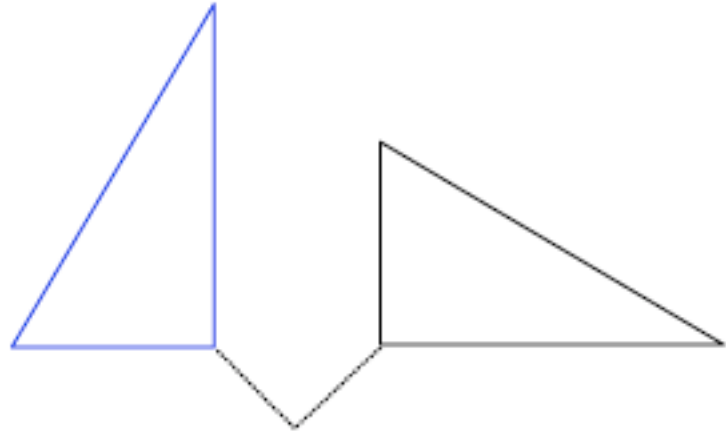
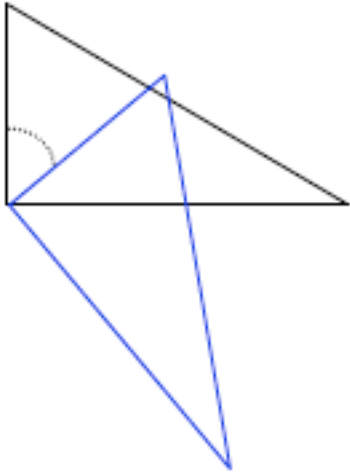
A central notion in this definition of shapes is ***pictorial equivalence***

for our purposes, **shape is ...**

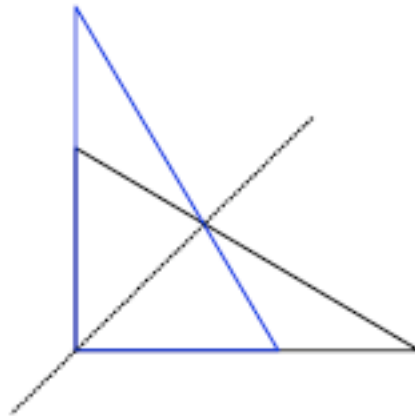
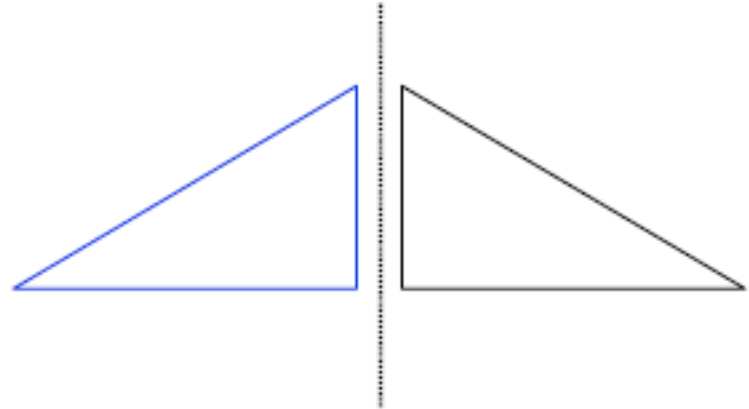
euclidean transformations



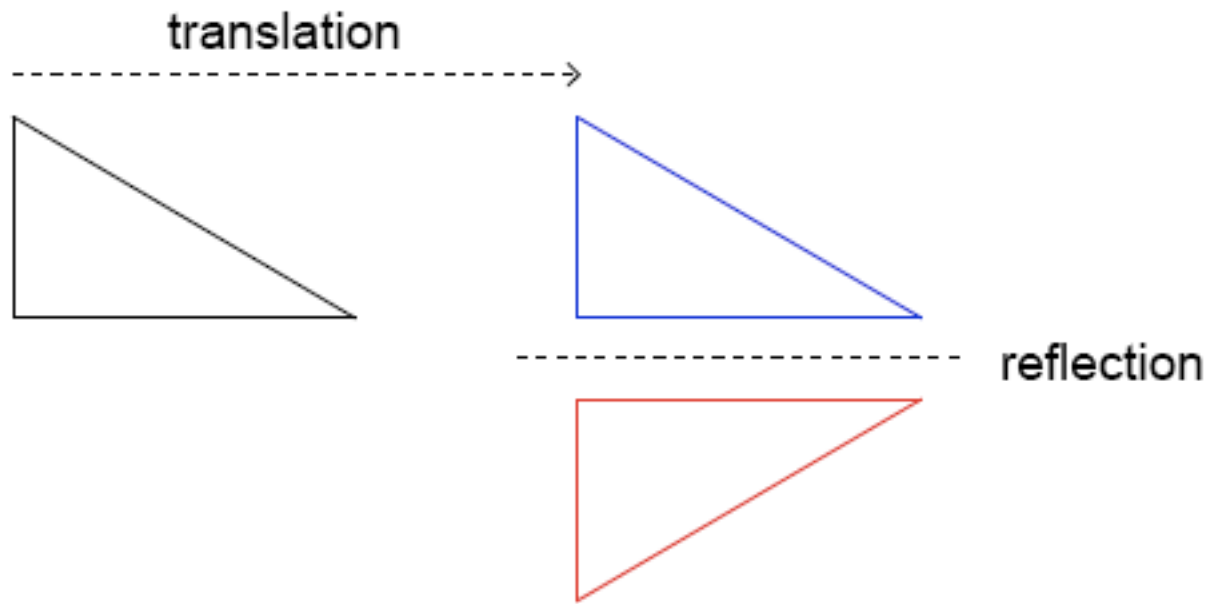
translation



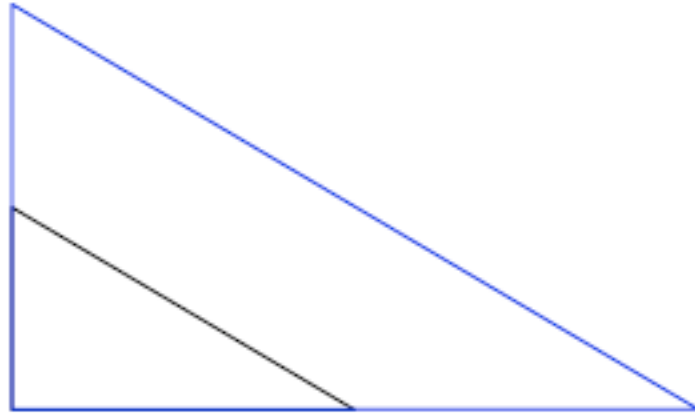
rotation



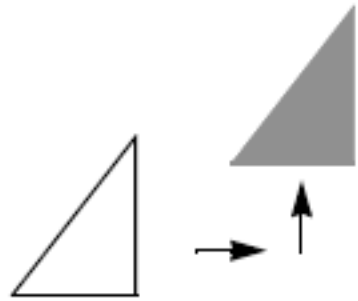
reflection



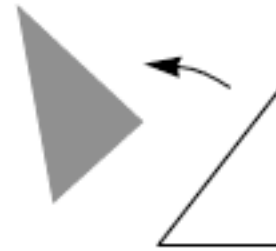
glide reflection – example of a combination



