# Lecture 1

# Finance Project

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# Goals of the course

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- Teach the students to *systematically design* algorithms and systems for problems in the finance domain.

- Taking finance ideas from the *academic* domain and making them real.

- **Maxim:** Learn by doing.

## Nature of the course

- Largely independent in nature.

- Jeff and I are here to guide you, but you will work as independent teams.

- Think of the course as a *structured independent study course.*

## What we will do in class

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- Jeff and I will go through a systematic design of a system for pricing *mortgage backed securities* or *MBSs*.

- Use the lectures as a guide for your project.

- There will be *five* phases to the project. These phases will be defined later.

# Logistics

- Students will form a team of 3-4.

- Each team will select a paper from a set provided in class.

- Each team will design a system based on the paper they select.

- Each team will go through the five phases (to be described later).

## Some useful tips

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- Pick a *balanced team.*

- Stick to the schedule for each phase.

## Grading

- Grading will be done depending on the outcome of each phase.

- No tests and homeworks.

## Five phases

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Description of these phases will be provided later, but here they are.

- Requirements phase (**Phase 1**).

- High-level design phase (**Phase 2**).

- Low-level design phase (**Phase 3**).

## Phases (Contd)

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- In class presentation (**Phase 4**).

- Prototype (**Phase 5**)

## Prototype

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- This will be a *scaled back* version of the design.

- Make as many limiting assumptions as possible, but state them carefully.

- You can use **`C,C++,JAVA`** to implement the prototype. Make sure you tell Jeff the environment you are using.

## Presentation

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- There will be one presentation per team.

- The presentation will be a synopsis of phase 1,2, and 3.

- Each presentation will be 20 minutes long.

- Last three lectures will be all presentations.

# Paper 1

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- J. Hull and A. White, Efficient Procedures for Valuing European and American Path-Dependent Options, *Journal of Derivative, No 1, Pages 21-31*, 1993.

## Paper 2

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- J. Hull and A. White, Valuing Derivative Securities Using the Explicit Finite Difference Method, *Journal of Financial and Quantitative Analysis, Vol 25. No 1, Pages 87-99*, 1990.

## Paper 3

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- J. Hull and A. White, Pricing Interest-Rate-Derivative Securities, *The Review of Financial Studies, Vol 3, No 4, Pages 573-592*, 1990.

# Paper 4

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- A. Li, P. Ritchken, and L. Sankarasubramanian, Lattice Models for Pricing American Interest Rate Claims, *Journal of Finance, Vol L, No 2, Pages 719-736*, 1995.

## Paper 5

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- P. Ritchken and L. Sankarasubramanian, Volatility Structures of Forward Rates and the Dynamics of the Term Structure,*Mathematical Finance, Vol 5, No 1, Pages 55-72*, 1995.

## Paper 6

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- P. Ritchken and L. Sankarasubramanian, Pricing the Quality Option in Treasury Bond Futures, ,*Mathematical Finance, Vol 2, No 3, Pages 197-214*, 1995.

## Paper selection

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- Don't choose papers 5 or 6 unless you are comfortable with stochastic calculus.

- Focus on the techniques and algorithms in the paper. It is OK if you don't understand all the mathematical derivations.

## Goals of reading the paper

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- Decide what *financial instruments* you want to price after reading the paper. Pick 2-3 instruments. You will be required to understand these instruments completely.

- You should have a clear idea about the algorithm proposed in the paper.

- Make a note of advantages/disadvantages of this technique/algorithm.

## Requirements document

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- Describe the financial instrument in great detail.

- Describe the assets the instruments depend upon. State the assumptions on the prices of these assets. Describe the cash-flow characteristics.

- Describe the financial instruments and there cash-flow characteristics.

## Requirements document

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- In an abstract sense we are describing what is the semantics of each operation that the user can do.

- In this *very specific example* this amounts to defining the precise semantics of *mortgage backed securities (MBSs)*.

## Describing Mortgages

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- *Fixed Rate:* The annual interest rate of the mortgage stays fixed through out the life of the mortgage.

- *Adjustable Rate Mortgages (ARMs):* The annual interest rate can be adjusted by the loaning agency.

## Fixed Rate Mortgages

- Let $MB_0$ be the original mortgage balance.

