Application: SWAPS

	Fixed	Floating
A	10%	l+0.3%
B	11.2%	l+1%

- Cost to A:

- Cost to B:

t=1 t=2 t=3

$$CF_0 = [c - l_0]D_1 + [(1 + c)D_2 - D_1] + [(1 + c)D_3 - D_2]$$

Fixed Income Portoflio Characteristics

- The Problem:
- Yield curve is:
  - 6 months 4%
  - 1 year 5%
  - 18 month 6%
- Bond A: 20% , 18 months
- Bond B: 20%, 6 months

Approximating Changes in Bond Price

- Price of a Bond is a function of yield.

$$\Delta P \approx \frac{\partial P(y)}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 P(y)}{(\partial y)^2}(\Delta y)^2$$

Duration

- What is $\frac{\partial P(y)}{\partial y}$?
- First the *Macaulay Duration*

$$P = \frac{c}{1+y/2} + \frac{c}{(1+y/2)^2} + \ldots \frac{c+F}{(1+y/2)^T}$$

$$\alpha_t = \frac{1}{P}\frac{c}{(1+y/2)^t}$$

$$D = \alpha_1 * (1) + \alpha_2 * (2) + \ldots \alpha_T * T$$

- The Modified Duration:

$$D* = \frac{-1}{P(y)}\frac{\partial P(y)}{\partial y}$$
$$D* = \frac{D}{1+y/2}$$

- 1 year 8% coupon priced at par

Convexity

- What is $\frac{\partial^2 P(y)}{(\partial y)^2}$?

$$C(y) = \frac{1}{P} * \frac{\partial^2 P}{(\partial y)^2}$$

- Each bond is a portfolio of zeros:

$$P(y) = \frac{100}{(1+y/2)^{2t}}$$

$$C(y) = \frac{t(t+1/2)}{(1+y/2)^2}$$

- 1 year 8% coupon priced at par.

Immunization

- How do you get $\Delta P(y) = 0$?

$$\Delta P(y) \approx -D * P(y)\Delta y + \frac{P(y) * C(y)(\Delta y)^2}{2}$$

- **Immunization Strategies**
- Match Duration and Convexity of Assets $\geq$ convexity of Liabilities
- Maximise Convexity
- Minimise Transaction Costs

Factor Yield Models

$$y_{it} = A_i + \beta_{i1} F_{it} + \ldots \beta_{ik} F_{kt} + \epsilon_{it}$$

- $K = 1$
- $y_i = A_i + \beta_i F_t$
- $\Delta y_i =$

$$\Delta P(y) \approx -P(y) D * (y) \beta_i (\Delta F) + \frac{C(y) P(y) \beta_i^2 (\Delta F)^2}{2}$$

1 Combining 2 Bond changes

$$\begin{aligned}\Delta P_1 + \Delta P_2 \approx & -[P_1 D_1 * \beta_1 + P_2 D_2 * \beta_2]\Delta F \\ + & \frac{1}{2}[C_1 P_1 \beta_1^2 + C_2 P_2 \beta_2^2](\Delta F)^2\end{aligned}$$

- **Example**
- Immunize against ΔF
  - $P_1 = \$10 \times 10^6$
  - $D_1* = 4$
  - $D_2* = 8$
  - $\beta_1 = 1.1$
  - $\beta_2 = 1.2$
- Suppose $C_1(P_1)\beta_1^2 + C_2 P_2 \beta_2^2 < 0$
- $$\frac{\Delta P_1 + \Delta P_2}{P_1 + P_2} \approx -[0]\Delta F + \frac{C_1 P_1 \beta_1^2 + C_2 P_2 \beta_2^2}{P_1 + P_2}\Delta F^2$$
- Immunize against ΔF^2