

# **48-747 Shape Grammars**

*KINDERGARTEN GRAMMARS*



*... mother found 'Gifts'. And 'gifts' they were. Along with the gifts were the system, as a basis for design and the elementary geometry behind all natural birth of Form"*

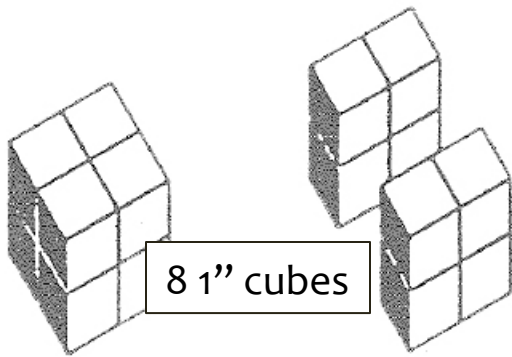
Developed by Friedrich Fröbel

Well known to designers because its formative influence on FLW

Based on a series of **geometrical gifts**

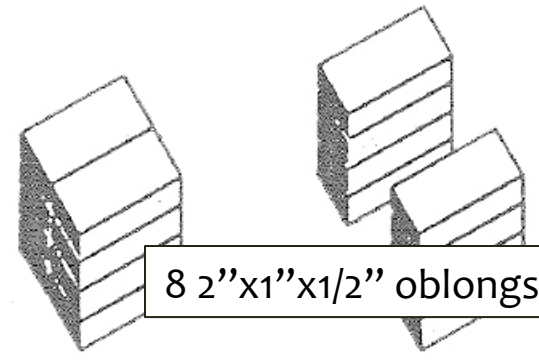
+ a system of **categories of form**

the **kindergarten method**



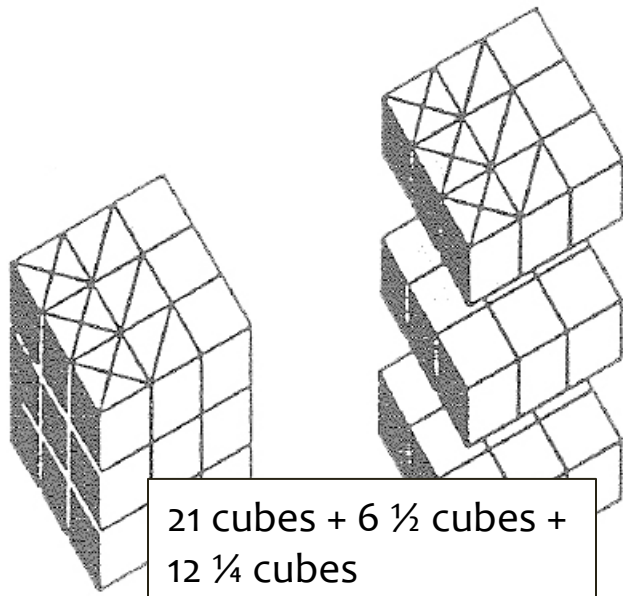
8 1" cubes

Gift 3



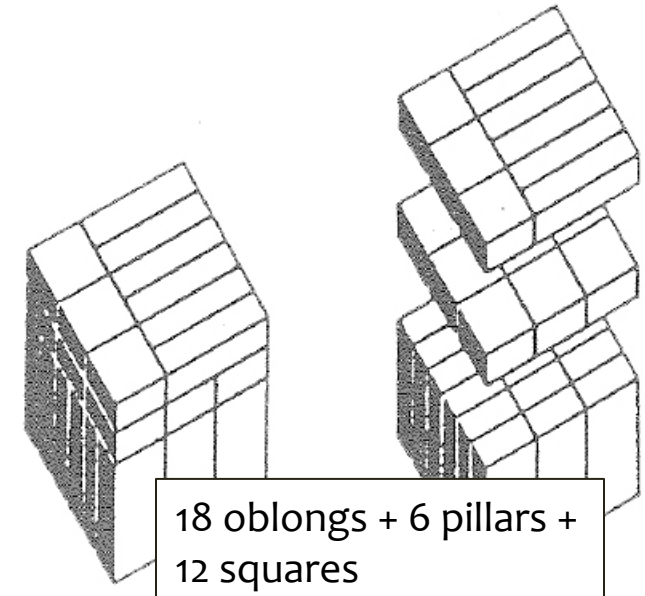
8 2"x1"x1/2" oblongs

Gift 4



21 cubes + 6 1/2 cubes +  
12 1/4 cubes

Gift 5



18 oblongs + 6 pillars +  
12 squares

Gift 6

the gifts

form a vocabulary – the building blocks  
e.g., cubes, 1/2 cubes, 1/4 cubes, oblongs and so on

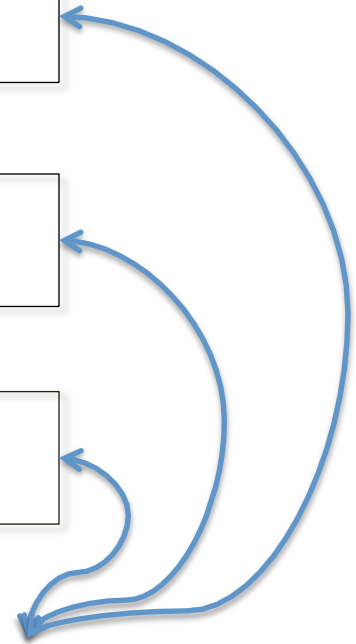
Verbal  
exposition  
and  
Visual  
exploration