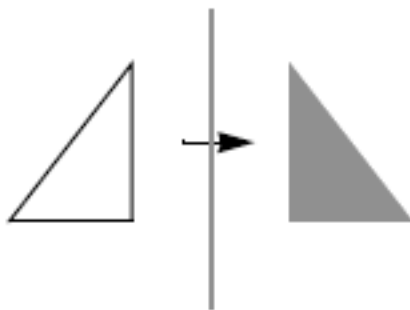
scale



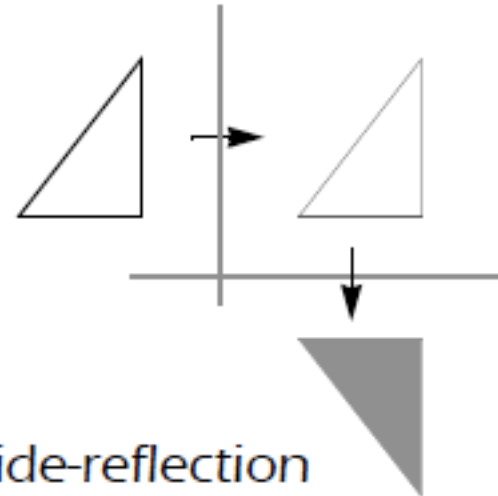
Translation



Rotation

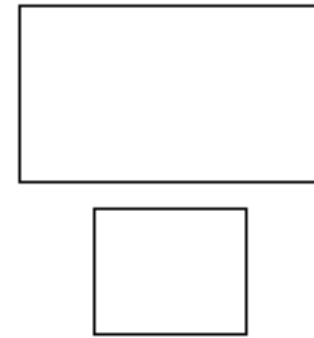
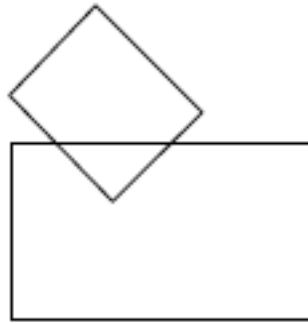
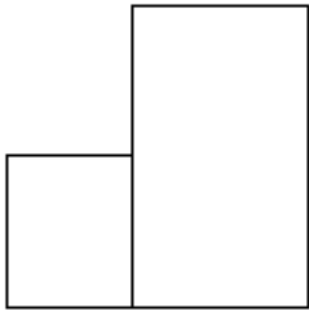


Reflection

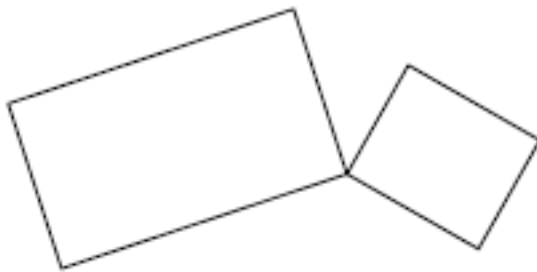


Glide-reflection

euclidean transformations + scale



When two or more shapes combine they form a *spatial relation*
That is, a set of shapes specifies a spatial relation



