Another Interpretation of the CAPM

$$\max_{\alpha_1,\alpha_2} V(ER_p, Var(R_p))$$

- F.O.C. $\Longrightarrow$ CAPM
- CAPM $\Longrightarrow$ Market Portfolio is Mean Variance Efficient

$$\sigma = \sqrt{x_i^2\sigma_i^2 + x_p^2\sigma_p^2 + 2x_i x_p cov(r_i, r_p)}$$

$$\frac{d\sigma}{dx_i} = \frac{1}{2}[\sigma^2]^{\frac{-1}{2}}[2x_i\sigma_i^2 + 2x_p cov(r_i, r_p)]$$

$$\frac{d\sigma}{dx_i} \mid_{x_i=0} = \frac{x_i\sigma_i^2 + x_p cov(r_i, r_p)}{\sigma_p}$$

Factor Models

$$r_{it} = A_1 + \beta_{i1} F_{1t} + \beta_{i2} F_{2t} + u_{it}$$

$$\begin{aligned} E(u_{it}) &= \\ Cov(u_{it}, F_{1t}) &= \\ Cov(u_{it}, F_{2t}) &= \end{aligned}$$

Implications of Factor models for Variance/Covariance

- 2 Factors: F_{1t}, F_{2t}
- 2 Assets: r_t^a, r_t^b

$$
\begin{aligned}
r_t^a &= \\
r_t^b &= \\
\\
var(r_t^a) &= \\
&= E[\beta_1^a(F_{1t} - EF_1) + \beta_2^a(F_{2t} - EF_2) + (u_t^a - Eu^a)]^2 \\
&= \\
&= \\
&= (\beta_1^a)^2 var(F_1) + (\beta_2^a)^2 var(F_2) + var(u_a) + 2\beta_1^a\beta_2^a cov(F_1, F_2) \\
cov(r^a, r^b) &= \\
&= \\
&= \\
&= \beta_1^a\beta_1^b var(F_1) + 2[\beta_1^a\beta_2^b] cov(F_1, F_2) + \beta_2^a\beta_2^b var(F_2)
\end{aligned}
$$

Example: Factor Model

- 2 factors: inflation and the market
- 2 stocks United and Brittanic

	Market	Inflation	Residual
U	0.5	1.2	0.03
B	1.5	0.4	0.05

- Variance of market=6
- Variance inflation= 3
- Covariance = 1

Risk Management and Portable Alphas

- 2 factors
- $r_{1t} = 1.3F_{1t} + 2.4F_{2t}$
- $r_{2t} = 0.8F_{1t} + 5.3F_{2t}$
- Want 2.3 on F_1 and 4.0 on F_2

APT

- $Er_i = \lambda_0 + \lambda_1 \beta_{i1} + \lambda_2 \beta_{i2} + \ldots$

- $Er_a - r_f = 10\%$
- $\beta_A = 0.8$
- $Er_b - r_f = 14\%$
- $\beta_b = 1.2$