Lecture 1

Finance Project

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Goals of the course

- Teach the students to *systematically design* algorithms and systems for problems in the finance domain.

- Taking finance ideas from the *academic* domain and making them real.

- **Maxim:** Learn by doing.

Nature of the course

- Largely independent in nature.

- Jeff and I are here to guide you, but you will work as independent teams.

- Think of the course as a *structured independent study course.*

What we will do in class

- Jeff and I will go through a systematic design of a system for pricing *mortgage backed securities* or *MBSs*.

- Use the lectures as a guide for your project.

- There will be *five* phases to the project. These phases will be defined later.

Logistics

- Students will form a team of 3-4.

- Each team will select a paper from a set provided in class.

- Each team will design a system based on the paper they select.

- Each team will go through the five phases (to be described later).

Some useful tips

- Pick a *balanced team*.

- Stick to the schedule for each phase.

Grading

- Grading will be done depending on the outcome of each phase.

- No tests and homeworks.

Five phases

Description of these phases will be provided later, but here they are.

- Requirements phase (**Phase 1**).

- High-level design phase (**Phase 2**).

- Low-level design phase (**Phase 3**).

Phases (Contd)

- In class presentation (**Phase 4**).

- Prototype (**Phase 5**)

Prototype

- This will be a *scaled back* version of the design.

- Make as many limiting assumptions as possible, but state them carefully.

- You can use `C,C++,JAVA` to implement the prototype. Make sure you tell Jeff the environment you are using.

Presentation

- There will be one presentation per team.

- The presentation will be a synopsis of phase 1,2, and 3.

- Each presentation will be 20 minutes long.

- Last three lectures will be all presentations.

Paper 1

- J. Hull and A. White, Efficient Procedures for Valuing European and American Path-Dependent Options, *Journal of Derivative, No 1, Pages 21-31*, 1993.

Paper 2

- J. Hull and A. White, Valuing Derivative Securities Using the Explicit Finite Difference Method, *Journal of Financial and Quantitative Analysis, Vol 25. No 1, Pages 87-99*, 1990.

Paper 3

- J. Hull and A. White, Pricing Interest-Rate-Derivative Securities, *The Review of Financial Studies, Vol 3, No 4, Pages 573-592*, 1990.

Paper 4

- A. Li, P. Ritchken, and L. Sankarasubramanian, Lattice Models for Pricing American Interest Rate Claims, *Journal of Finance, Vol L, No 2, Pages 719-736*, 1995.

Paper 5

- P. Ritchken and L. Sankarasubramanian, Volatility Structures of Forward Rates and the Dynamics of the Term Structure,*Mathematical Finance, Vol 5, No 1, Pages 55-72*, 1995.

Paper 6

- P. Ritchken and L. Sankarasubramanian, Pricing the Quality Option in Treasury Bond Futures, ,*Mathematical Finance, Vol 2, No 3, Pages 197-214*, 1995.

Paper selection

- Don't choose papers 5 or 6 unless you are comfortable with stochastic calculus.

- Focus on the techniques and algorithms in the paper. It is OK if you don't understand all the mathematical derivations.

Goals of reading the paper

- Decide what *financial instruments* you want to price after reading the paper. Pick 2-3 instruments. You will be required to understand these instruments completely.

- You should have a clear idea about the algorithm proposed in the paper.

- Make a note of advantages/disadvantages of this technique/algorithm.

Requirements document

- Describe the financial instrument in great detail.

- Describe the assets the instruments depend upon. State the assumptions on the prices of these assets. Describe the cash-flow characteristics.

- Describe the financial instruments and there cash-flow characteristics.

Requirements document

- In an abstract sense we are describing what is the semantics of each operation that the user can do.

- In this *very specific example* this amounts to defining the precise semantics of *mortgage backed securities (MBSs)*.

Describing Mortgages

- *Fixed Rate:* The annual interest rate of the mortgage stays fixed through out the life of the mortgage.

- *Adjustable Rate Mortgages (ARMs):* The annual interest rate can be adjusted by the loaning agency.