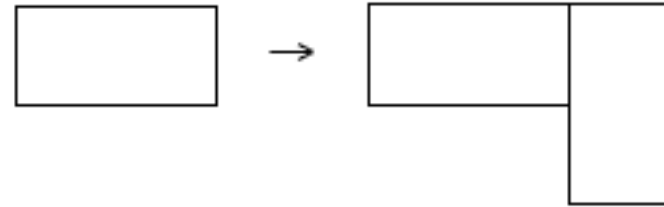
spatial relations

Shape	A, B
Relation	$A+B$
Rule	$A \rightarrow A + B$
	$B \rightarrow A + B$

shape rules



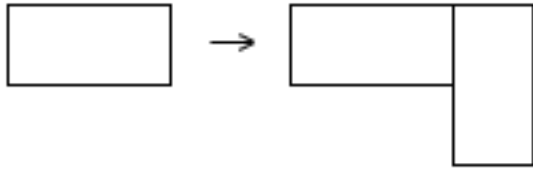
Relation



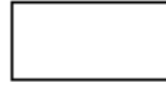
Rule

from **relation** to **rule**

rule



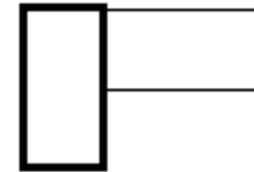
shape



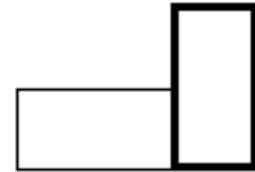
possible results



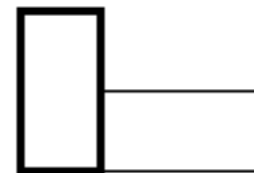
or



or



or

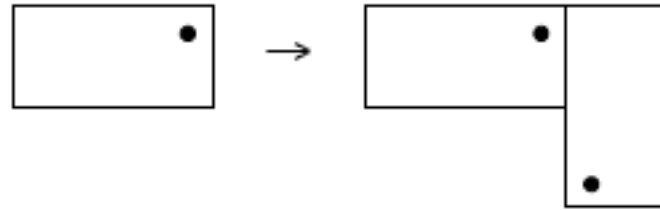


The application of Euclidean transformation does not alter the spatial relation

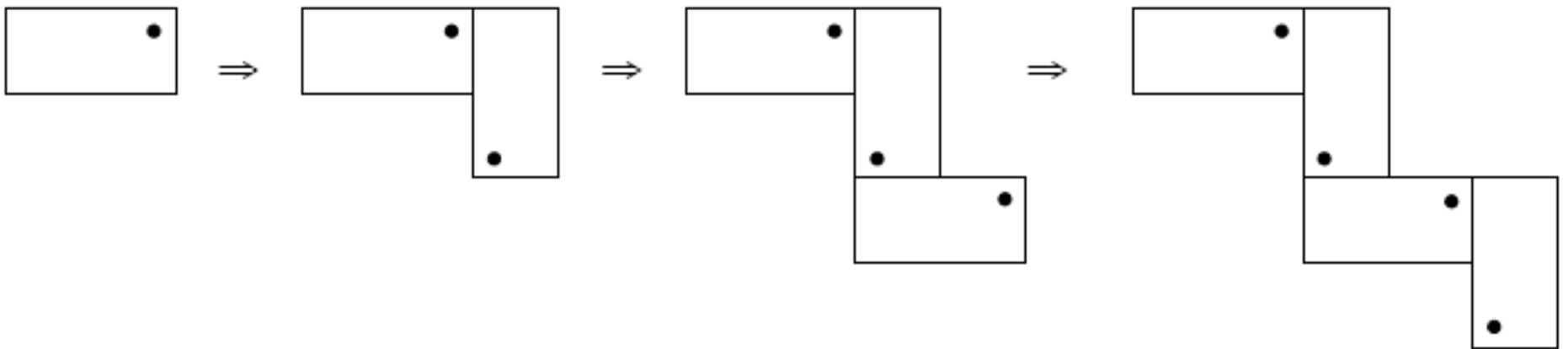
to **derivation**

labeling rules helps control rule application

Labeled rule

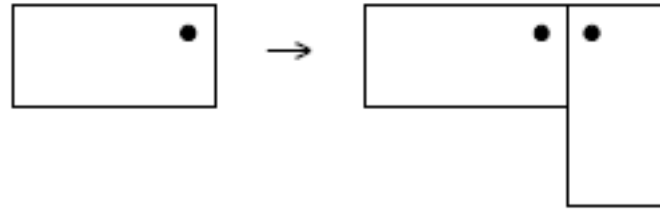


derivation

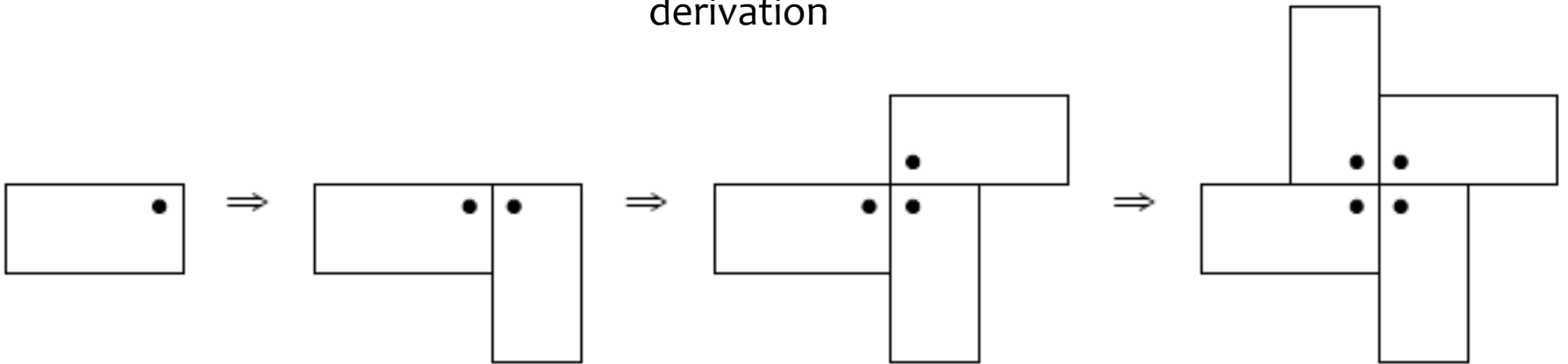


labeled rule based **derivation**

Labeled rule

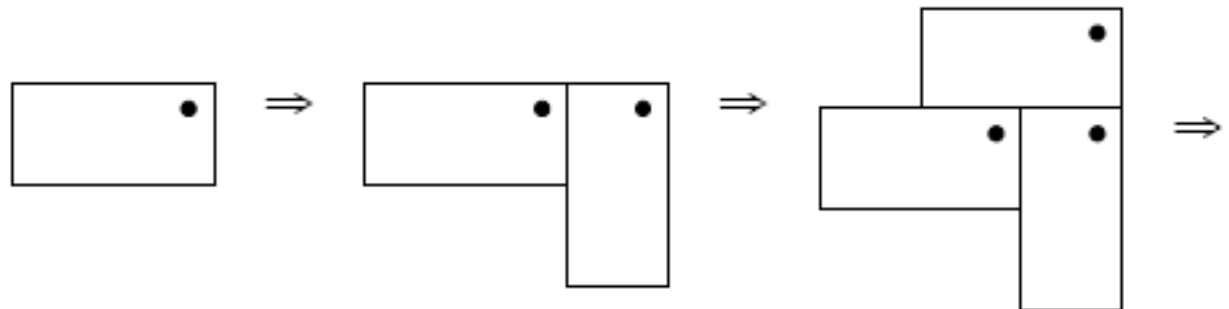
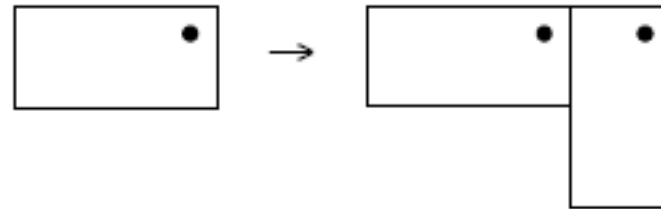


derivation

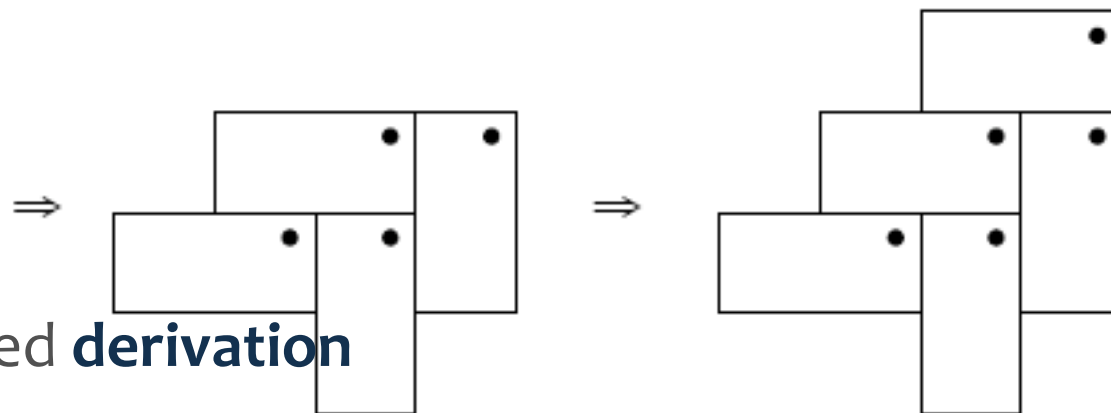


labeled rule based **derivation**

Labeled rule

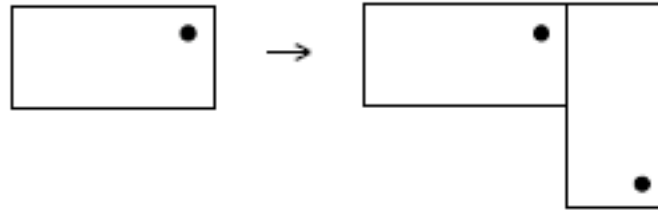


derivation

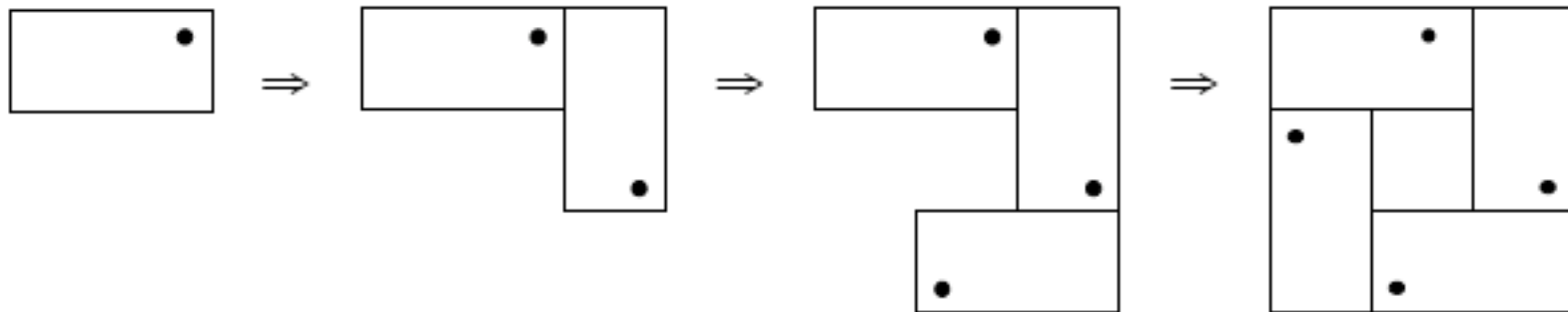


labeled rule based **derivation**

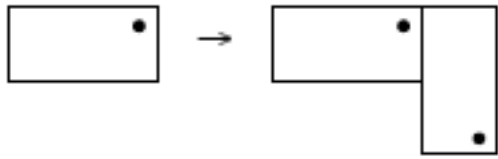
Labeled rule



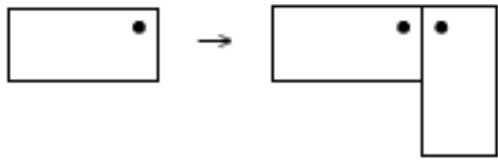
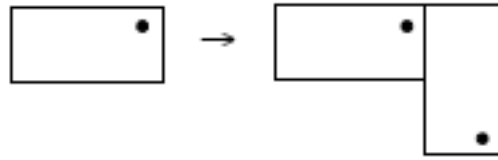
derivation



labeled rule based **derivation**

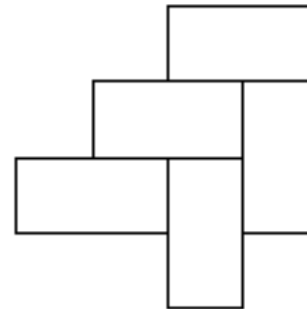
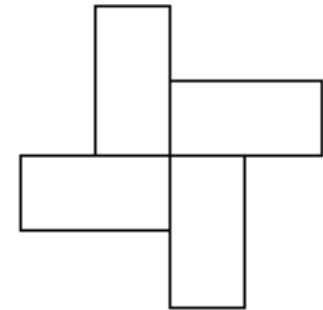
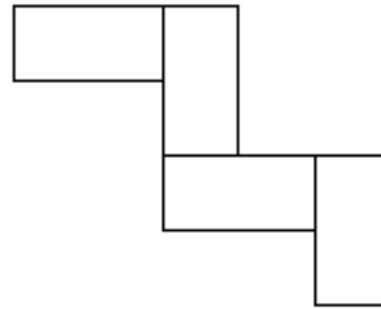
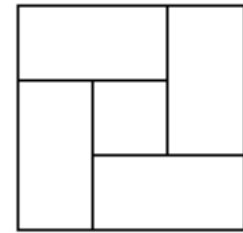


rules



shape grammar

designs



shape grammar

vocabulary

shapes made up from these vocabulary

In the general case, shapes are parameterized schemes

production rules

(or rules of change, encapsulate a spatial relation)

seed shape (we have to start somewhere)

