Portfolio Mechanics

CAPITAL ALLOCATION

- 2 assets, one risky, one risk free
- r_f return on the risk–free asset
- r_r return on the risky asset
- r_p return on a portfolio made of the risky asset and risk free
- σ_r^2 standard deviation of the risky asset
- σ_p^2 standard deviation of the portfolio
- y weight of the risky asset in the portfolio

$$
\begin{aligned}
y &= \\
r_p &= \\
\sigma_r^2 &= \\
\sigma_p^2 &=
\end{aligned}
$$

ASSET ALLOCATION

- 2 assets A and B, both risky
- returns: r_a, r_b
- standard deviations: σ_a, σ_b
- weight in asset A: x_A
- weight in asset B: $x_B =$

$$
\begin{aligned}
r_p &= \\
Er_p &= \\
\sigma_p^2 &=
\end{aligned}
$$

CORRELATION COEFFICIENT

$$\rho_{A,B} = \text{ Correlation coefficient between A, B}$$
$$\rho_{A,B} =$$

EXAMPLE

	Er (%)	σ (%)
A	10	16
B	6	12

ρ_{AB}	σ_{AB}	$x_A^2\sigma_A^2 + x_B^2\sigma_B^2$	σ_p^2
1			
0			
-1			

BENEFITS OF DIVERSIFICATION

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1, i \neq j}^{n} \sum_{j=1}^{n} w_i w_j cov(r_i, r_j)$$

If $w_i = \frac{1}{n}$

$$\sigma_p^2 = \sum_{i=1}^{n} \frac{1}{n^2} \sigma_i^2 + \sum_{i=1, i \neq j}^{n} \sum_{j=1}^{n} \frac{1}{n^2} cov(r_i, r_j)$$

If

$$\overline{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2$$

$$\overline{c}ov = \frac{1}{n(n-1)} \sum_{i=1, i \neq j}^{n} \sum_{j=1}^{n} cov(r_i, r_j)$$

Then:

$$\sigma_p^2 = \frac{1}{n} \overline{\sigma}^2 + \frac{n-1}{n} \overline{c}ov$$

$$\min_{\{x_i\}_{i=1}^n} \sigma_p^2$$
$$\sum_{i=1}^n x_i = 1$$

$$\min_{\{x_i\}_{i=1}^n} \sigma_p^2$$
$$\sum_{i=1}^n x_i = 1$$
$$Er_p = \bar{r}$$

$$\max_{\{x_i\}_{i=1}^{n}} \frac{Er_p - r_f}{\sigma_p}$$

$$s.t. \sum_{i=1}^{n} x_i = 1$$

$$V = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 & \sigma_{14}^2 \\ \sigma_{21}^2 & \sigma^2 & \sigma^2 & \sigma_{24}^2 \\ \sigma^2 & \sigma^2 & \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 & \sigma^2 & \sigma_{44}^2 \end{bmatrix}$$

$$e = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} Er_1 \\ Er_2 \\ Er_3 \\ Er_4 \end{bmatrix}$$