# Application: SWAPS

Swaps are private agreements between 2 companies to exchange cash flows in the future. They are either negotiated bilaterally or arranged through a swap broker. Companies enter into swaps because they face different interest rates in the market and there are gains from trade.

The simplest form of swap is *plain vanilla*. Suppose that there are two companies, A and B. In a plain vanilla swap, A pays B cash flows equal to interest at a floating rate on some *notional* principal for a specified period of time. In exchange B pays A cash flows equal to interest at a fixed rate on the notional principal. This notional principal does not change hands, it is just the sum on which the interest payments are calculated.

Why would a company receive a fixed rate? (essentially to convert to floating) — a company with a heavy percentage of long term fixed rate debt that is worried about a slowdown in the economy and falling interest rates. A company who pays fixed might have floating rate debt and be worried about increasing interest rates. Similarly, a company with fixed rate assets and floating rate liabilities might agree to pay fixed.

The floating rate payments are often based on LIBOR (London Inter–Bank Offer rate), federal funds, commercial paper rates, T–bill rates, (and many others).

How does one value these cash flows? Suppose that you are in a swap agreement to get fixed rate $c$ percent and to pay the floating rate, which is set on the previous period's LIBOR so you pay $l_0$ in period 1, $l_1$ in period 1 and in general, $l_t$ in period $t$. What are the cash flows?

At t=1 you get $c-l_0$, at t=2 you get $c-l_1$ and in general at time $t$ you get $c-l_{t-1}$. In order to find the present value of these cash flows, define $D_1$ as the price today of \$1 in period 1 and $D_2$ as the price today of \$1 in period 2 and in general, $D_t$ as the price today of \$1 in period $t$. The present value of the cash flows that come from the swap are then:

$$PVCF = (c - l_0)D_1 + [(1 + c)D_2 - D_1] + [(1 + c)D_3 - D_2] \dots [(1 + c)D_T - D_{T-1}]$$

or, more concisely:

$$PVCF = (c - l_0)D_1 + \sum_{\tau=2}^{T}(1 + c)D_\tau - D_{\tau-1}$$

Notice that if you divide the sum through by $D_\tau$, you get:

$$PVCF = (c - l_0)D_1 + \sum_{\tau=2}^{T}[(1 + c) - \frac{D_{\tau-1}}{D_\tau}]D_\tau$$

This is known as pricing off the forward curve because the forward rate between $\tau$ and $\tau + 1$ is

$$[1 + f_{\tau,\tau+1}] = \frac{1 + r_{\tau+1})^{\tau+1}}{(1 + r_\tau)^\tau} = \frac{D_\tau}{D_{\tau+1}}$$

How would you figure out a mutually beneficial swap between two counterparties? First, determine the net payments. In the previous example, the party receiving fixed pays $c - l_t$. This must be less than the amount that he would pay if he was not in the swap. One can obtain a similar condition for the party paying fixed. These two conditions jointly determine a mutually beneficial swap. Notice, that there is typically more than one swap that will make both parties better off.