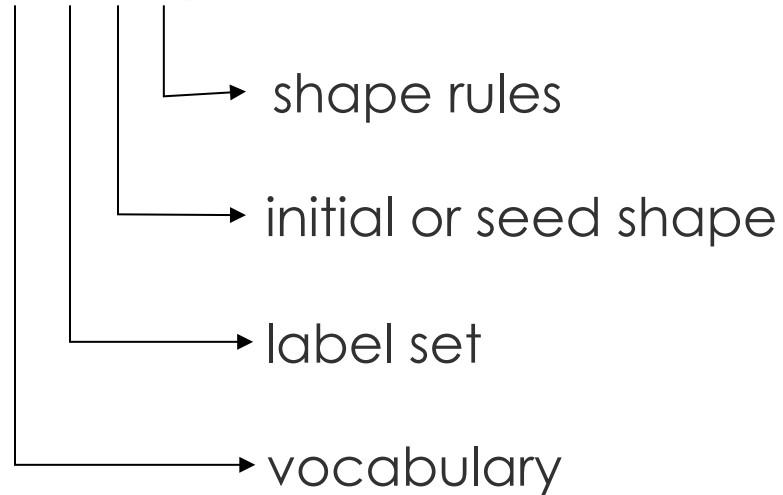
+ a “notion” of **rule application**

shape grammar embodies **change** implies **generation**

shape grammar $G = (S, L, I, R)$

initial (seed) shape

belongs to the *universe* of labeled shapes made up of shapes in S and labels in L



R contain rules of the form $a \rightarrow b$ where **a** and **b** belong to the universe of labeled shapes made of shapes in S and labels in L except **a** cannot be empty

formally: a shape grammar is

Vocabulary is a limited set of shapes no two of which are similar.

The vocabulary provides the basic building blocks by means of which shapes can be generated through shape arithmetic and geometric (euclidean) transformations.

vocabulary

If we are given a set of shapes S , then we can create a set U called the **universe** of S in the following manner:

The empty shape is in U

Every shape in S is in U

What can you say about the universe of the set of shapes consisting just one shape, a single line of unit length, $\{(0,0),(1,0)\}$?

f and g , $f(s)+g(t)$ is in U

U is thus closed under shape addition and the Euclidean transformations.

universe

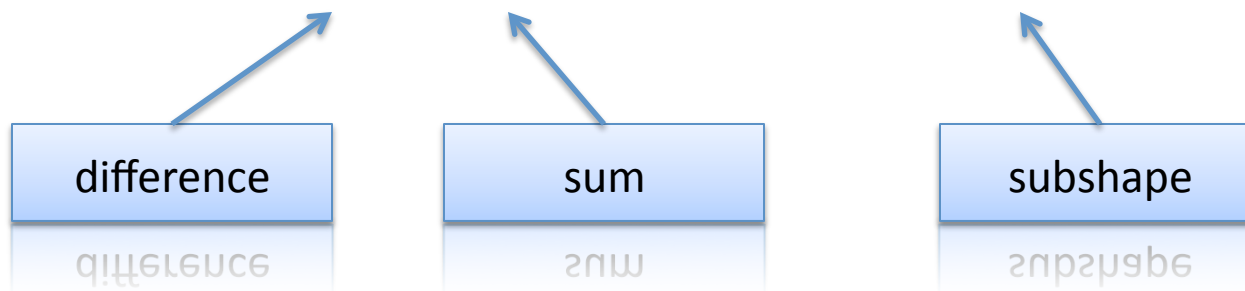
A rule is *applicable* to the *current shape*
which is either the initial shape
or a shape produced from the initial shape
whenever the left hand side of the rule ‘*occurs*’ in the object
in which case
it is *replaced* by the right hand side of the rule under rule
application

shape rule application

A rule $a \rightarrow b$ is applied only if a 'occurs' in the given shape u under some 'transformation' T in which case $T(a)$ is substituted by $T(b)$ in the current shape

Rule application

$$v \leftarrow u - T(a) + T(b) \text{ provided } T(a) \leq u$$



We describe this as $u \Rightarrow v$

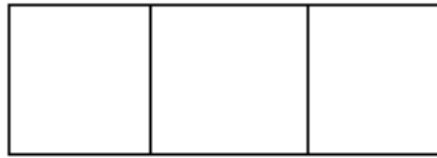
rule application

Implicit in this definition is the fact that 'parts' of shapes are recognizable in *arbitrary* ways

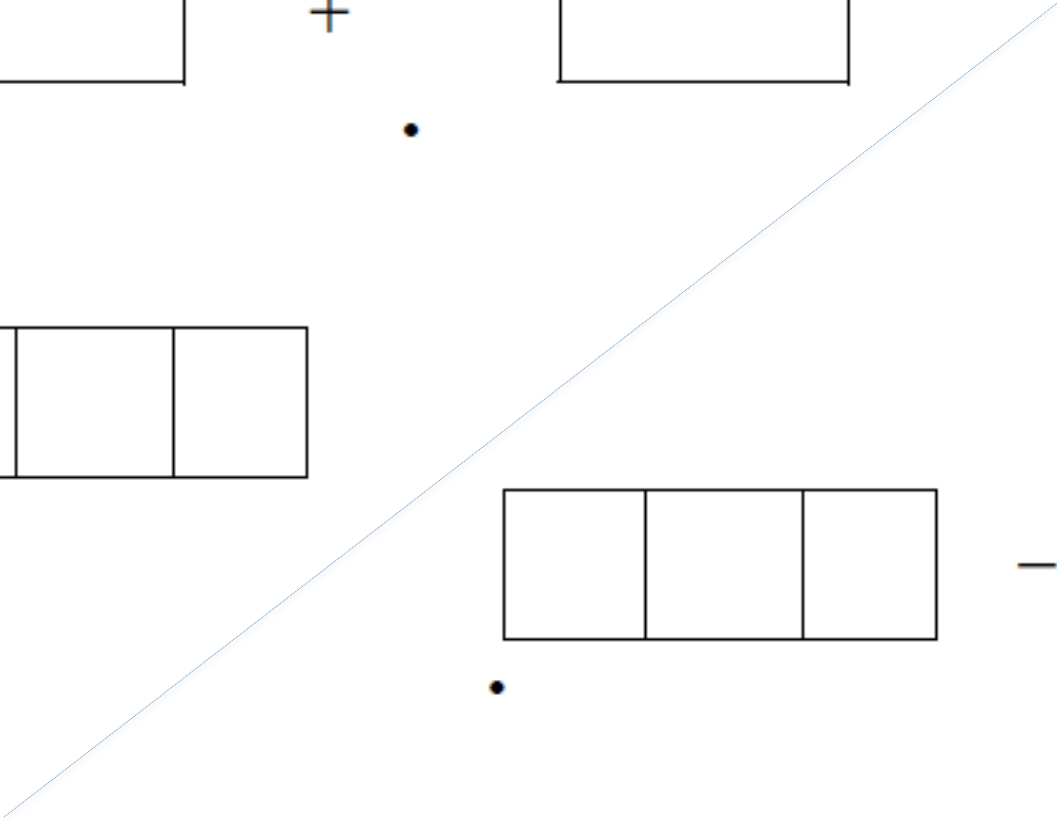
shape



+



-



addition and subtraction

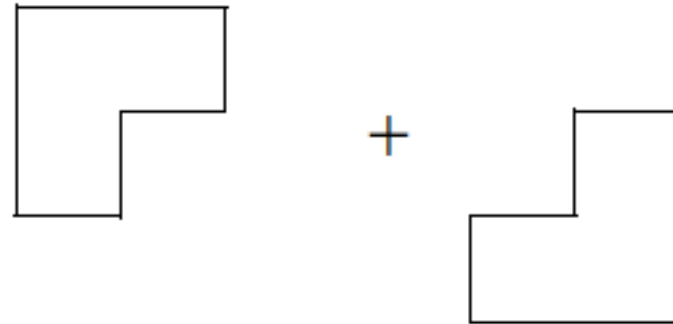
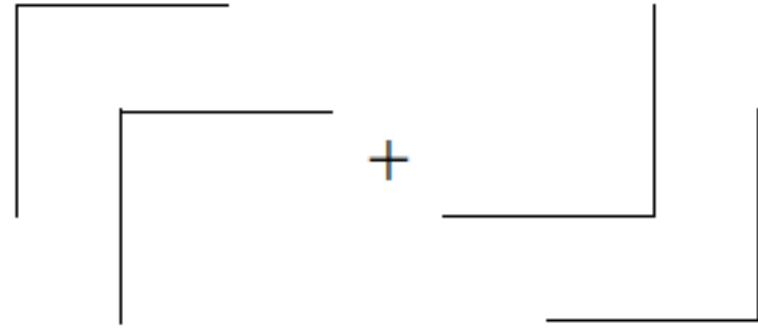
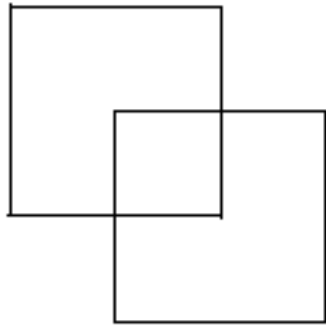