Fixed Rate Mortgages

- Let MB_0 be the original mortgage balance.

- Let c be the simple monthly interest rate.

- Let MP be the monthly mortgage payment.

- Let n be the number of months.

Relationship between MP and MB_0

- The following equation should hold between MP and MB_0:

$$MB_0 = MP \sum_{i=1}^{n} (1+c)^{-i}$$
$$= MP \frac{1-(1+c)^{-n}}{c}$$

- Hence the monthly mortgage payment MP is given in terms of the mortgage amount MB_0 using the following formula:

$$MP = MB_0 \tfrac{c(1+c)^n}{(1+c)^n - 1}$$

Principal at time t

- Let MB_t the remaining mortgage balance at time t.

- We have the following relationship between MB_t and MP.

$$MB_t = MP \frac{1 - (1+c)^{-(n-t)}}{c}$$

- So we have the following equation between MB_0 and MB_t:

$$MB_t = MB_0 \frac{(1+c)^n - (1+c)^t}{(1+c)^n - 1}$$

Breaking the mortgage payments

- At time t the mortgage balance is MB_{t-1} $(t \geq 1)$.

- The interest I_t on this is mortgage balance is:
$$cMB_{t-1}$$

- The mortgage payment MP at time t is broken into two parts: *interest payment* I_t and *payment applied towards principal* P_t. We have the following equation:
$$MP = I_t + P_t$$

Scenarios

- A requirements document for a large software system has a huge number of scenarios.

- Basically, *scenarios* describe what should happen in specific cases.

- For example, in the requirements document for an online brokerage system a scenario might describe what should happen when a user logs on and buys a stock.

Examples

- In this case, scenarios are simply examples of cash flows.

- Consider a mortgage of $100,000$, annual mortgage rate ($12c$) of 9.5% and time period of 30 years (360 months).

- Check that the monthly mortgage rate MP is 840.85.

Example continued

- Check that $I_0 = 791.67$ and $P_0 = 49.19$.

- Check that $I_{215} = 574.95$ and $P_{215} = 265.90$.

- I_t is a decreasing function of t and P_t is an increasing function of t. (`Why?`).

ARMs

- ARMs start out with an initial interest rate.
- ARMs interest rate can be adjusted by a margin m at a frequency specified in the contract.
- *Lifetime cap* c_L: This is an upper bound that the interest rate cannot exceed.
- *Lifetime floor* c_F: This is a lower bound on the interest rate.

ARMs

- Let us the interest rate is $c(t-1)$ at time $t-1$ and we are adjusting at time t,

- The new interest rate $c_(t)$ is given by the following cases:
  - if $x(t)+m > c(t-1)$
  $$\min\left[x(t)+m, c_L, c(t-1)+c_P\right]$$
  - if $x(t)+m \leq c(t-1)$
  $$\min\left[x(t)+m, c_F, c(t-1)-c_P\right]$$

Explanation of terms

- $x(t)$: Underlying index specified in the contract. Two widely used indices are cost of funds index (COFI) and a constant maturity (one year or five year) Treasury index.

- c_L and c_F are the lifetime cap and floor respectively.

- c_P denotes the ARMs periodic cap, i.e., cannot adjust by more than this amount.

New Mortgage Payment

- Assume that current time is t, the interest rate, mortgage balance at time $t-1$ are $c(t-1)$ and MB_{t-1}.

- The new *adjusted* interest rate is $c(t)$.

- The new mortgage payment $MP(t)$ is given by the following expression:
$$MB_{t-1}\frac{1-(1+c(t))^{-(n-t)}}{c(t)}$$

- Everything else stays the same.

Action Items

- Pick your teams (3-4 students) and send e-mail to Jeff or me.

- Pick a paper (Norene Mears has a copy of each paper) and copy it. *Return the master copy.*

- Papers should be in London and New York in 2-3 days.

- Read the paper and decide on the financial instruments you are going to price.

- Start reading about the financial instruments you are going to price.