SOLUTIONS ASSIGNMENT 1

1) I did this using EXCEL — you could also actually do the optimization problem by hand. First, I defined a 5×1 vector of weights, called w. I typed in the 5×5 variance/covariance matrix called V and the 5×1 vector of expected returns called R.

   a) I calculated $w^T V w = 15.70844$ where each of the weights is 0.2 and $w^T R = 10.228$. Hence, the standard deviation of the equally weighted portfolio is 3.963438.

   b) Now, the problem is to find the portfolio that gives you a mean of 15% but has the lowest possible standard deviation. To do this, minimize the standard deviation (or variance) of the portfolio subject to the constraint that the expected return cell is 15 and subject to the constraint that the weights sum to 1. $w_1 = 1.402933$, $w_2 = -0.00473$, $w_3 = -1.59337$, $w_4 = 1.940491$, $w_5 = -0.74532$. Notice that in order to get the high return you will short sell asset 2,3, and 5 (i.e. their weights are negative). The value of the variance for these weights is 20.70366. Hence, the standard deviation is 4.550128.

   c) If you don't want to short sell any of the third asset then you have to solve the problem with a constraint on the third asset so that $w_3 \geq 0$. In which case, the weights turn out to be : $w_1 = 1.621453$, $w_2 = -1.0637$, $w_3 = 0$, $w_4 = 2.313419$ and $w_5 = -1.87117$. The expected return of the portfolio is 15 (of course) and the variance is 37.19202. Notice that the minute you add a constraint in, the variance for the same mean goes up. Adding a constraint can't help you. The standard deviation is 6.098526

   d) If the risk–free rate is 5%, then the optimal risky portfolio comes from maximizing $\frac{Er_p - r_f}{\sigma_p}$, with the constraint that the weights sum to one. In this case, the weights are — $w_1 = 1.469775$, $w_2 = 0.123962$, $w_3 = -0.62295$, $w_4 = 0.32985$ and $w_5 = -0.30063$. The slope of the tangent line is 2.548. The optimal risky portfolio has expected return 11.28039 and standard deviation of 2.464668.

   e) If you want to get the 15% with the optimal risky portfolio and the risk free: If y is the weight in the risk–free security, Solve

$$y5 + (1 - y)11.28 = 15$$

   . The weight in the risk free asset will be -0.592. That is, you go and borrow the risk free asset to invest 1.592 in the optimal risky portfolio. The standard deviation of the

new portfolio is just $1.592 \times 2.465 = 3.924$ (I've rounded the numbers off). Notice that this risk is substantially less than the risk that you incur when you don't have a risk free asset — so compare this with your answer in b).

f) When I redid b), my weights were, $1.716528, -0.669, -1.41687, 2.174137$ and -0.8048. When I redid d), my weights were $1.764753, -0.53995, -0.41897, 0.528711, -0.33454$. Notice that when the covariance increased you actually decreased the standard deviation of the portfolio. This is because the weights of asset 1 and 2 were of opposite signs. By short selling 2 you are creating an asset that is *negatively* correlated with asset 1.

2) One way to eliminate the "wrong" portfolio is to see which one has the highest slope. You know that the optimal risky portfolio lies on the highest possible line that is tangent to the mean variance efficient frontier. So, compare:

$$\frac{10-5}{15} = 0.333$$

with

$$\frac{18-5}{17} = 0.765.$$

Clearly, portfolio B dominates portfolio A — it has a higher slope. So, A cannot possibly be the optimal risky one. It will turn out that this slope is one of the ways in which portfolios are evaluated: if you can show that one portfolio has a higher slope then the manager is doing a better job.

3) If the two assets are perfectly negatively correlated, then you can write the standard deviation as a perfect square so:

$$x_L\sigma_L - x_M\sigma_M$$

The lowest possible standard deviation you can get is 0, so find the weights so that a portfolio of the two assets has a zero standard deviation:

$$x_L\sigma_L - x_M\sigma_M = 0.$$

A little bit of algebra yields the result that

$$x_L = \frac{0.8}{1.4}$$

and

$$x_M = \frac{0.6}{1.4}.$$

You can easily check that these weights give a zero standard deviation to the portfolio. Negative correlation will turn out to be VERY important. If you want to hedge a portfolio (make sure that you are not exposed to risk) then to eliminate risk you construct a portfolio that has a zero standard deviation —essentially by finding a negatively correlated asset.