Forms of Beauty

Forms of Life

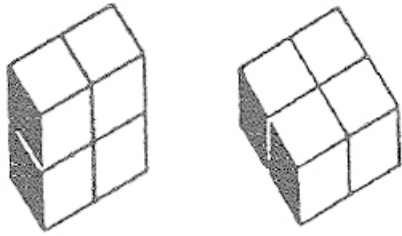
Forms of Knowledge



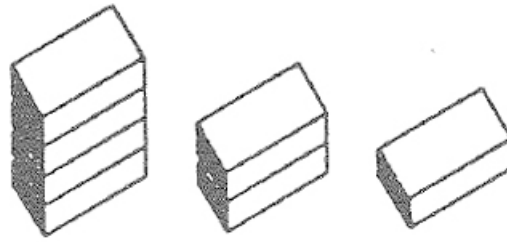
based on the *geometrical gifts*

+ a system of **categories of form**

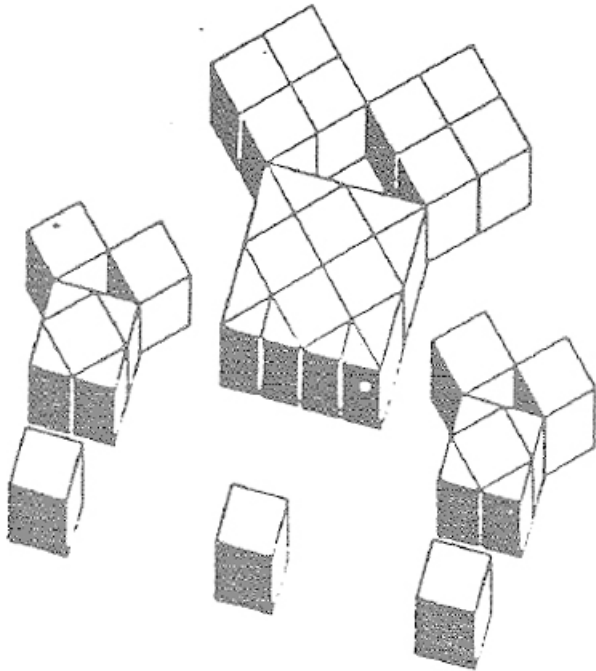
the **kindergarten** method



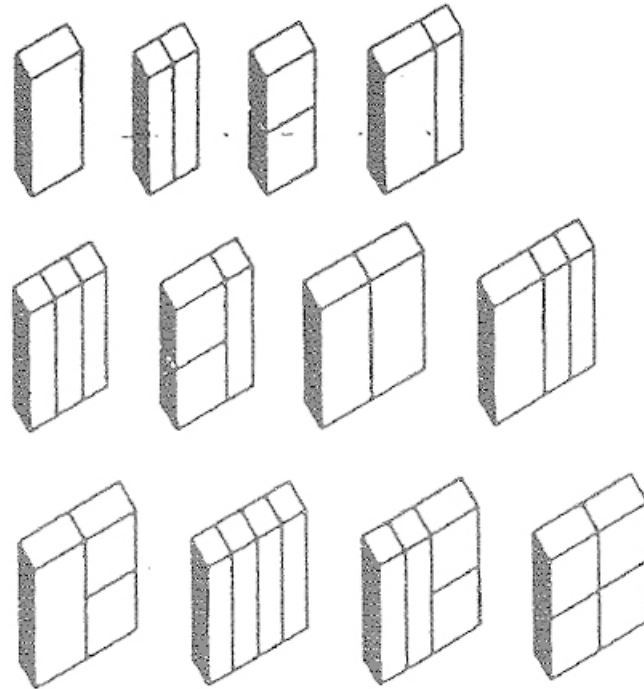
Gift 3: equivalence



Gift 4: division by 2

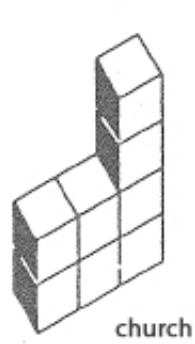


Gift 5: pythagorus' theorem



Gift 6: expanding series

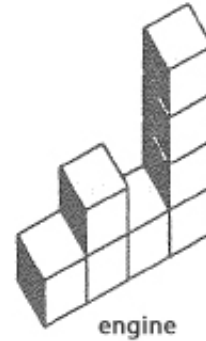
forms of knowledge



church



station house

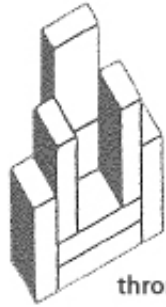


engine

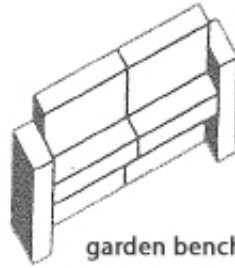


armchair

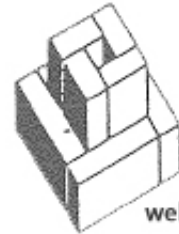
Gift 3



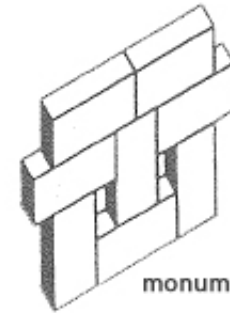
throne



garden bench

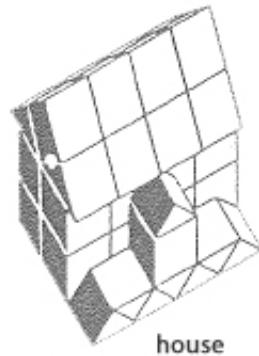


well

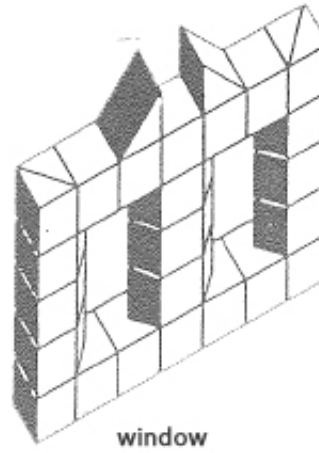


monument

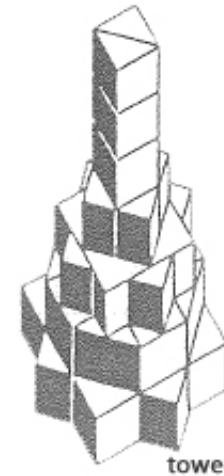
Gift 4



house



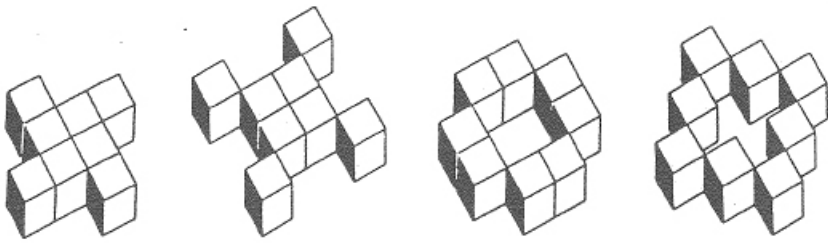
window



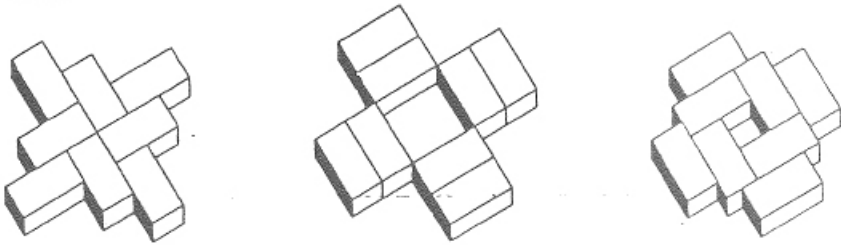
tower

Gift 5

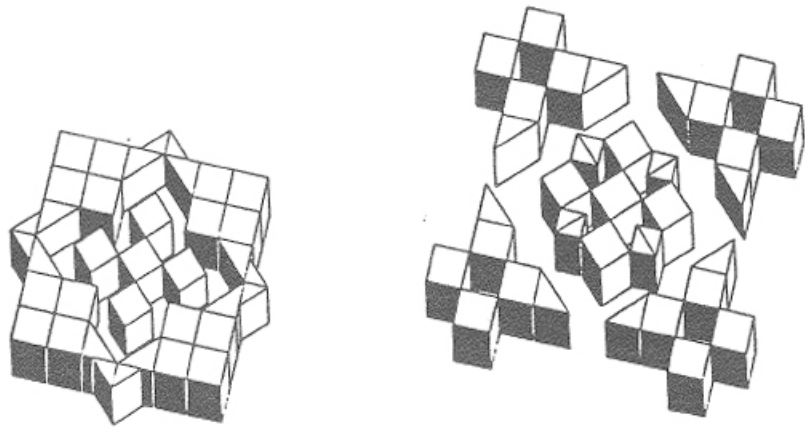
forms of life



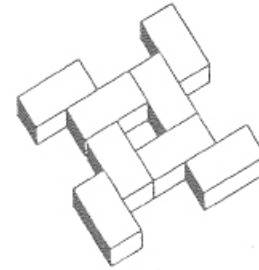
Gift 3



Gift 4



Gift 5



Gift 6

forms of beauty



Categories of forms suggest possibilities of design, in principle, the combination suggests a language (of designs)

Gifts constitute a vocabulary that gives rise to an pedagogical analogue in ...

the **studio** method

## *Vocabulary*

Architectural and structural elements

## *Categories*

Architectural programmes

Building types

Historical styles

Symbolic references

Aesthetic doctrines (manifestos)

*the studio encourages  
**FREE PLAY** within these  
constraints*

*abstractly, **TRANSITIONS**  
from shapes (forms) to  
shapes (forms)*

the **studio** method

suggest a course of actions –

starting with a ***vocabulary*** (shapes)

- essentially, compositions– from shape to shape  
(free play with forms)

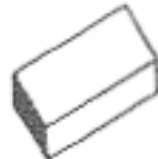


***relationships*** between shapes (forms)

**transitions**



cube  
Gift 3



oblong  
Gift 4



cube  
Gift 5



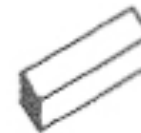
half-cube



quarter-cube



oblong  
Gift 6

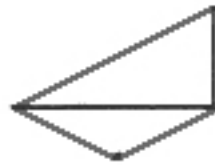
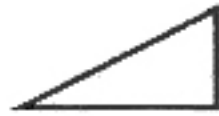


pillar

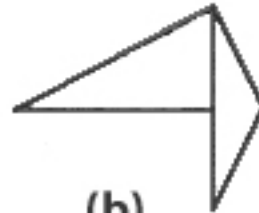


square

**vocabularies**



(a)



(b)



(c)

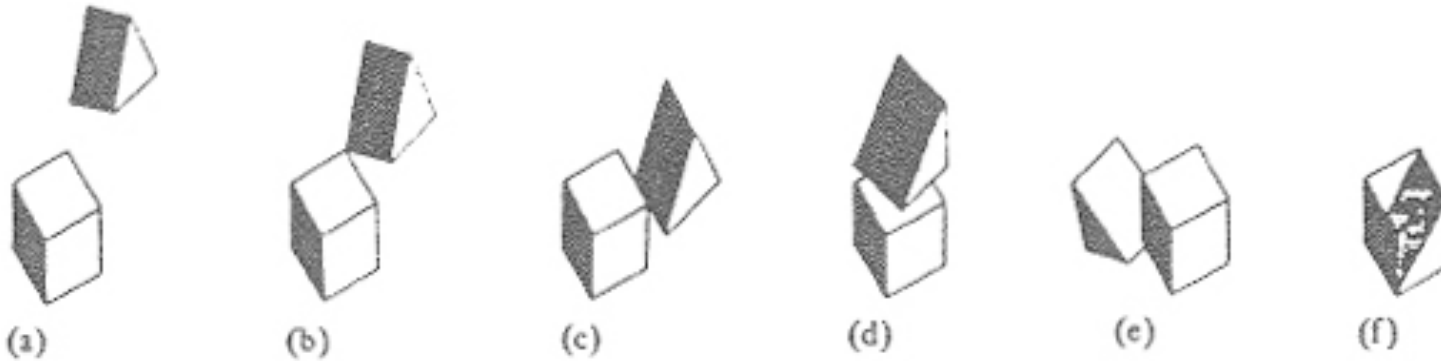


(d)



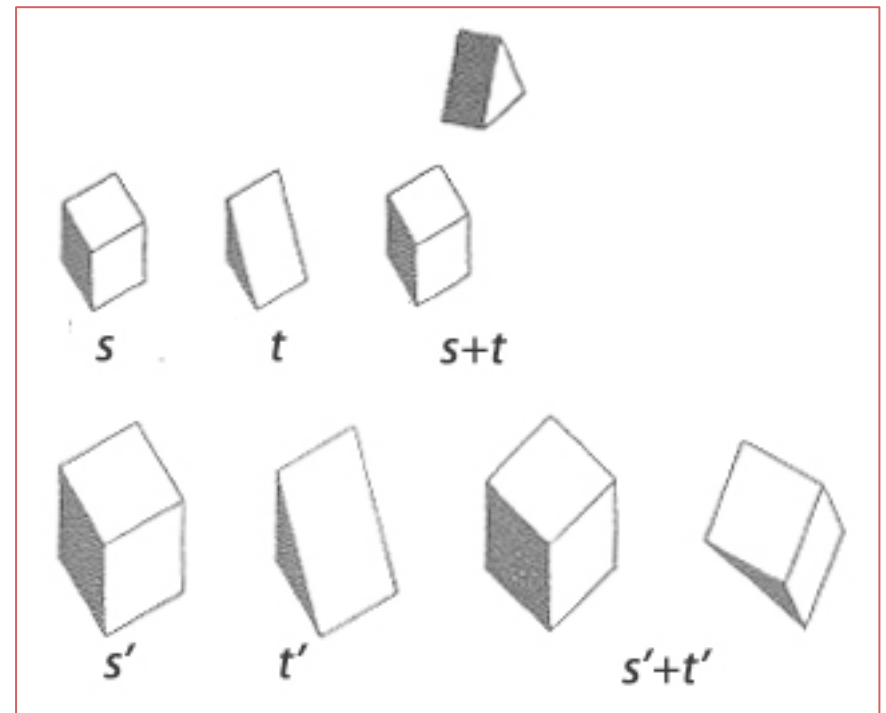
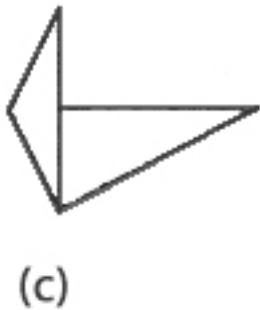
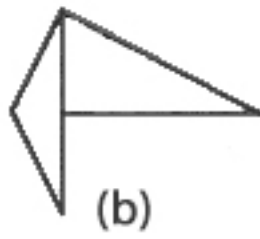
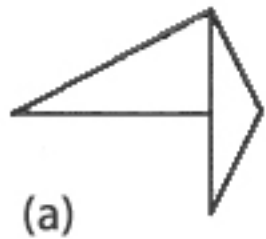
(e)

**spatial relations** arise whenever there are two or more shapes



**spatial relation** between a cube and a quarter-cube

the number of shapes in the relations are identical and there is *a geometrical transformation* that maps every shape in one relation to a corresponding shape in the other relation



**equivalent** spatial relations arise *whenever*

*Face to face. That is right.  
Edge now are meeting quite.  
Edge to face now we will lay,  
Face to edge will end the play.*



- Spatial relations specified by two blocks
- A face overlaps another face so that the faces share a vertex and edges intersecting at this point coincide
- The blocks do not interpenetrate

**lets us play**



# Gift 4

## Gift 3



cube



cube, cube: 1, 1



oblong



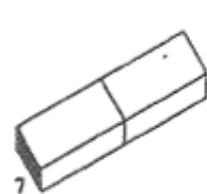
1  
oblong, oblong: 1, 1



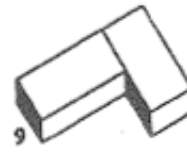
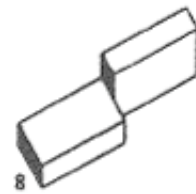
3  
oblong, oblong: 1, 2