- Let $c$ be the simple monthly interest rate.

- Let $MP$ be the monthly mortgage payment.

- Let $n$ be the number of months.

## Relationship between $MP$ and $MB_0$

- The following equation should hold between $MP$ and $MB_0$:

$$\begin{aligned} MB_0 &= MP \sum_{i=1}^{n} (1+c)^{-i} \\ &= MP \frac{1-(1+c)^{-n}}{c} \end{aligned}$$

- Hence the monthly mortgage payment $MP$ is given in terms of the mortgage amount $MB_0$ using the following formula:

$$MP = MB_0 \frac{c(1+c)^n}{(1+c)^n - 1}$$

## Principal at time $t$

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- Let $MB_t$ the remaining mortgage balance at time $t$.

- We have the following relationship between $MB_t$ and $MP$.

$$MB_t = MP\frac{1-(1+c)^{-(n-t)}}{c}$$

- So we have the following equation between $MB_0$ and $MB_t$:

$$MB_t = MB_0\frac{(1+c)^n-(1+c)^t}{(1+c)^n-1}$$

## Breaking the mortgage payments

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- At time $t$ the mortgage balance is $MB_{t-1}$ $(t \geq 1)$.

- The interest $I_t$ on this is mortgage balance is:
$$cMB_{t-1}$$
- The mortgage payment $MP$ at time $t$ is broken into two parts: *interest payment* $I_t$ and *payment applied towards principal* $P_t$. We have the following equation:
$$MP = I_t + P_t$$

## Scenarios

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- A requirements document for a large software system has a huge number of scenarios.

- Basically, *scenarios* describe what should happen in specific cases.

- For example, in the requirements document for an online brokerage system a scenario might describe what should happen when a user logs on and buys a stock.

## Examples

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- In this case, scenarios are simply examples of cash flows.

- Consider a mortgage of $100,000$, annual mortgage rate $(12c)$ of $9.5\%$ and time period of 30 years (360 months).

- Check that the monthly mortgage rate $MP$ is 840.85.

# Example continued

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- Check that $I_0 = 791.67$ and $P_0 = 49.19$.

- Check that $I_{215} = 574.95$ and $P_{215} = 265.90$.

- $I_t$ is a decreasing function of $t$ and $P_t$ is an increasing function of $t$. (`Why?`).

## ARMs

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- ARMs start out with an initial interest rate.

- ARMs interest rate can be adjusted by a margin $m$ at a frequency specified in the contract.

- *Lifetime cap* $c_L$: This is an upper bound that the interest rate cannot exceed.

- *Lifetime floor* $c_F$: This is a lower bound on the interest rate.

## ARMs

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- Let us the interest rate is $c(t-1)$ at time $t-1$ and we are adjusting at time $t$,

- The new interest rate $c_(t)$ is given by the following cases:

  – if $x(t) + m > c(t-1)$

  $$\min\left[x(t) + m, c_L, c(t-1) + c_P\right]$$

  – if $x(t) + m \leq c(t-1)$

  $$\min\left[x(t) + m, c_F, c(t-1) - c_P\right]$$

## Explanation of terms

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- $x(t)$ : Underlying index specified in the contract. Two widely used indices are cost of funds index (COFI) and a constant maturity (one year or five year) Treasury index.

- $c_L$ and $c_F$ are the lifetime cap and floor respectively.

- $c_P$ denotes the ARMs periodic cap, i.e., cannot adjust by more than this amount.

## New Mortgage Payment

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- Assume that current time is $t$, the interest rate, mortgage balance at time $t-1$ are $c(t-1)$ and $MB_{t-1}$.

- The new *adjusted* interest rate is $c(t)$.

- The new mortgage payment $MP(t)$ is given by the following expression:
$$MB_{t-1}\frac{1-(1+c(t))^{-(n-t)}}{c(t)}$$

- Everything else stays the same.

## Action Items

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- Pick your teams (3-4 students) and send e-mail to Jeff or me.

- Pick a paper (Norene Mears has a copy of each paper) and copy it. *Return the master copy.*

- Papers should be in London and New York in 2-3 days.

- Read the paper and decide on the financial instruments you are going to price.

- Start reading about the financial instruments you are going to price.