A shape is a *subshape* of another
if all the lines in the first shape are lines in the second

A subshape identifies a *part* of a shape

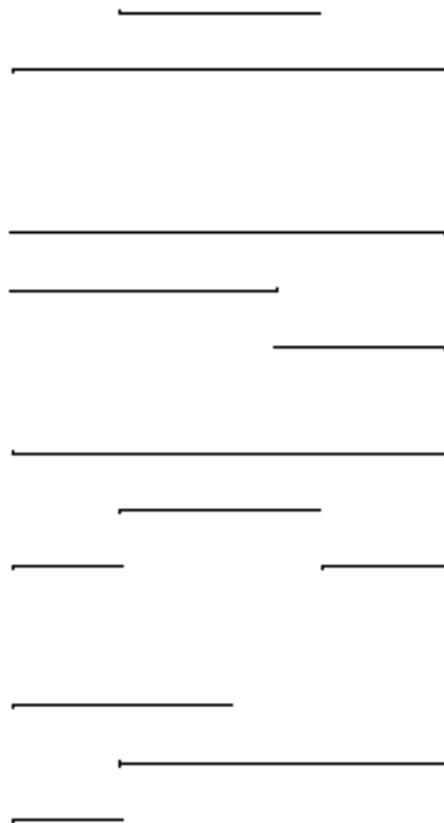
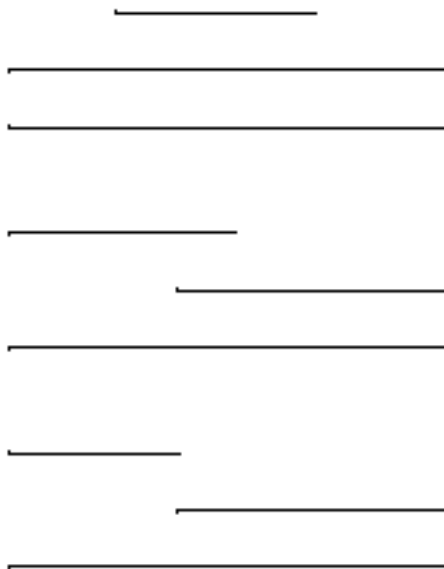
A shape has *indefinitely many* subshapes

subshapes

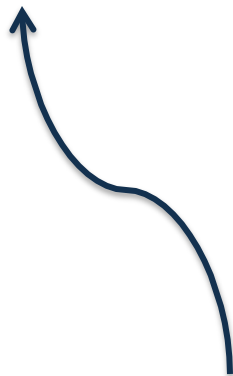


decomposing a shape

u
v
u+v

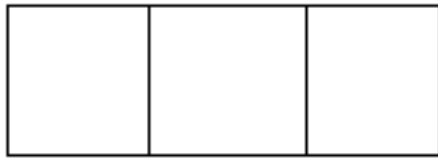


reduction rules for **add** and **subtraction**

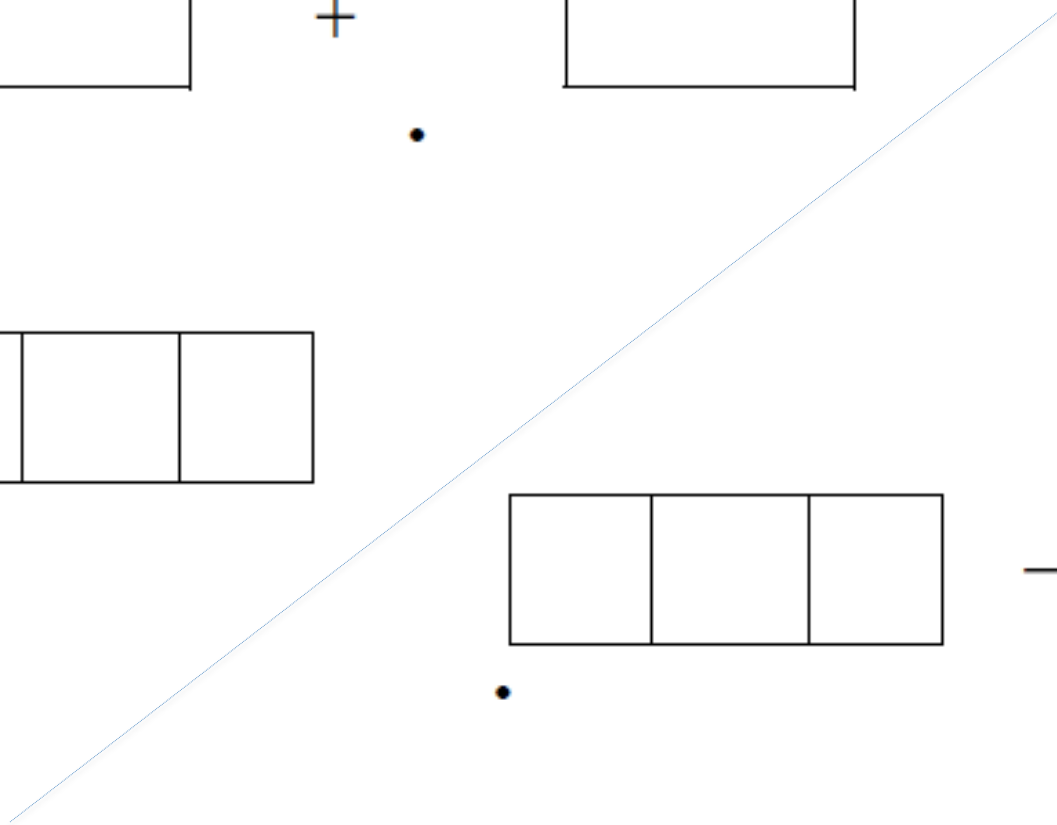




+



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addition and subtraction



a metaphor for shape grammars

a pencil –

lead at one end to **add** marks

an eraser at the other to **subtract** marks

lead side has a **shape**

eraser side has another **shape**, not necessarily the same

together they specify a **rule**

– that is, **see** something

take it away and its place do (**make/add**) something else

metaphor for shape grammars

deriving designs as a sequence of designs

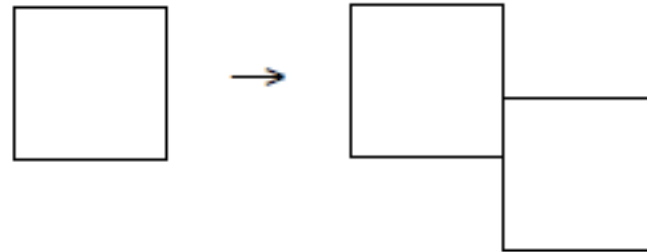
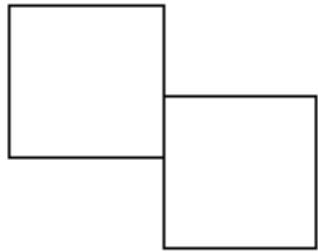
based on the work of Terry Knight <http://www.mit.edu/~tknight/IJDC/>

By labeling shapes we can “scaffold” the process of generating shapes

The *language* of a grammar G , is the set of all shapes (i.e., without non-terminals) that are produced from the initial shape through rule application