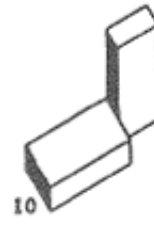
5  
oblong, oblong: 1, 3



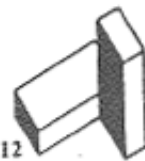
7  
oblong, oblong: 2, 2



9  
oblong, oblong: 2, 3



11  
oblong, oblong: 3, 3



# Gift 5



1  
cube



2  
half-cube



3  
quarter-cube



1  
cube, cube: 1, 1



2  
cube, half-cube: 1, 2



3  
cube, half-cube: 1, 3



4  
cube, half-cube: 1, 4



5  
cube, quarter-cube: 1, 5



6  
cube, quarter-cube: 1, 6



7  
cube, quarter-cube: 1, 7



9  
half-cube, half-cube: 2, 2



10  
half-cube, half-cube: 2, 3



11  
half-cube, half-cube: 2, 3



12  
half-cube, half-cube: 2, 3



13  
half-cube, half-cube: 2, 4



14  
half-cube, half-cube: 3, 3



15  
half-cube, half-cube: 3, 3



16  
half-cube, half-cube: 3, 3



17  
half-cube, half-cube: 3, 4



18  
half-cube, half-cube: 3, 4



19  
half-cube, half-cube: 3, 4



20  
half-cube, half-cube: 4, 4



21  
half-cube, half-cube: 4, 4



22  
half-cube, quarter-cube: 2, 5



23  
half-cube, quarter-cube: 2, 5



24  
half-cube, quarter-cube: 2, 5



25  
half-cube, quarter-cube: 2, 6



26  
half-cube, quarter-cube: 2, 7



27  
half-cube, quarter-cube: 3, 3



28  
half-cube, quarter-cube: 3, 3



29  
half-cube, quarter-cube: 3, 3



30  
half-cube, quarter-cube: 3, 4



31  
half-cube, quarter-cube: 3, 4



32  
half-cube, quarter-cube: 3, 7



33  
half-cube, quarter-cube: 3, 7



34  
half-cube, quarter-cube: 3, 7



35  
half-cube, quarter-cube: 3, 7



36  
half-cube, quarter-cube: 3, 7



37  
half-cube, quarter-cube: 3, 7



38  
half-cube, quarter-cube: 4, 5



39  
half-cube, quarter-cube: 4, 6



40  
half-cube, quarter-cube: 4, 6



41  
half-cube, quarter-cube: 4, 7



42  
half-cube, quarter-cube: 4, 7



43  
half-cube, quarter-cube: 4, 7



44  
half-cube, quarter-cube: 4, 7



45  
quarter-cube, quarter-cube: 5, 5



46  
quarter-cube, quarter-cube: 5, 5



47  
quarter-cube, quarter-cube: 5, 6



48  
quarter-cube, quarter-cube: 5, 6



49  
quarter-cube, quarter-cube: 5, 7



50  
quarter-cube, quarter-cube: 5, 7



51  
quarter-cube, quarter-cube: 6, 6



52  
quarter-cube, quarter-cube: 6, 6



53  
quarter-cube, quarter-cube: 6, 7



54  
quarter-cube, quarter-cube: 6, 7



55  
quarter-cube, quarter-cube: 6, 7



56  
quarter-cube, quarter-cube: 7, 7



57  
quarter-cube, quarter-cube: 7, 7



58  
quarter-cube, quarter-cube: 7, 7

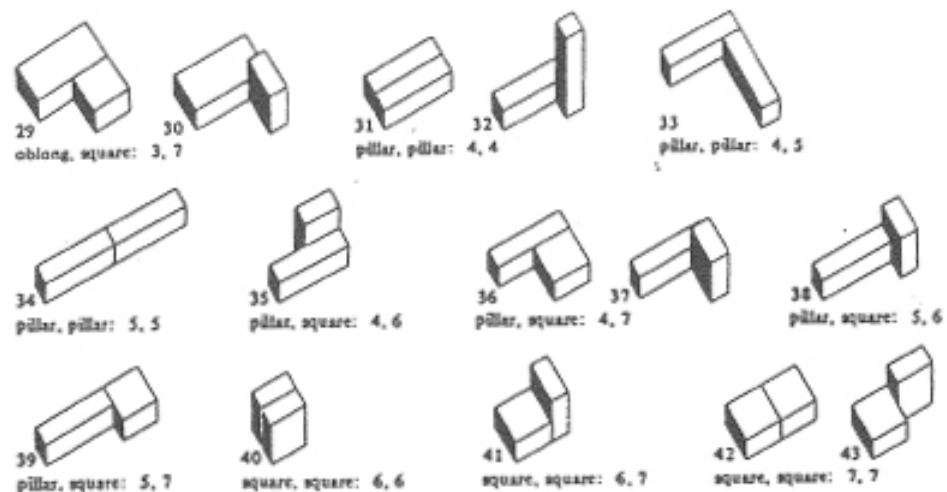
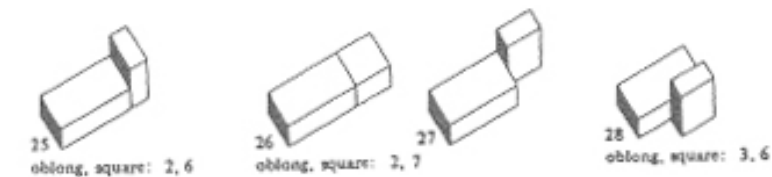
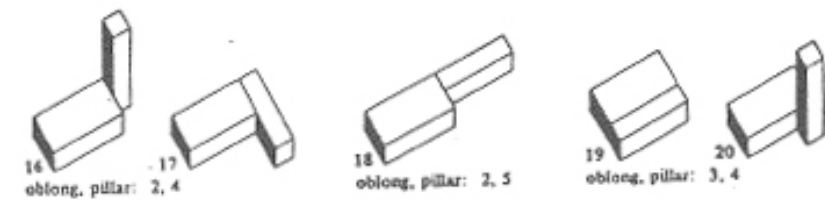
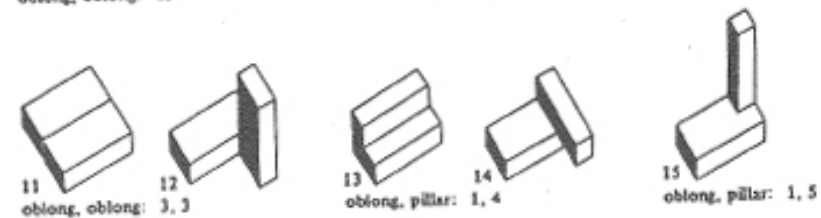
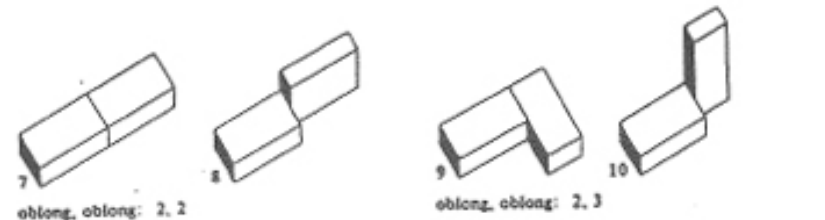
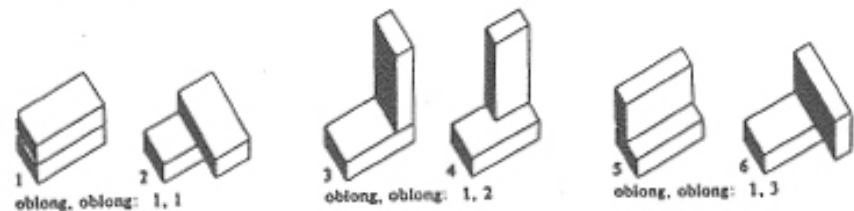
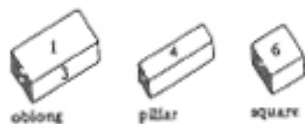


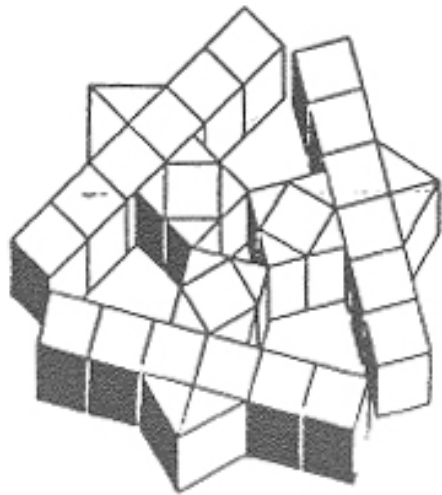
59  
quarter-cube, quarter-cube: 7, 7



60  
quarter-cube, quarter-cube: 7, 7

# Gift 6





5-1



5-3



5-6



5-8

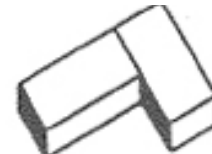


5-16

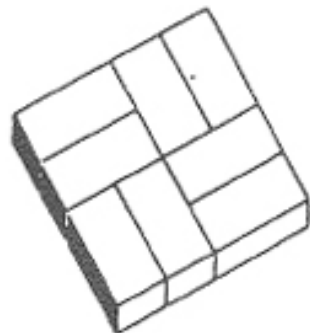
part of playing  
is **seeing**



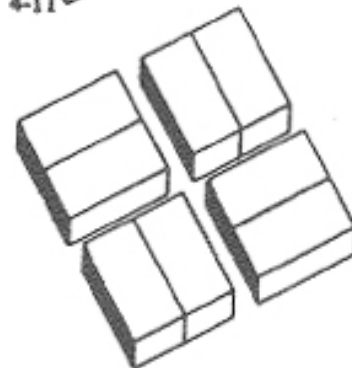
4-11



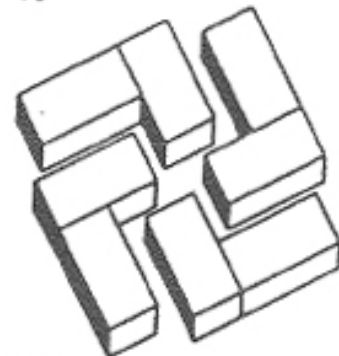
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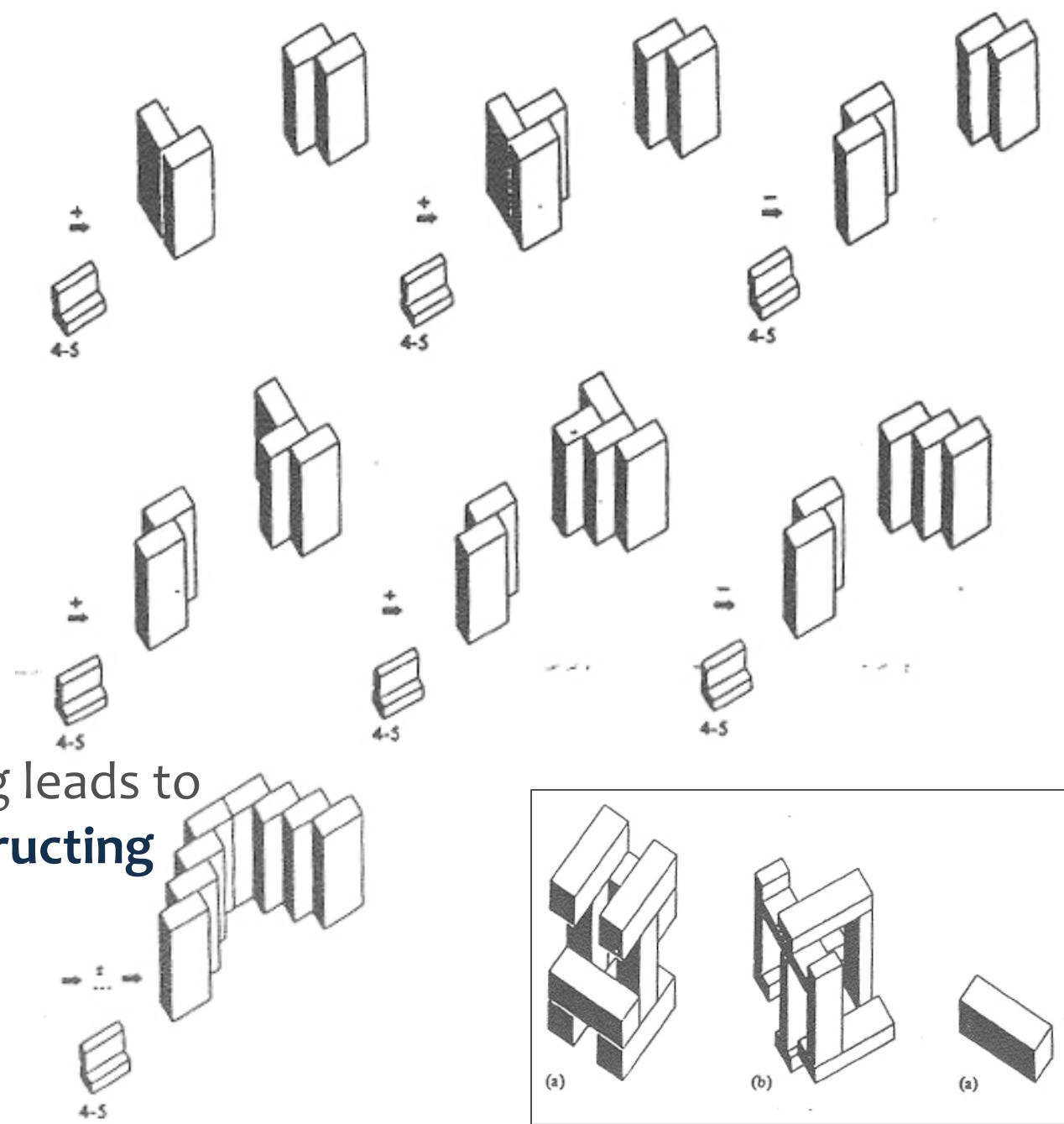
(a)



