# Intro to Prob and Stat I <br> GSIA, Carnegie Mellon University 45-733, Spring 2002 (mini 3) 

Final, Thursday Feb 28, 2002

Instructions You may use any books, notes, calculators, and other aids you like. You may not converse, nor may you cooperate.

Please complete all questions.
Each question is worth 20 points.
Please show all relevant work.
Please do use the reverse sides of pages, if necessary, but try to avoid using/attaching additional pages.

1. In American roulette, there is a wheel with 38 spaces on it. The spaces are numbered $1-36,0$, and 00 . There are eighteen red and eighteen black spaces, and the 0 and 00 spaces are green. At each play, each of the 38 spaces is equally likely to come up, and successive plays are independent.
There are many ways to bet roulette, but we will consider two ways. First, you may bet on either red or black. If the color you bet comes up, you win $\$ 1$. If it does not, you lose $\$ 1$. Second, you may bet on a particular number. If your number comes up, you win $\$ 35$. If your number does not, you lose $\$ 1$.
(a) (2 points) Betting black/red, what are the odds of winning?
(b) (2 points) Betting black/red, what is the expected value of playing?
(c) (2 points) Betting a specific number, what are the odds of winning?
(d) (2 points) Betting a specific number, what is the expected value of playing?
(e) ( 6 points) In which way of betting is your chance of being ahead after 4 plays higher. (Ahead does not include "even," it means having more money than you started with)
(f) (6 points) If the casino runs a roulette game 1000 times a day, what is the probability that it comes out ahead that day? Assume only one person at a time plays and all bets are $\$ 1$ on a specific number.
(additional work page)
2. Your company frequently introduces new products. As part of this process, the marketing department is asked to forecast sales for each product's first year. You have been asked to evaluate the performance of marketing in this function. You have collected the following data (all in millions of $\$$ ). You may assume that forecast and actual sales are distributed normally and independently.

|  | Forecast | Actual |
| ---: | ---: | ---: |
|  | 25 | 16 |
|  | 22 | 25 |
| 15 | 8 |  |
| 13 | 5 |  |
| 36 | 29 |  |
| 49 | 51 |  |
|  | 18 | 17 |
|  | 33 | 26 |
|  | 22 | 19 |
|  |  |  |
| mean | 25.9 | 21.8 |
| std dev | 11.5 | 13.6 |

(a) (5 points) Calculate a $90 \%$ confidence interval for actual mean sales. Interpret.
(b) (5 points) Calculate an $80 \%$ confidence interval for the actual variance of sales. Interpret.
(c) (5 points) Test, at the $10 \%$ level (2-sided) that mean actual sales are 29 .
(d) (5 points) If we were to test at the $5 \%$ level, what would happen?
(additional work page)
3. Please use the same data as in the previous problem.
(a) (10 points) Calculate a $90 \%$ confidence interval for the mean error in forecast.
(b) (10 points) Test, at the $95 \%$ level, the hypothesis that the forecasts are right on average.
(additional work page)
4. A health insurer is reviewing its contracts with hospitals. One important service it is considering is coronary artery bypass graft surgery ("heart bypass"). Death is an important complication of this surgery, so that death rates in hospital are an important quality indicator.
It is often claimed that "practice makes perfect" in this procedure, so you are to look into whether high volume providers (lots of practice) produce better outcomes. You compile the data available for your insureds and find:

| Category | Patients | Dead |
| ---: | ---: | ---: |
|  |  |  |
| High Volume | 1006 | 13 |
| Low Volume | 297 | 12 |

(a) (10 points) Test, at the $5 \%$ level, the claim that high and low volume hospitals have the same mortality rates. Interpret.
(b) (10 points) Compute an $80 \%$ confidence interval for the difference in mortality rates between high and low volume hospitals.
(additional work page)
5. As part of an effort to site a new plant, you perform a survey in Anytown, PA to assess local wage conditions. You survey, randomly, 100 workers in similar plants and find that they make, in wages and benefits, on average, $\$ 23.12 / \mathrm{hr}$ with a standard deviation of $\$ 6.75 / \mathrm{hr}$.
(a) (10 points) Make and interpret a $95 \%$ confidence interval for mean pay.
(b) (10 points) Your boss wants a narrower interval. What are your options for giving it to her?
(additional work page)