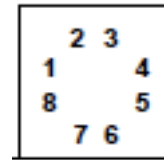
Language = { shape | initial-shape \Rightarrow^* shape }

language

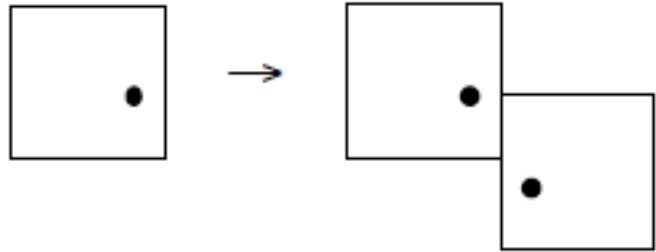
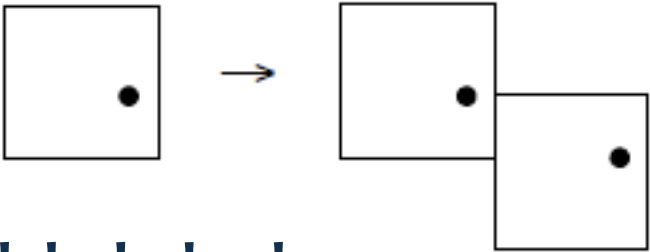
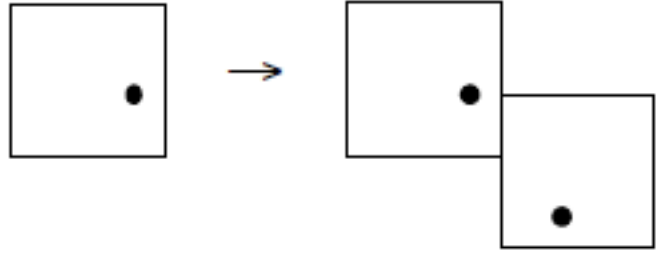
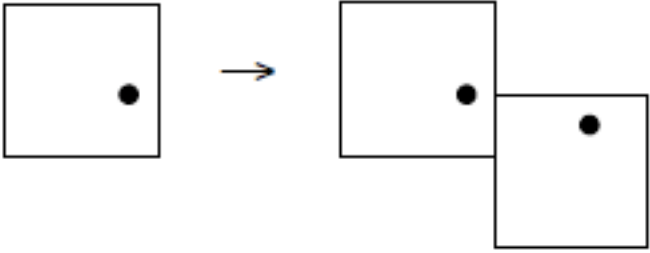
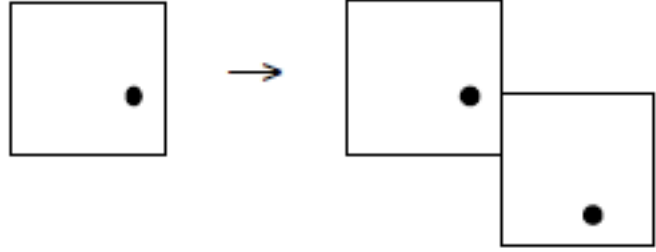
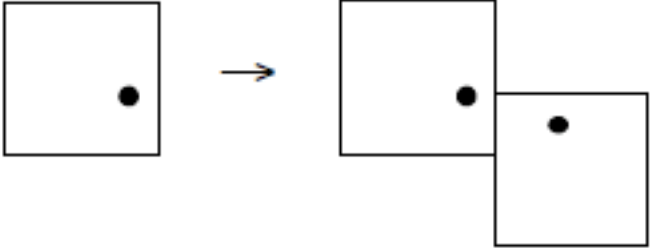
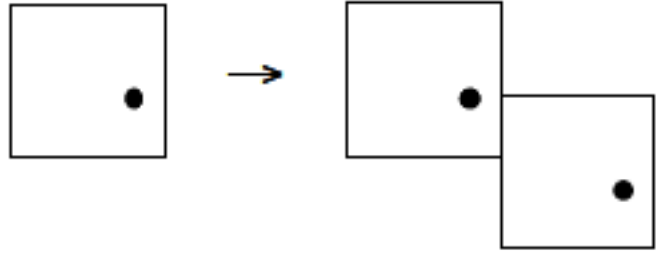
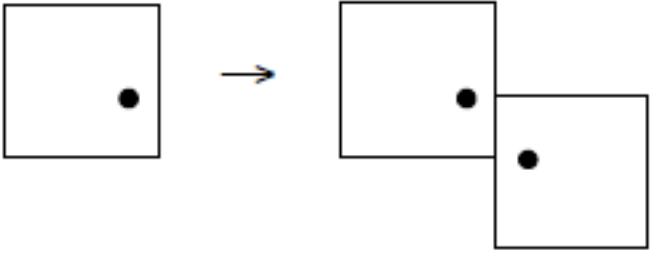


a shape relation and a corresponding rule

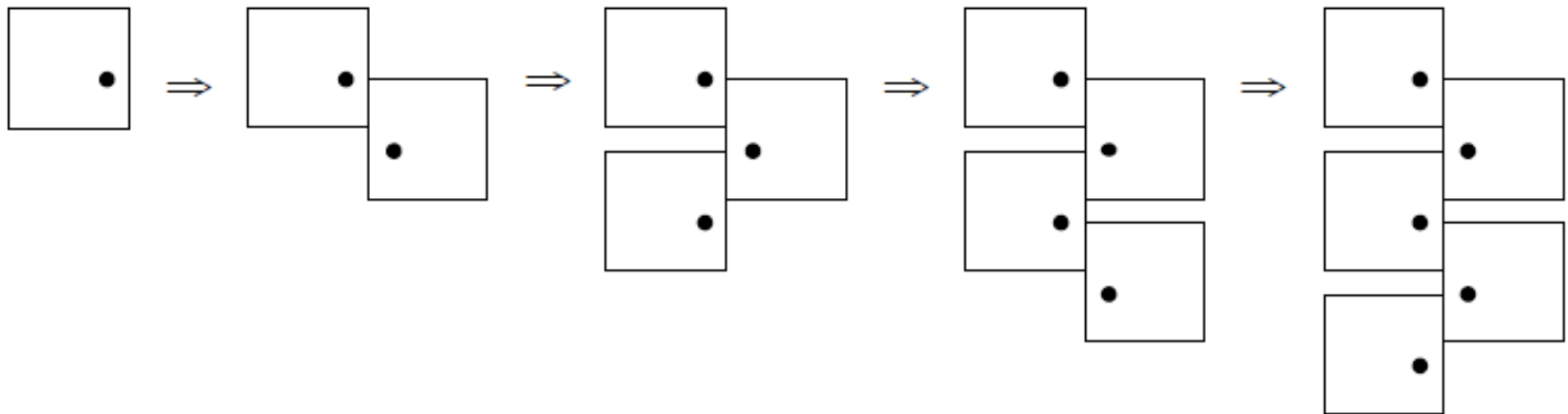
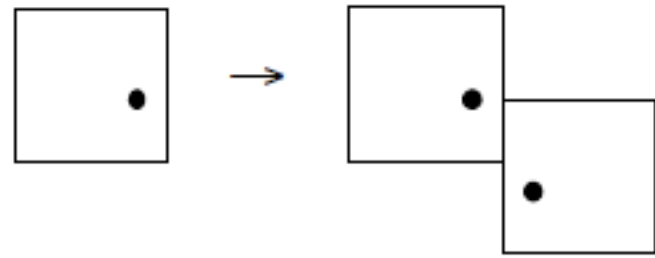
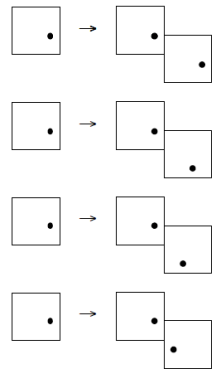
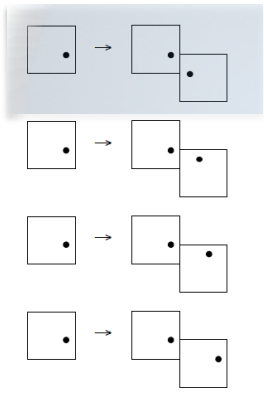
consider the symmetry of a square