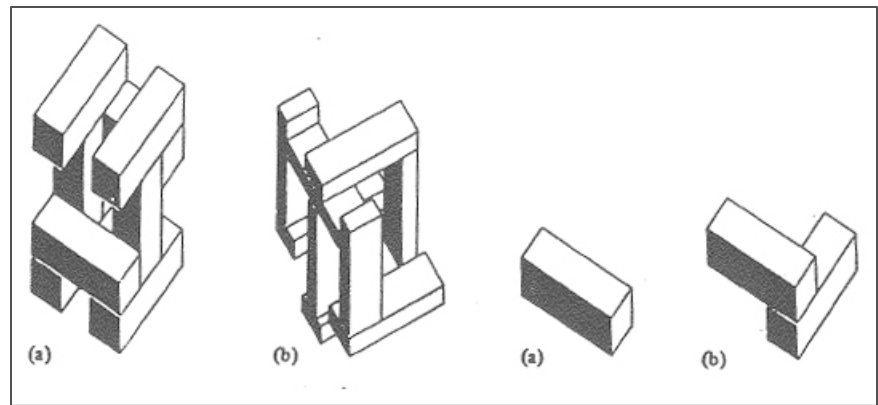
(b)



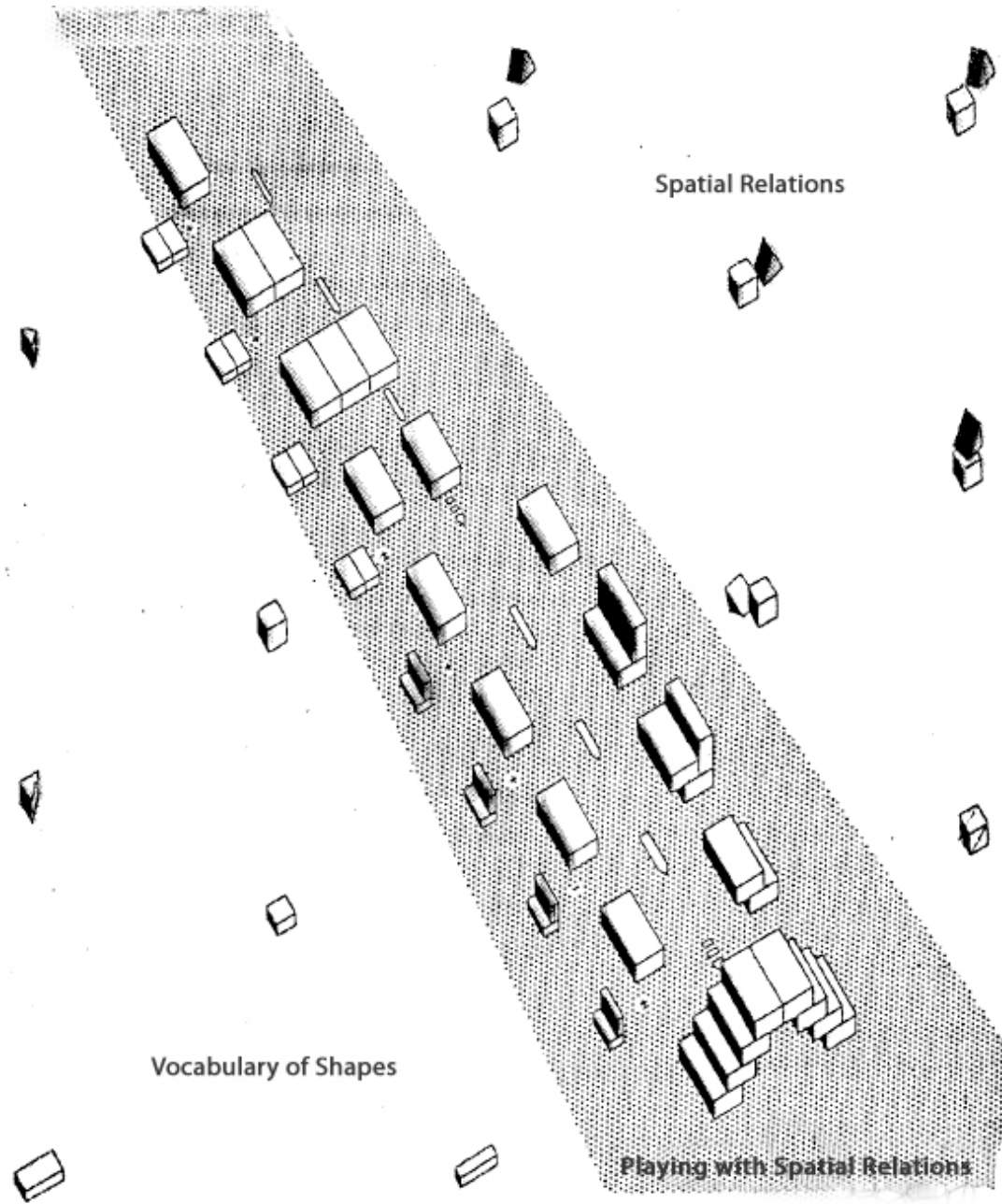
(c)



seeing leads to  
**constructing**



# playing with spatial relations



recapping the studio method via the kindergarten

suggest courses of actions – essentially, compositions –  
from shape to shape (forms)



