33-765 — Statistical Physics

Department of Physics, Carnegie Mellon University, Spring Term 2020, Deserno

Problem sheet #7

24. Examples of simple thermodynamic identities *(4 points)*

Rewrite the following thermodynamic derivatives until only the "standard" derivatives α, κ_T, c_P, and c_V (and possibly factors, such as T or P, or numbers, such as 2 or π) occur:

1. $\left(\frac{\partial T}{\partial P}\right)_{S,N} = \ ?$

2. $\left(\frac{\partial F}{\partial S}\right)_{T,N} = \ ?$

25. Maxwell relations and Jacobians in tedious disguise *(4 points)*

1. Prove the first $T\,\mathrm{d}S$ equation: $T\,\mathrm{d}S = Nc_V\,\mathrm{d}T + \frac{\alpha T}{\kappa_T}\,\mathrm{d}V$.
2. Prove the second $T\,\mathrm{d}S$ equation: $T\,\mathrm{d}S = Nc_P\,\mathrm{d}T - \alpha TV\,\mathrm{d}P$.

Hint: If you write the entropy $S(\cdot,\cdot,N)$ in the variables indicated by the $T\,dS$ equation(s), what would be its differential?

26. "A pearl of theoretical physics"... *(4 points)*

...that's what H.A. Lorentz called Boltzmann's following brilliant insight: Consider some mystery system, of which we only know that it is extensive, the chemical potential vanishes, and it satisfies the equation of state $PV = \frac{1}{3}U$.

1. Explain why in such a situation we must have $U(T,V) = V\,u(T)$.
2. Express the entropy as a function of temperature and volume. (This will involve $u(T)$, which you need not eliminate.)

   Hint: The Euler equation will prove useful, but you should explain, why you're allowed to use it!
3. Find a differential equation for $u(T)$ by pondering over the temperature dependence of the pressure. *(Hint: Maxwell!)*
4. Solve the differential equation and thus predict how the energy density and the entropy depend on the temperature.

27. Relation between the isothermal and the adiabatic compressibilities *(4 points)*

In analogy to the well-known relation between the isobaric and the isochoric heat capacities, c_P and c_V, derive the following very similar formula for the isothermal and adiabatic compressibilities, κ_T and κ_S:

$$\kappa_T - \kappa_S = \frac{TV\alpha^2}{Nc_P}\,.$$

28. Adiabatic compression *(4 points)*

1. Show that, quite generally, $\frac{\kappa_T}{\kappa_S} = \frac{c_P}{c_V} =: \gamma$, where γ is called the *adiabatic index*.
2. Calculate κ_T, c_V, c_P, and γ for the monoatomic ideal gas.
3. Show that for adiabatic (constant entropy) compression of an ideal gas we get $P \propto V^{-\gamma}$.