33-765 — Statistical Physics

Department of Physics, Carnegie Mellon University, Spring Term 2020, Deserno

Some study tips for the midterm

Since we will have our midterm on Wednesday, March 18th, I'm not going to put out a homework for Monday March 16th. Instead, you should use the time to study the material and practice some of the calculations we have encountered. It would be an excellent idea to revisit the homework we have done so far. On this sheet I'll list some (hopefully obvious) subjects and skills that are worth reviewing. I can't guarantee that this is exhaustive, but it almost surely very nearly is.

Subjects

1. Probability theory
   a) The basics: probabilities and probability densities; normalization; expectation value and variance; moments in general; Bayes theorem;
   b) Minimally more advanced: pairwise and mutual independence; covariance and correlation; binomial distribution;
   c) The sweet stuff: transformation theorem; characteristic functions and their use; central limit theorem; Jensen's inequality; knowing the Gamma function;
2. Differentials
   a) What they are; difference between exact and inexact ones; integrability conditions; integrating factors;
3. Legendre transforms
   a) Definition and properties; the differential of a Legendre transform;
4. Thermodynamics
   a) The entropy of an ideal gas, and how it relates to maximizing the probability of a macrostate;
   b) Understanding the first law, $\mathrm{d}U = \text{đ}Q + \text{đ}W$, and the connection between heat and entropy, $\text{đ}Q = T\,\mathrm{d}S$.
   c) Equilibrium conditions;
   d) Thermodynamic potentials and their derivatives; extremum principles and stability conditions;
   e) $\mathrm{d}U = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N$, and appreciating the enormous number of things that follow from this.
   f) Thermodynamic processes in general and heat engines in particular; the role played by running (a) reversible and (b) cyclic; heat engines running backwards (*i. e.*, heat pumps and refrigerators);
   g) Definitions of standard thermodynamic "response functions" (isobaric specific heat, isothermal compressibility, *etc.*); the powerful weapons known as "Maxwell relations" and "Jacobians";

Skills

1. Perform standard calculations with probabilities and probability densities (*e. g.*, finding normalization constants, calculating moments); use the transformation theorem; use Jensen's inequality;
2. Test differentials for exactness; find integrating factors for those that are not exact;
3. Find Legendre transforms for convex or concave functions;
4. Do calculations for thermodynamic processes (*e. g.*, adiabatic expansion of a gas, or running a heat engine between finite or infinite reservoirs);
5. Find equations of states from thermodynamic potentials; derive thermodynamic identities;