relationships between shapes (forms) → PLAY

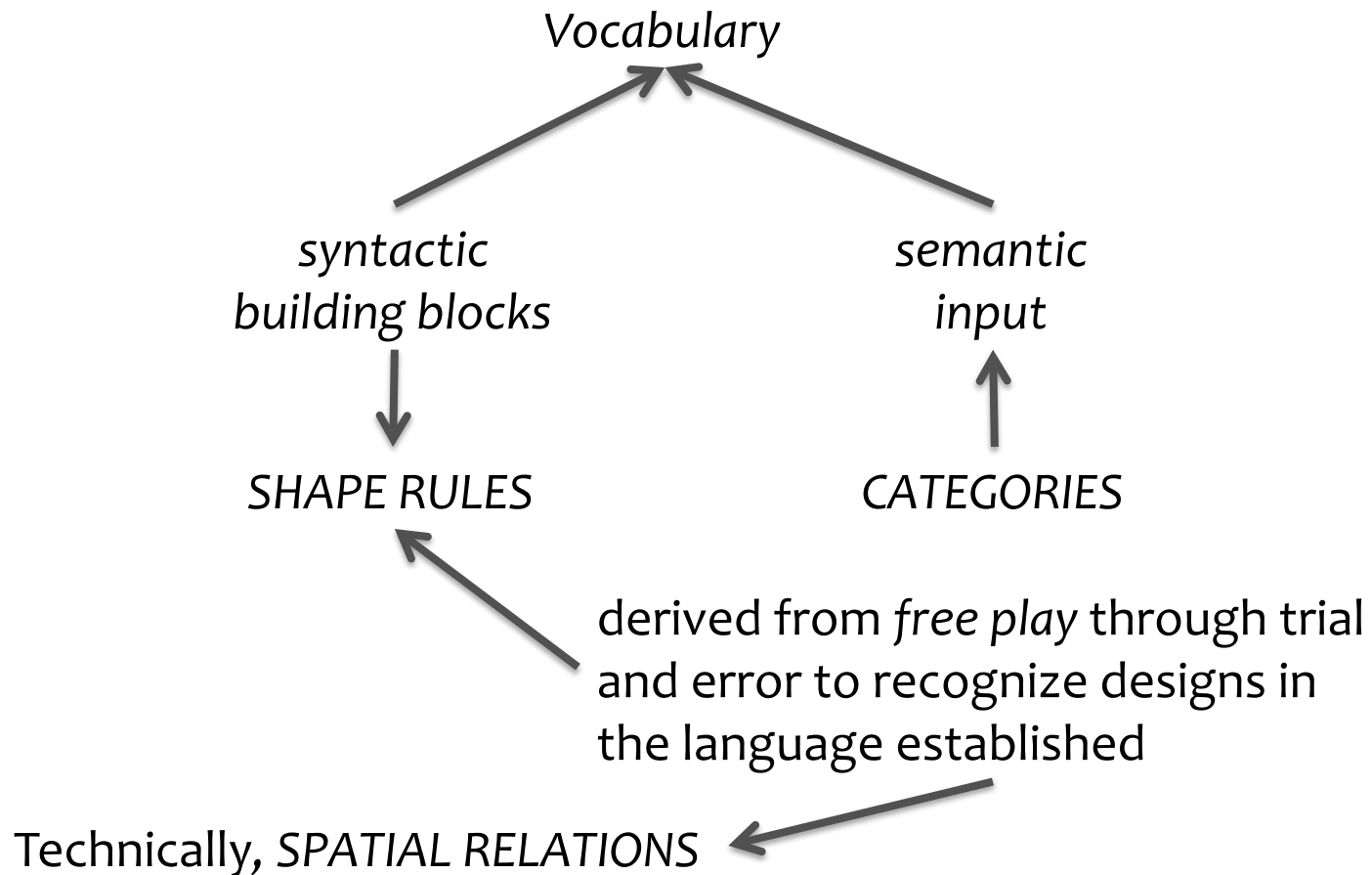


SHAPE RULES

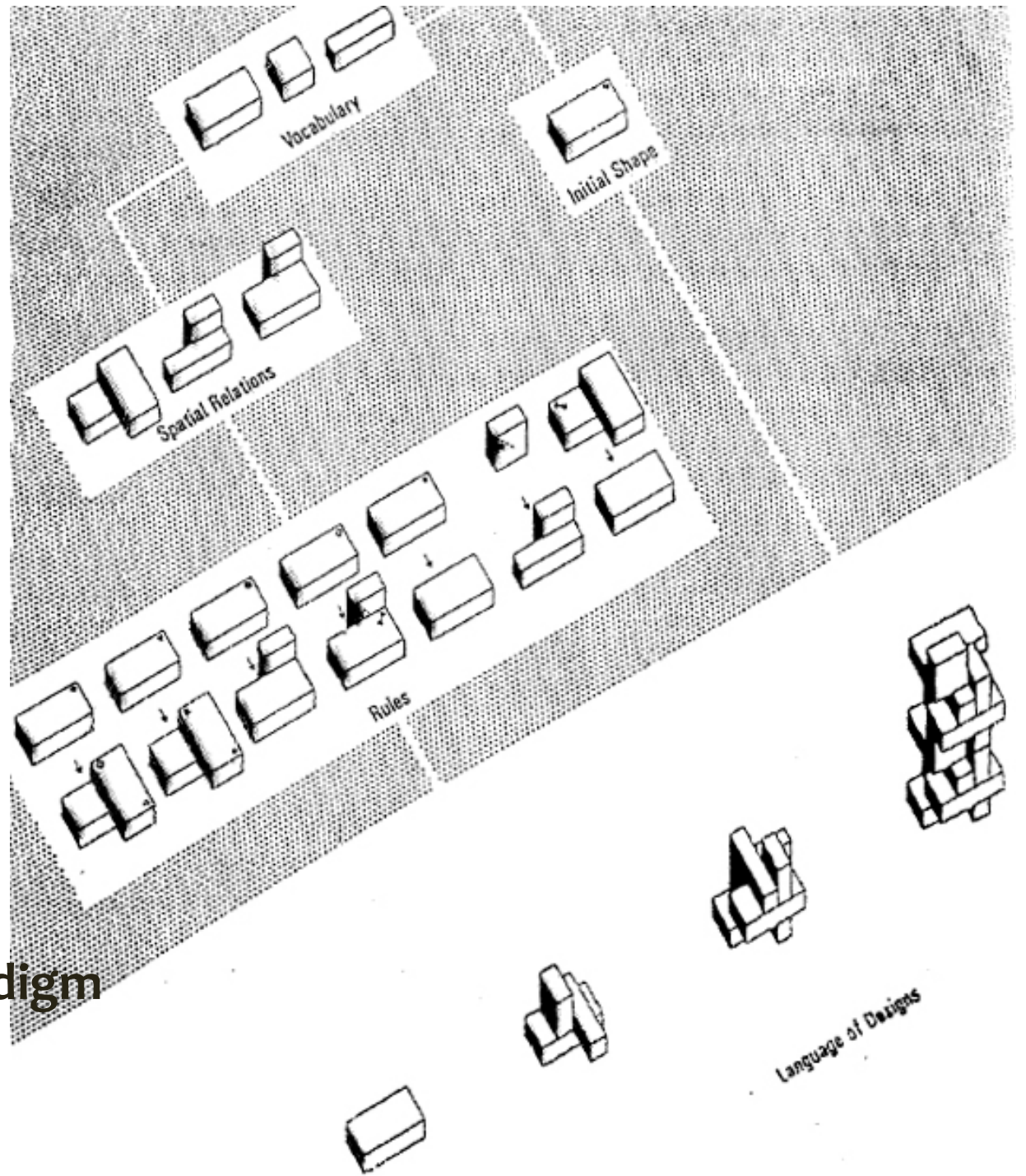
**transitions**



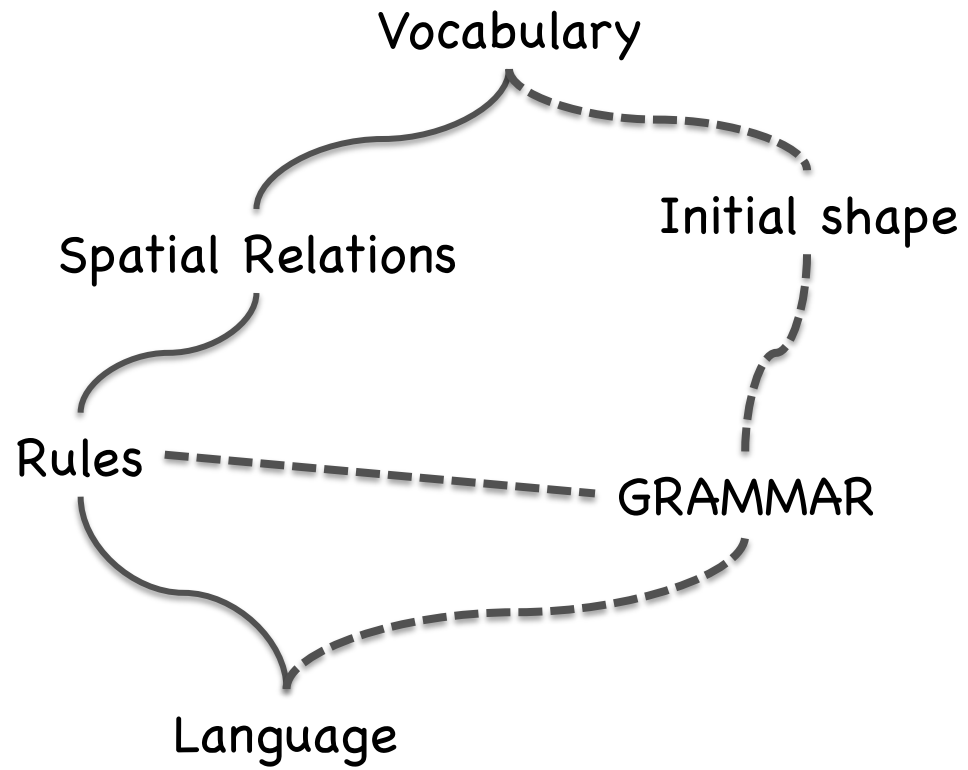
# CONSTRUCTIVE PARADIGM



**rules** when coupled with **categories**, which offer meaning & purpose, give rise to a ...



a constructive paradigm



the **grammar paradigm**

## why rules ? ...

Rules offer greater precision and control than spatial relations

Rules are simpler than designs because they are localized

Rules increase the power of observation

Rules offer explicit and detailed descriptions of knowledge

Rules shift from simple possibilities to a realm of knowledge  
(i.e., languages of designs)

Rules allow for the exploration of alternatives

Rules can be modified systematically to incorporate new ideas  
and changing circumstances

so ... lets look at **rules**

fall into two categories:

**Additive rules**

$$x \rightarrow s + t$$

$$x \in \{s, t\}$$

**Subtractive rules**

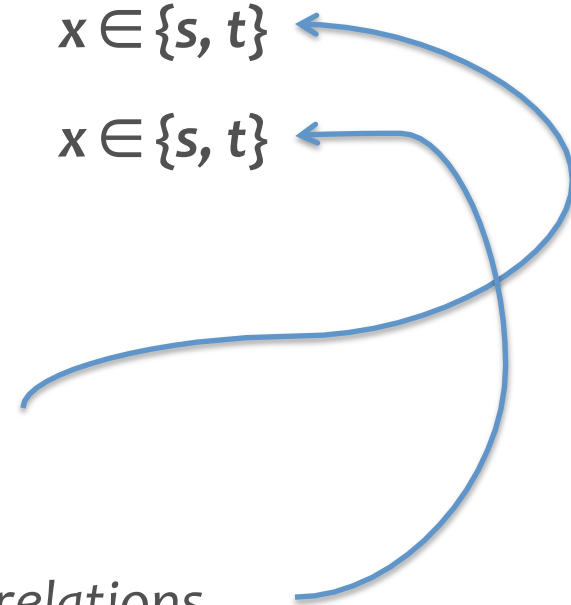
$$s + t \rightarrow x$$

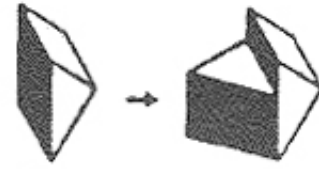
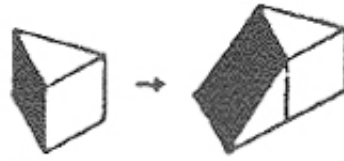
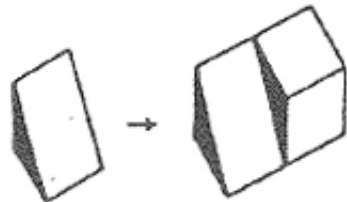
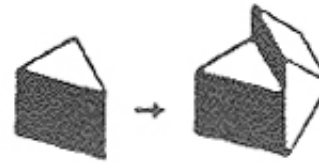
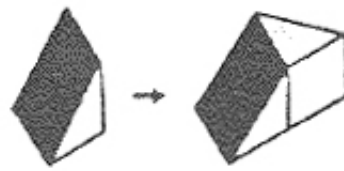
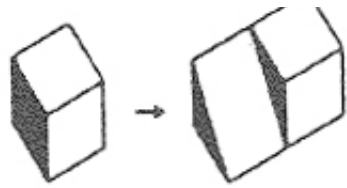
$$x \in \{s, t\}$$

“forming” spatial relations

“breaking” spatial relations

**rules**

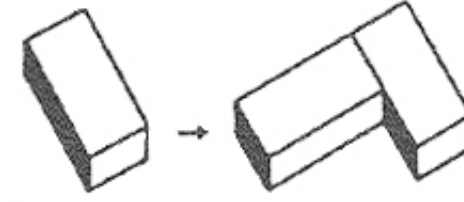
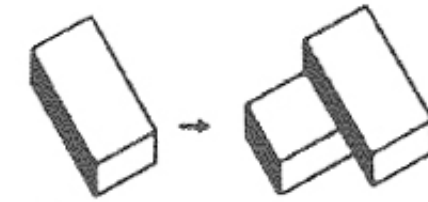
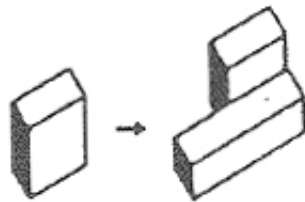
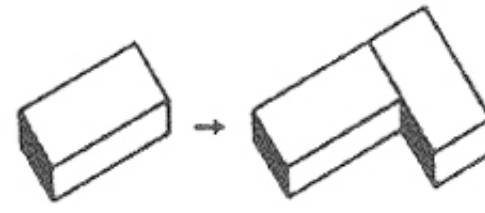
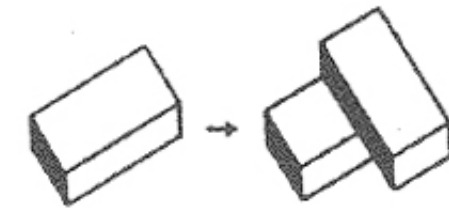
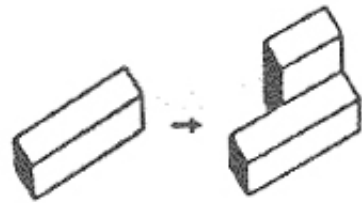




(a)

(b)

(c)

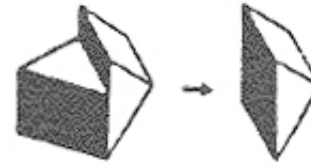
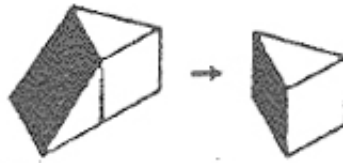
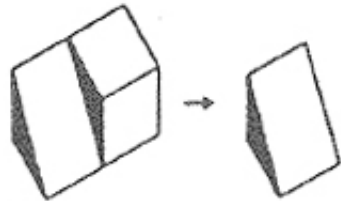
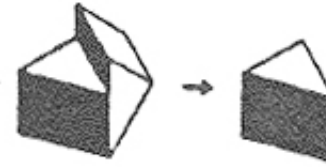
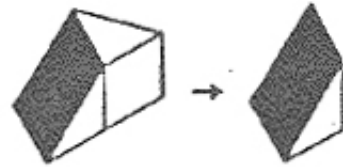
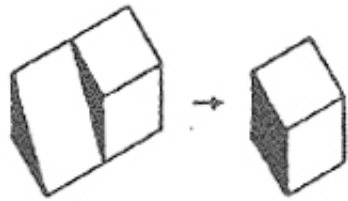