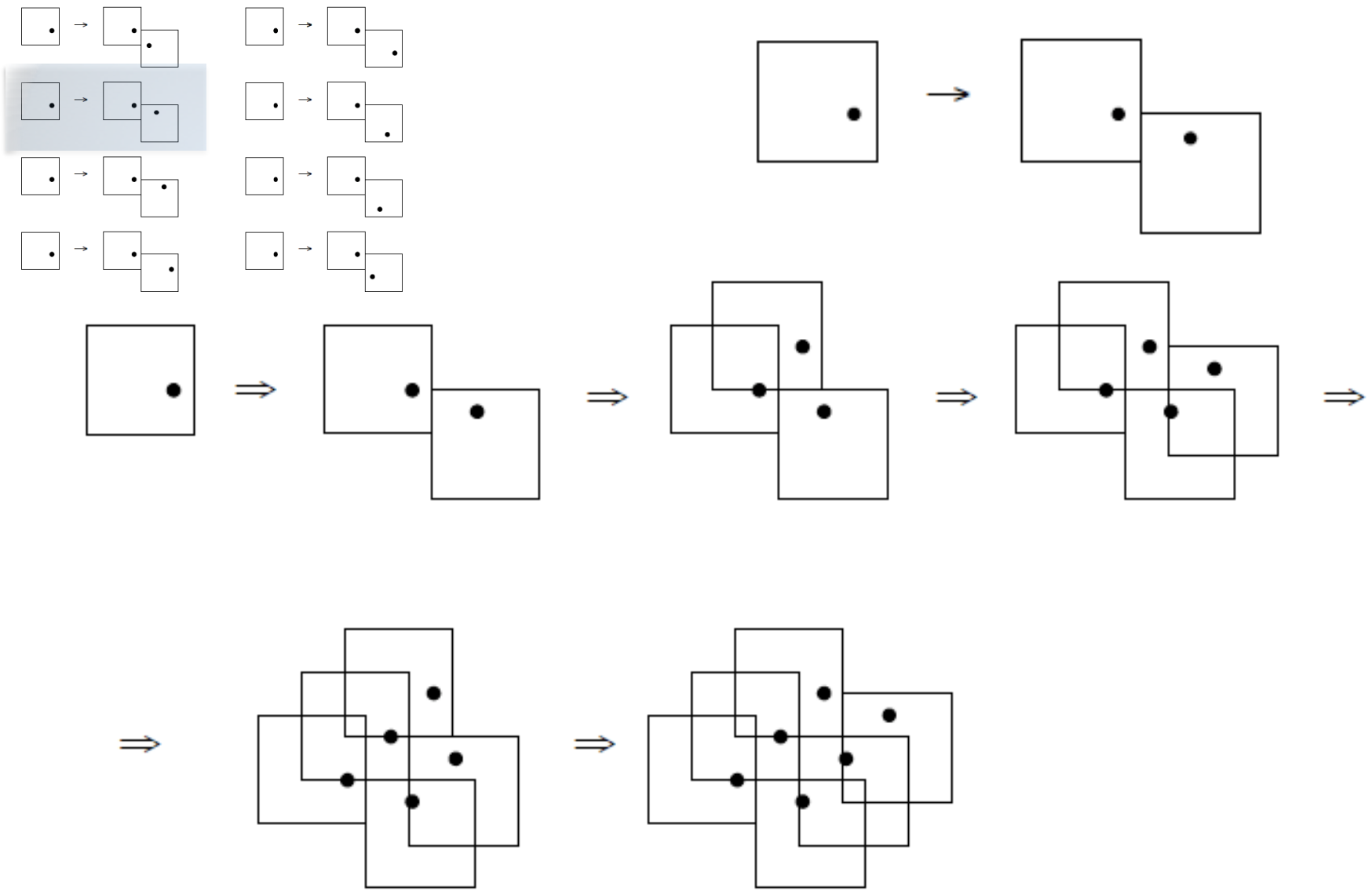
some possible labeling positions for a square



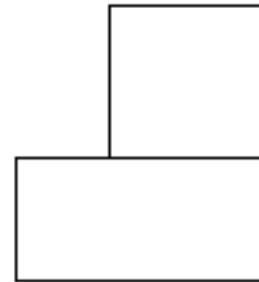
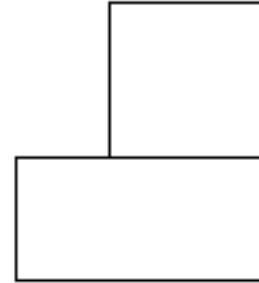
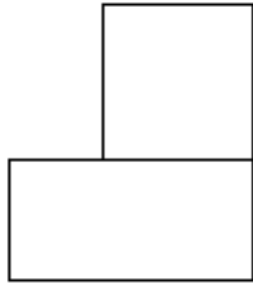
labeled rules



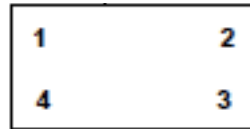
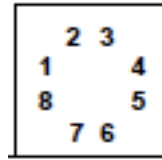
a labeled rule and sample derivations



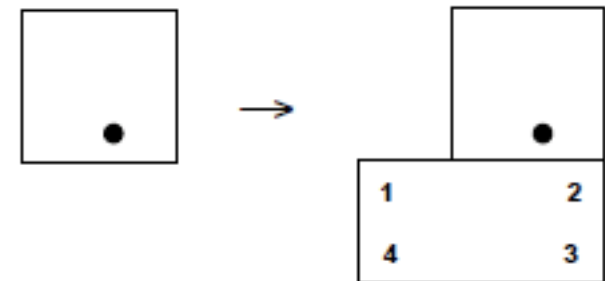
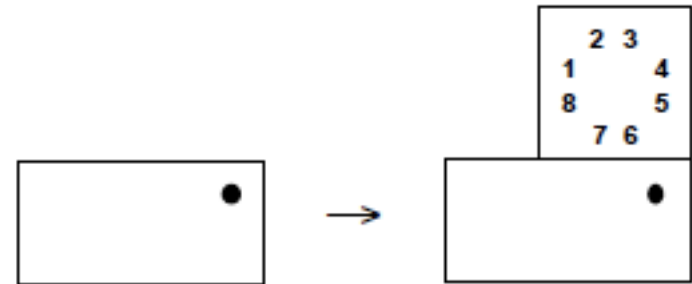
another labeled rule and sample derivations



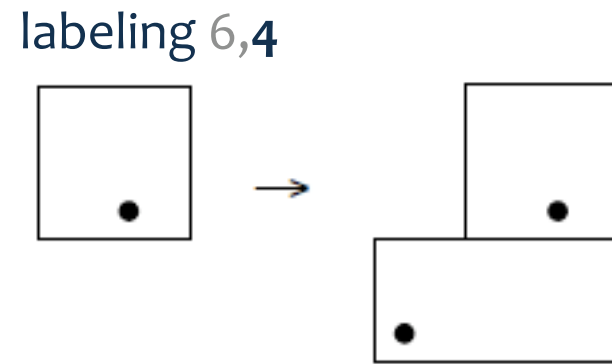
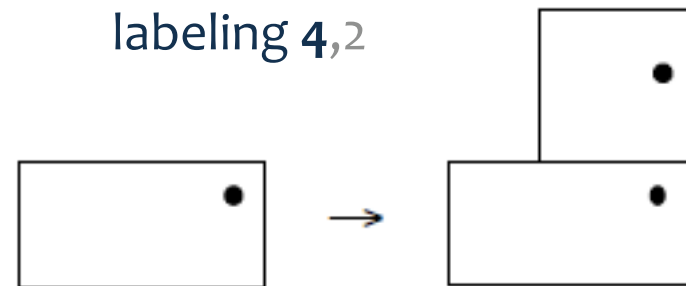
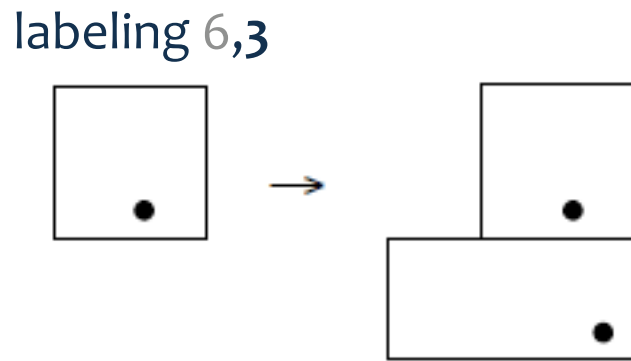
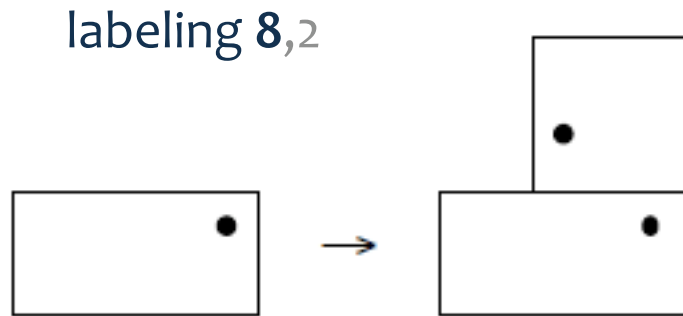
another shape relation and two corresponding rules



