SOLUTIONS FOR ASSIGNMENT 2

There were two distinct parts to this assignment — the CAPM and factor model.

1)

a) Once again, I calculated this in EXCEL. The standard deviation using the command for a sample for the REIT is 9.231 and the standard deviation for the Brown group is 8.167. The Vanguard has a standard deviation of 4.606. This makes sense because the Vanguard index is a diversified portfolio, neither the REIT nor the Brown are well diversified. The REIT has the highest standard deviation.

b) I calculated one portfolio that had a weight of 0.99 in the index and 0.01 in the REIT. This portfolio had a standard deviation of 4.568 . I also calculated a portfolio of 0.99 in the index and 0.01 in the Brown group. This had a standard deviation of 4.614. Notice that the composite portfolio with the REIT has a lower standard deviation than the composite portfolio with the brown group. The REIT is uncorrelated with the Vanguard, and it provides further gains from diversification.

c) I calculated the beta estimates for the two stocks as 0.147 for the REIT and 1.163 for the Brown group.

The beta of an asset is the covariance of the asset with the market portfolio. From part b) you know that a portfolio made up of the REIT and the index has a lower standard deviation than the portfolio made up of the index and the Brown stock. We know that if you add a security to a portfolio, the change in standard deviation of the portfolio depends not on the standard deviation of the asset that you are adding but how the asset covaries with the portfolio, therefore if you add REIT and you get a low standard deviation, then the REIT must have a low covariance with the Vanguard Group i.e. a low beta. The reverse argument works for the Brown group.

NOTICE: It is important to recognize that your results are **ESTIMATES**, that is they are the best guess of the true population values. If we wanted to do a t–test of the significance of the estimated variables: we have 24 observations 2 degrees of freedom at the 95 percent confidence level, the t–value is 2.074.

Hence, the true value of the brown beta is somewhere in the interval:

$$1.164 + / - (2.074)(0.285)$$

Hence, the value for brown could be as high as 1.754 or as low as 0.574. (The standard error of the beta estimate is 0.285). The REIT has an even higher standard error 0.426. As above you can

construct an interval in which the true parameter value lies with 95 percent probability:

$$0.147 + / - (2.074)(0.426)$$

Hence, the REIT could be as low as -0.7365 and as high as 1.0305.

Were you bothered by the fact that the intercept on the beta regressions were not 0? Think about the CAPM and what regression you actually performed. (The intercept is not supposed to be 0).

2) This questions illustrates both the convenience and the pitfalls of using a factor approach.

a) I estimated the variance of each of the securities and the covariances (using the sample command).

$$V = \begin{pmatrix} 332.39 & 377.93 & 355.82 & 205.80 \\ 377.93 & 684.77 & 513.36 & 319.04 \\ 355.82 & 513.36 & 539.17 & 292.06 \\ 205.80 & 319.04 & 292.06 & 333.86 \end{pmatrix}$$

b) I get $\beta_A = 2.01, \beta_B = 2.92, \beta_C = 2.81, \beta_D = 2.02$, where β_A is the factor loading on asset A.

c) The variance of my idiosyncratic shocks are: $\sigma_A^2 = 155.05, \sigma_B^2 = 309.79, \sigma_C^2 = 192.67, \sigma_D^2 = 155.31$. (Notice these are LARGE).

d)

$$V = \begin{pmatrix} 332.386 & 257.88 & 247.89 & 177.94 \\ 257.88 & 684.77 & 360.47 & 258.76 \\ 247.89 & 360.47 & 539.17 & 248.74 \\ 177.94 & 258.76 & 248.74 & 333.86 \end{pmatrix}$$

The variances are identical, but the covariances are too small. Essentially, the factor approach supposes that the factor explains *all* the uncertainty associated with returns, and that the residuals are correlated. If they are positively correlated, then the covariances will be underestimated using a factor approach. This is a weakness of the factor approach. If there is a substantial correlation between the residuals, then the variance covariance matrix could be quite far off from the directly estimated version.