(d)

(e)

(f)

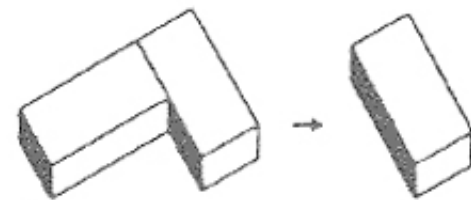
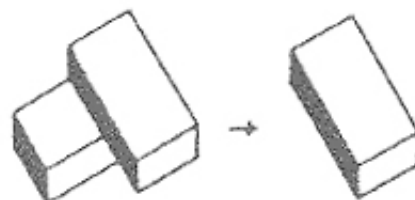
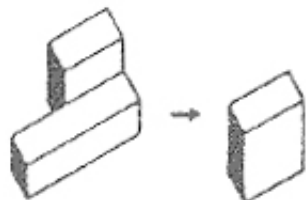
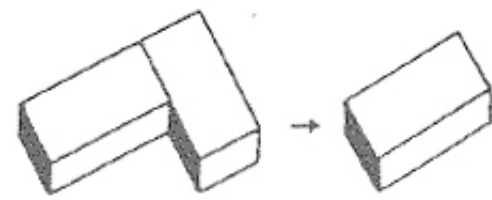
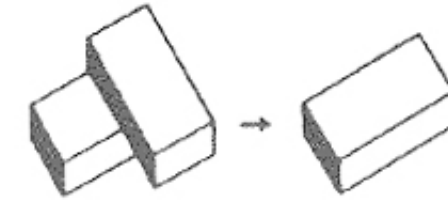
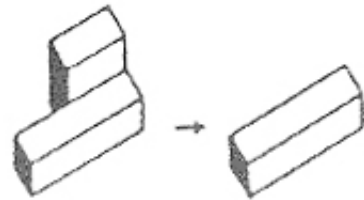
**additive rules**



(a)

(b)

(c)

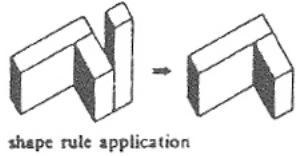
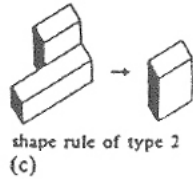
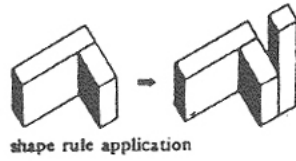
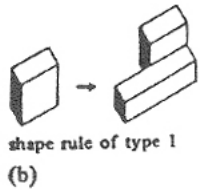


(d)

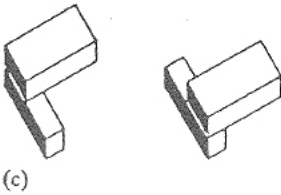
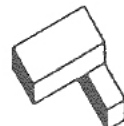
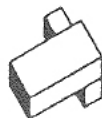
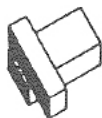
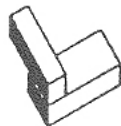
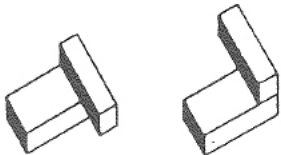
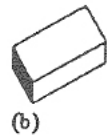
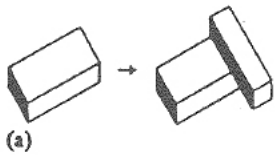
(e)

(f)

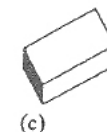
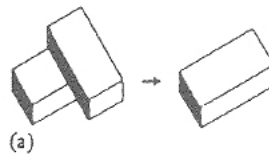
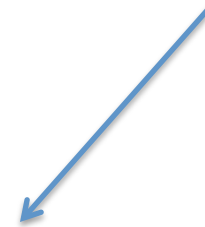
**subtractive rules**



from the same relation



under different transformations



rule application



Technically, equivalent to  
labeled elements

Rules can be aided  
by annotations

Labeled points

Rules can be aided by annotations

**Additive rules**

$$\langle x, P \rangle \rightarrow \langle s + t, Q \rangle \quad x \in \{s, t\}$$

**Subtractive rules**

$$\langle s + t, P \rangle \rightarrow \langle x, Q \rangle \quad x \in \{s, t\}$$

**annotated rules**

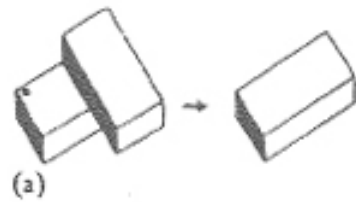
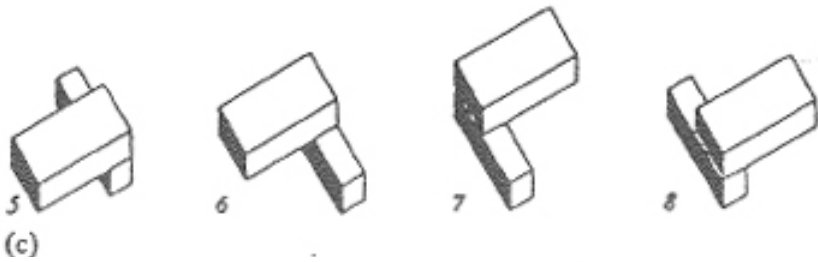
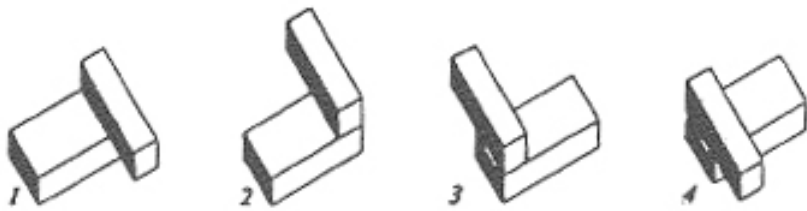
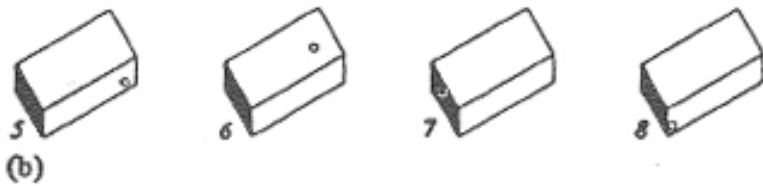
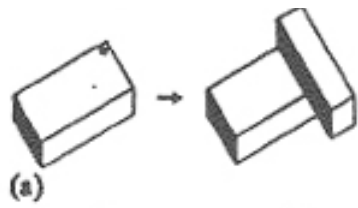
Annotations serve three purposes:

- add or destroy symmetry
- increase or decrease ambiguity
- help deal with interpenetration

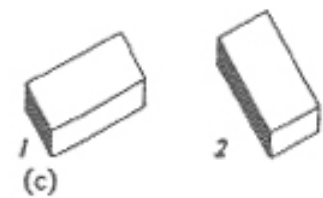
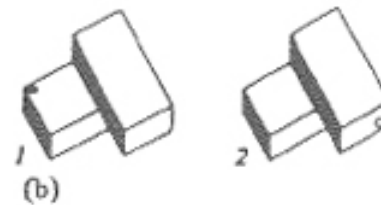
Labels are generally used to avoid such problems

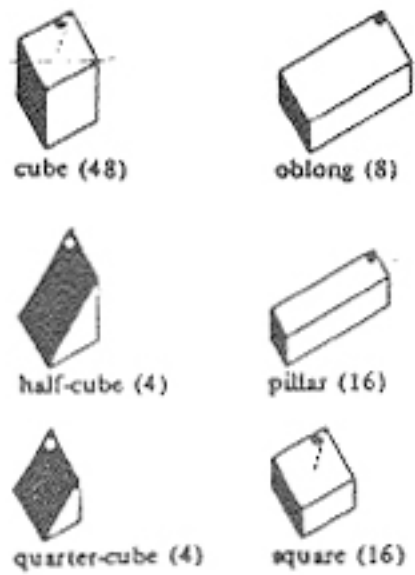
Labels are also used to demarcate stages in this play

**annotations and labels**

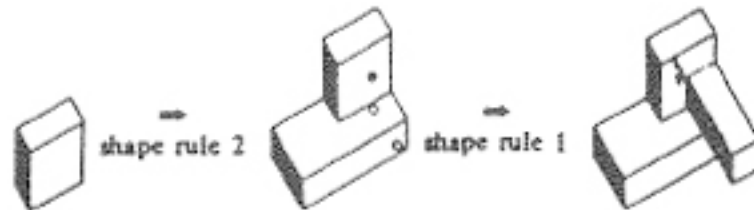
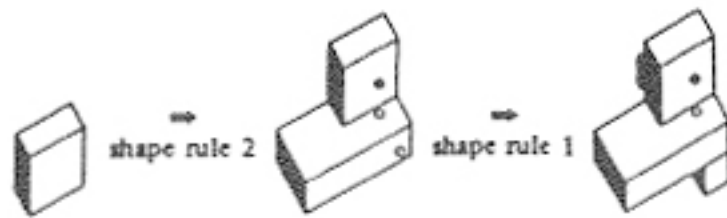
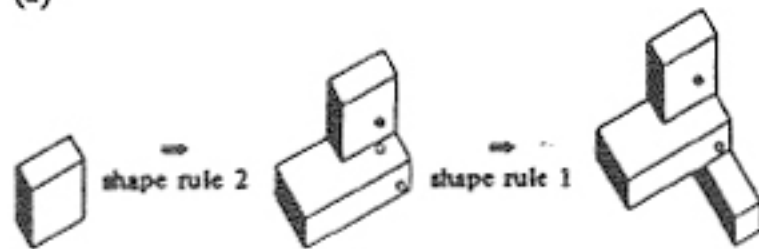