consider the symmetry of a square
and the rectangle



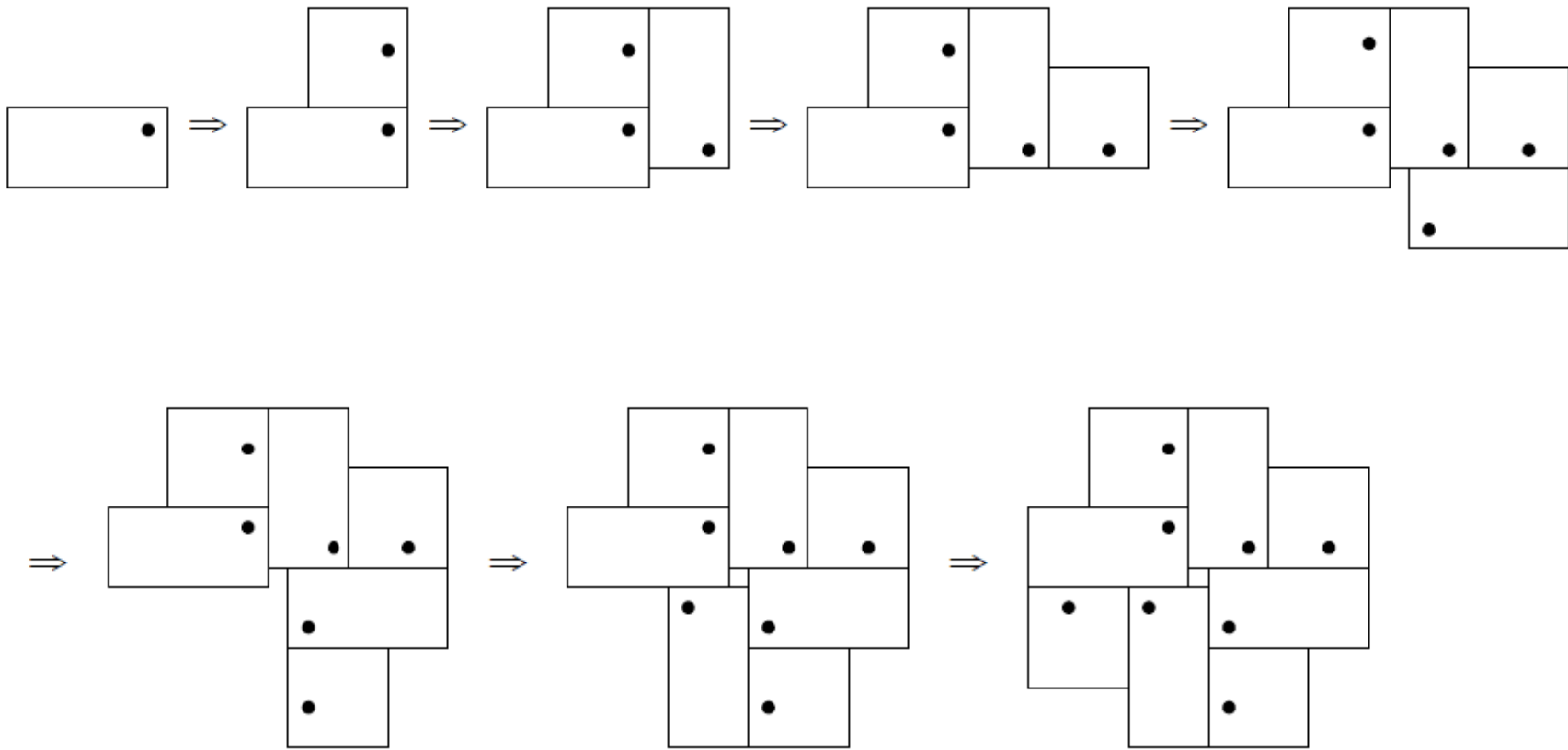
some possible **labeling positions**
and possible **shape rules**



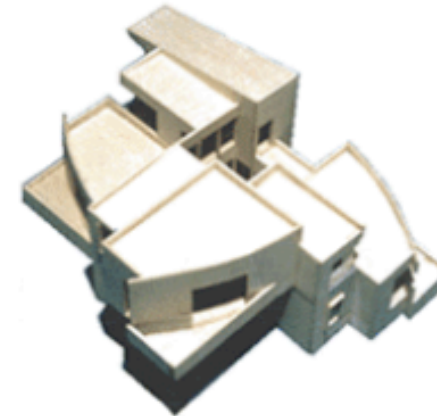
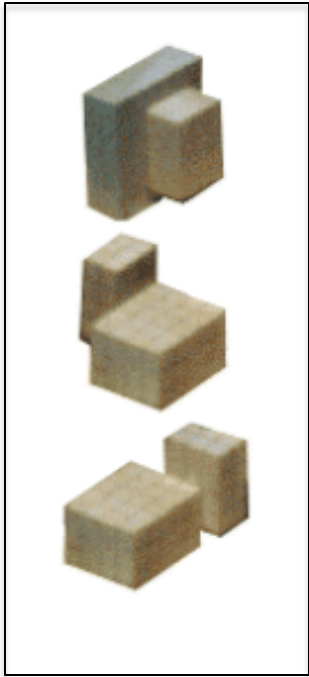
	2	3	
1		4	
8		5	
7	6		

1		2
4		3

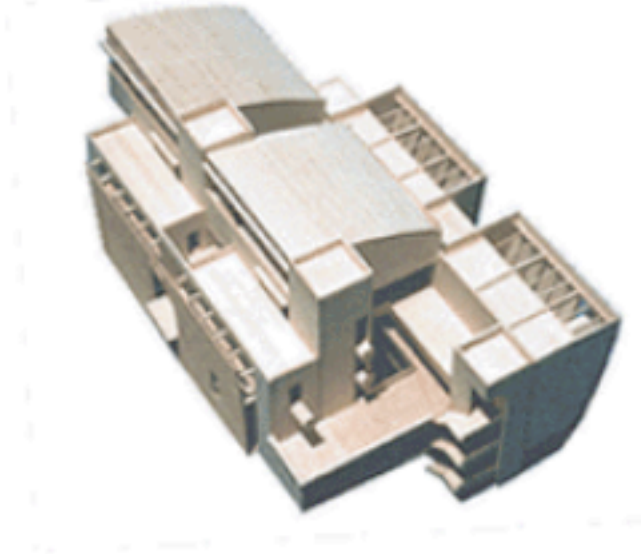
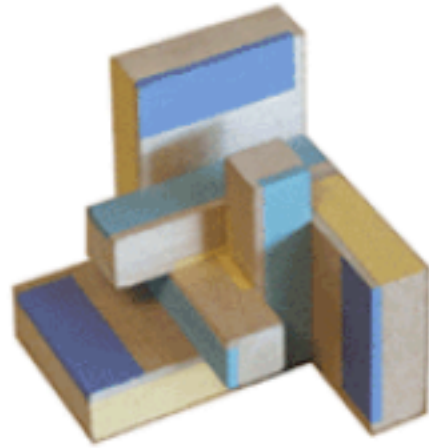
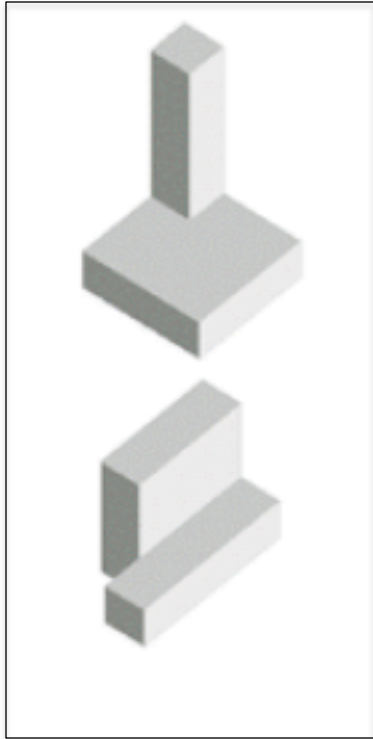
example **labeling**



derivation using rule with 4,4 labeling



application in a real design context



inspiration

every house worth considering as a work of art
must have a grammar of its own

”grammar” in this sense, means the same thing in any
construction, whether it be of words or of stone or wood

**the worlds we study can be understood by
capturing underlying relationships**

that enter into the construction of the thing

the “grammar” of the house is the manifest articulation of
its parts. this will be the “speech” it uses

to be achieved, *construction must be grammatical*

frank lloyd wright *The Natural House*