**rule application** according  
to the *symmetry of the lhs*

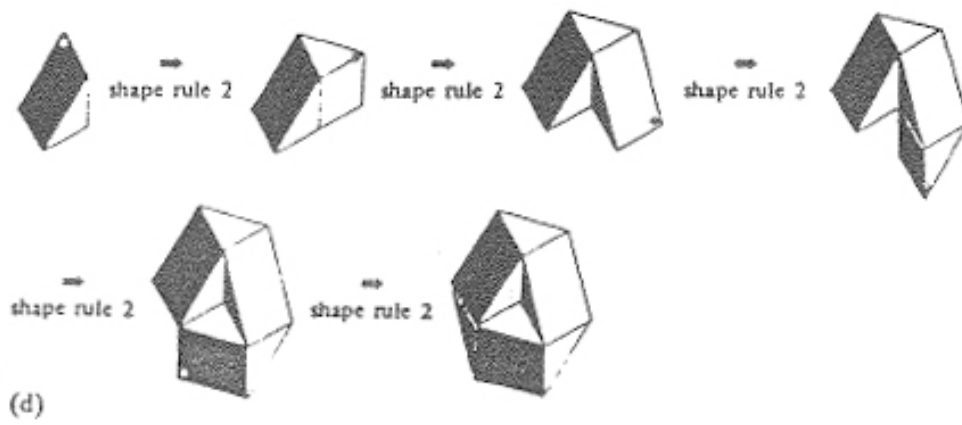
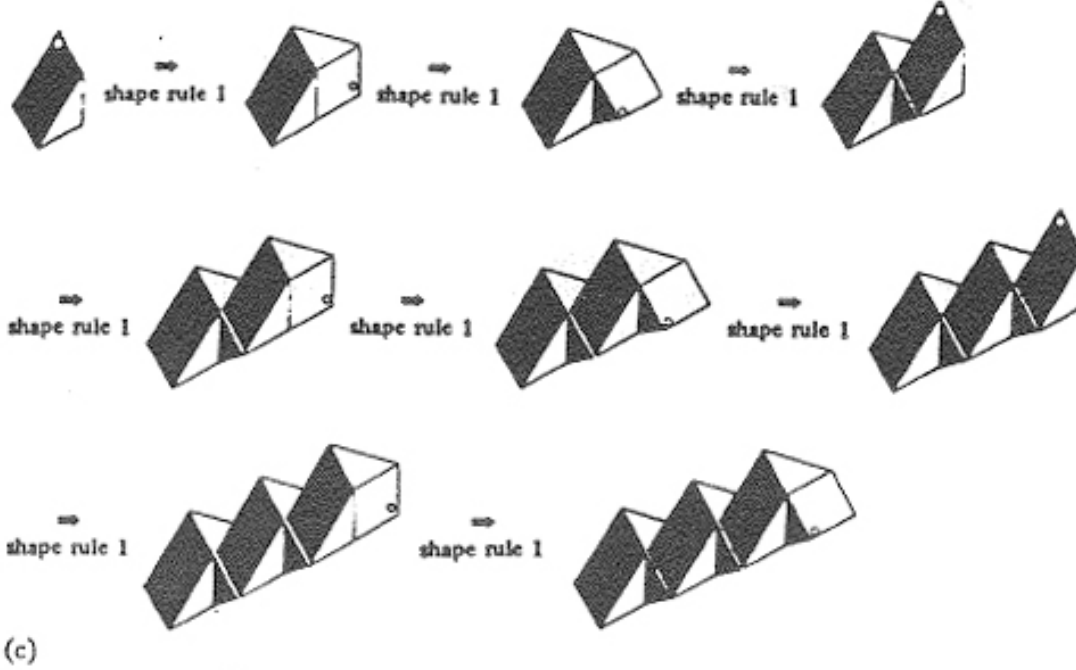
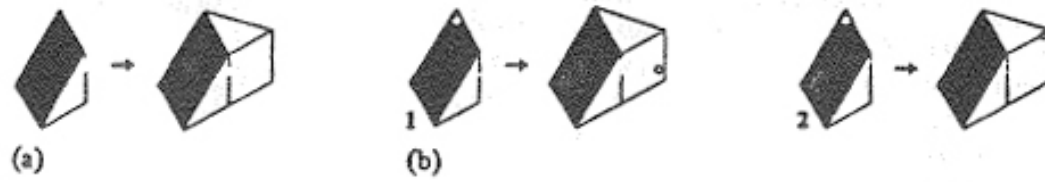


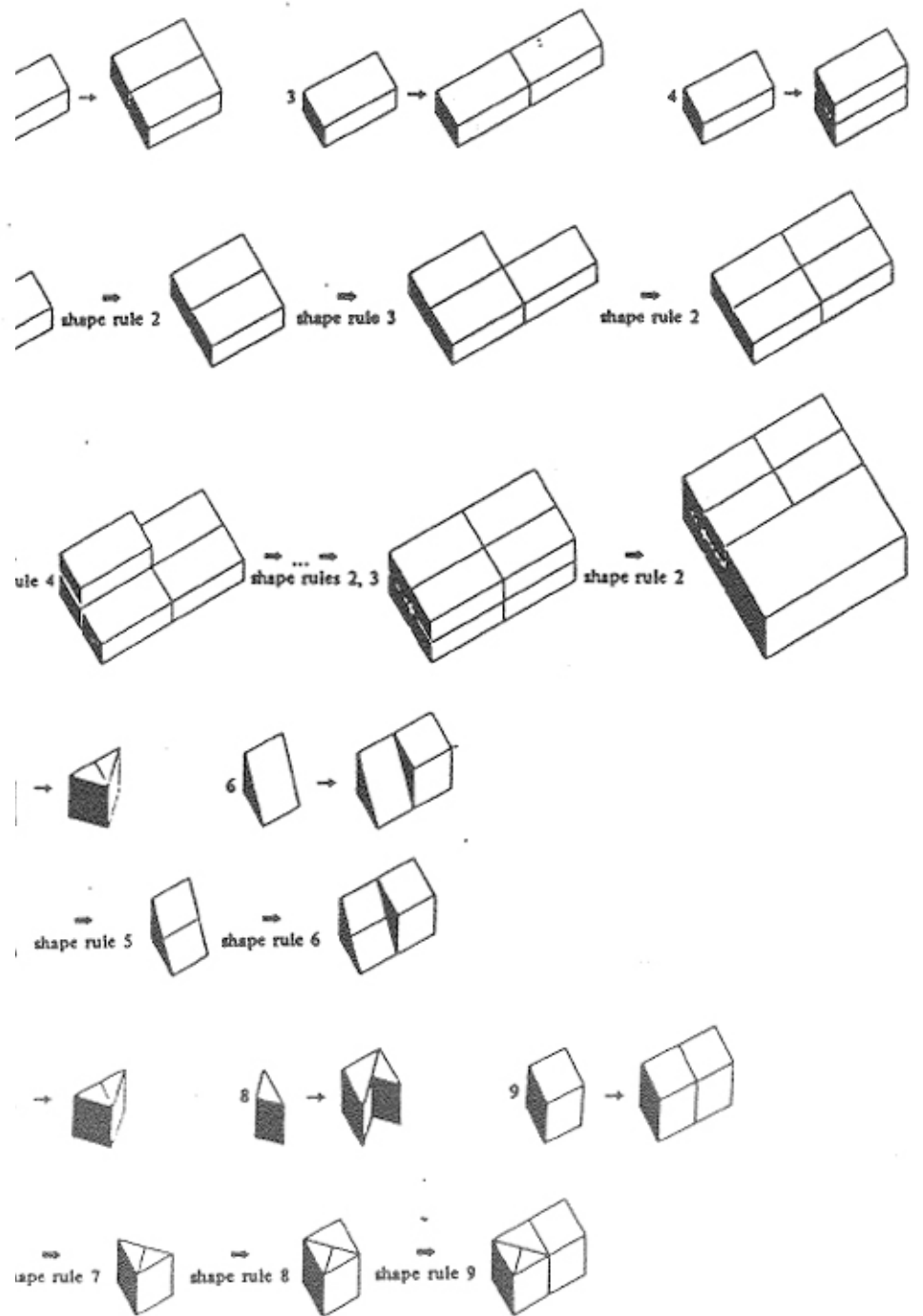
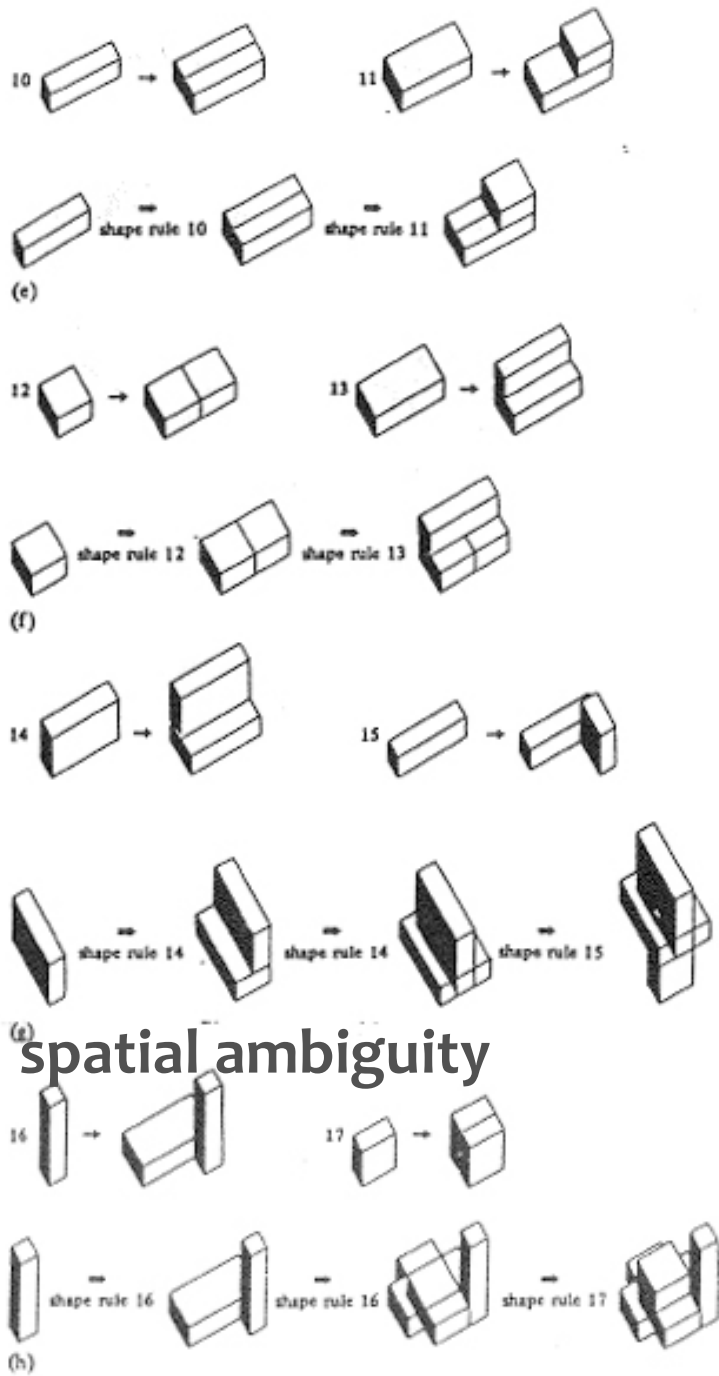


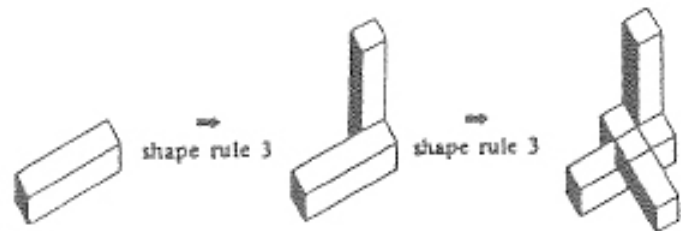
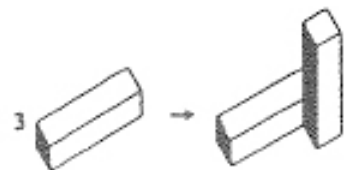
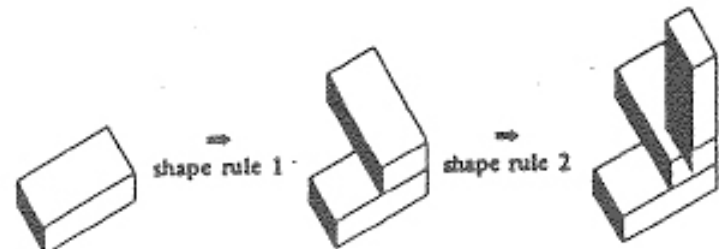
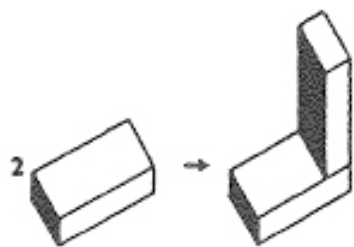
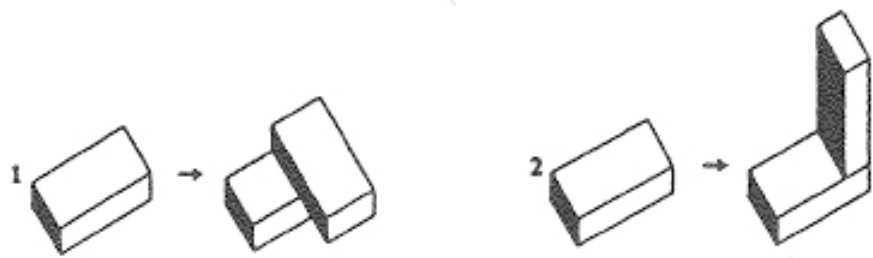
(a)



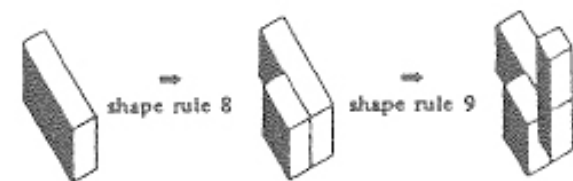
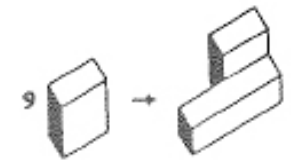
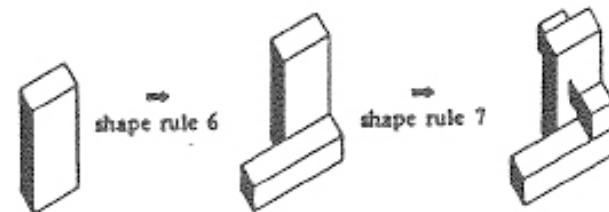
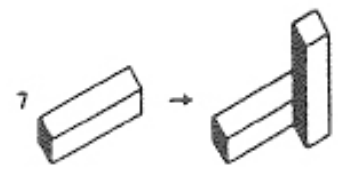
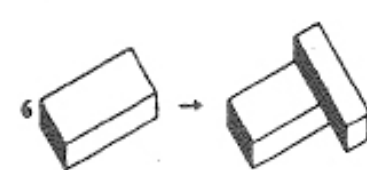
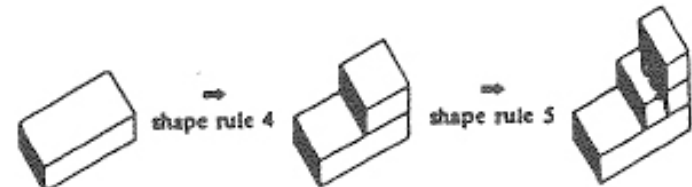
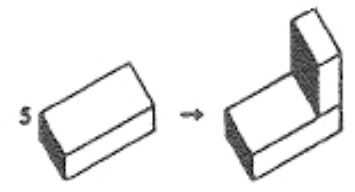
(b)



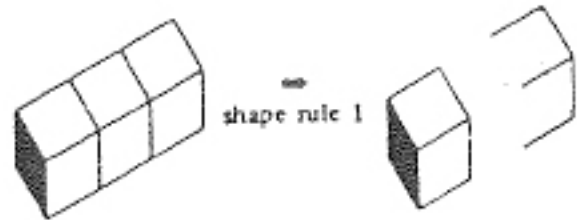
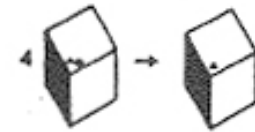
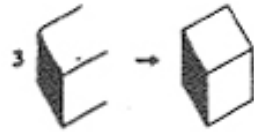
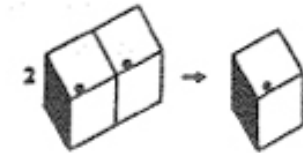
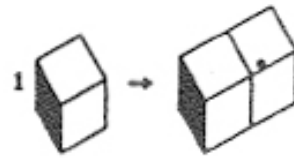
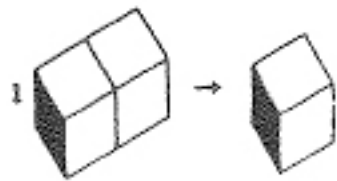




interpenetration



(e)



5  $\langle \emptyset, \{(0, 0, 0):\# \} \rangle \rightarrow \langle \emptyset, \{(x, y, z):\# \} \rangle$

6  $\langle \emptyset, \{(0, 0, 0):\# \} \rangle \rightarrow \langle \emptyset, \{(0, 0, 0):\Delta \} \rangle$

7  $\langle \emptyset, \{(0, 0, 0):\Delta \} \rangle \rightarrow \langle \emptyset, \{(x, y, z):\Delta \} \rangle$

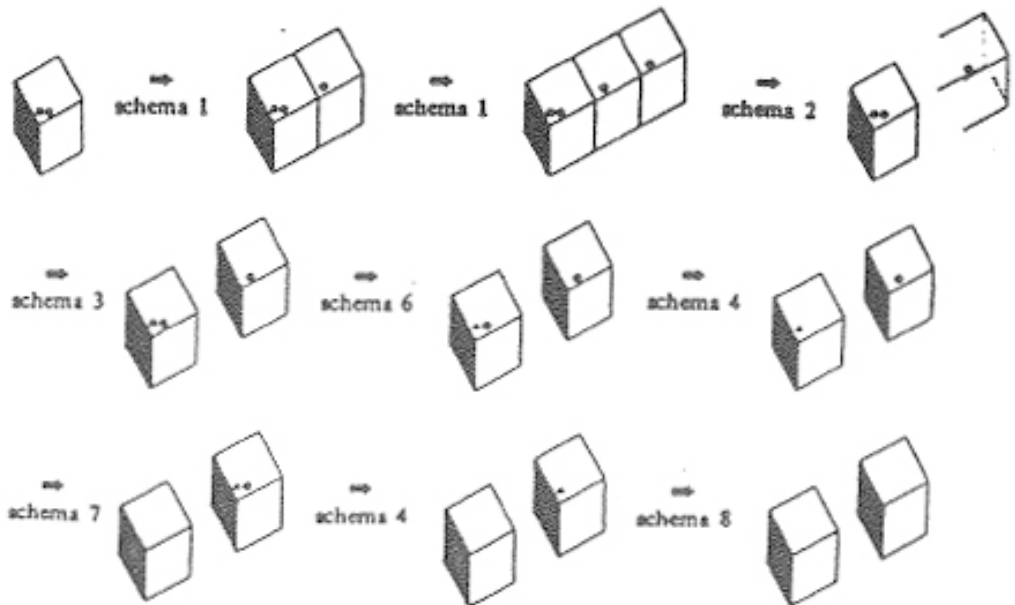
8  $\langle \emptyset, \{(0, 0, 0):\Delta \} \rangle \rightarrow \langle \emptyset, \emptyset \rangle$

shape rule schemata



initial shape

funny things happen  
under subtractive rules

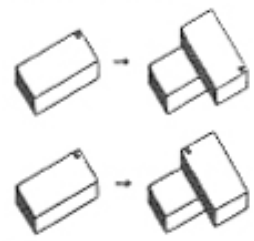




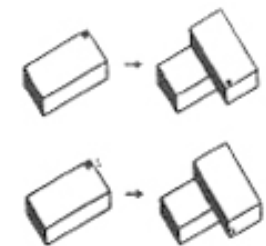
*Given a corpus of designs find the **simplest** grammar that specifies the designs*

Solution to the problem involves identifying hidden structures more often than not, vestiges of these hidden structures have been erased

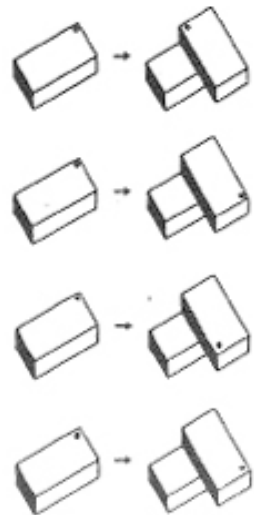
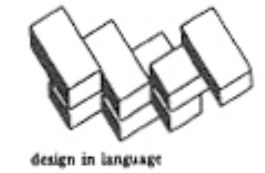
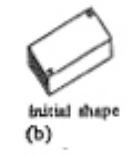
**subtractive rules** compound the inference problem



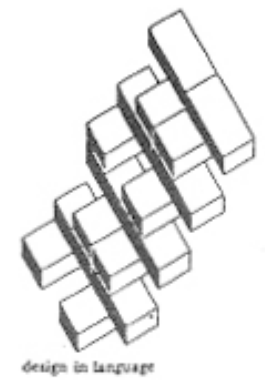
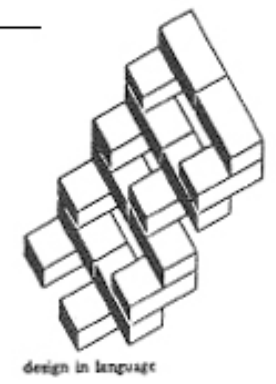
$G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 $G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 shape rules



$G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 $G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 shape rules

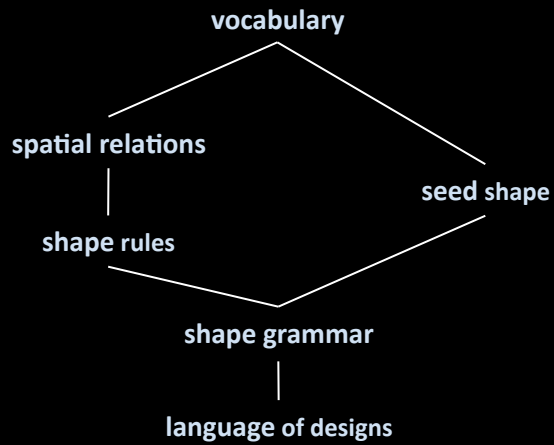


$G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 $G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 $G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 $G_p, \{(0, 0, 0):\bullet\} \rightarrow G_p, \emptyset$   
 shape rules  
(a)



two grammars

more grammar examples in the kindergarten paper



motivated by Frederick Froebel's Kindergarten model of free play and creativity through his gifts

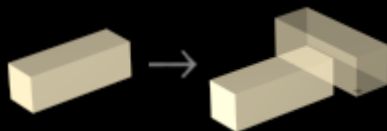
vocabulary  
+ spatic



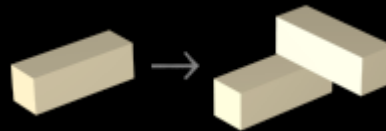
a design



free play



additive shape rule



subtractive shape rule



language



paradigm: child's play

*Play is the purest, the most spiritual, product of man at this stage,  
and is at once the prefiguration and imitation of the total human  
life, –of the inner, secret, natural life in man and in all things.*

*It produces, therefore, joy, freedom, satisfaction, repose within  
and without, peace with the world.*

*The springs of all good rest within it and go out from it.*

**on *Play* in *The Education of Man* by Friedrich Fröbel**