33-759 Introduction to Mathematical Physics
Fall Semester, 2005
Assignment No. 2.
Due Friday, Sept. 9

READING: Kreyszig Secs. 12.6, 12.7; 14.1 through 14.4

READING AHEAD: Kreyszig Secs. 12.8, 12.10; 13.1 through 13.3; 15.1 through 15.4

EXERCISES:

1. Turn in at most one page, and not less than half a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course.

2. You should be able to do exercises 1 to 16 at the end of Sec. 14.1. Check that this is so by choosing a couple of them at random. Turn in solutions to 6 and 18.

3. OPTIONAL. Prove formula (6**) on p. 745 of Kreyszig for the radius of convergence of a power series. It is not very difficult if you know the definition of lim sup.

4. You should be able to do exercises 1 to 16 at the end of Sec. 14.2. Check that this is so by choosing a couple of them at random. Turn in solutions to 8 and 18. Beware using a simplistic application of the ratio test. Summing the series numerically for various choices of z is one way to check your answer.

5. Work out the first three nonzero terms in the power series of

$$f(z) = \sinh(z)/\sin(z)$$

about $z = 0$. Do this "by hand" to show that you know how to do it. You may check your answer with Maple/Mathematica if you want to.

6. Kreyszig 14.3 No. 4. Once you have found the series with simpler coefficients, sum it to obtain a (simple) function in closed form, and then apply calculus to work out the closed form of the function represented by the original series. Find the first few terms in the power series expansion of this last function in order to check your work.

7. a) Find all the zeros of $\sin z$ in the complex plane. [Hint: Find a quadratic equation for $w = e^{iz}$.]

b) Find all the zeros and all the poles of $\tan z$ in the complex plane, and the order of each zero and pole. [Hint: If you know something about $\sin z$, what do you know about $\cos z$?]

c) Same thing for $\tanh z$. [Hint: Easy.]

8. Kreyszig 12.7 No. 4 and No. 8. As well as expressing your answers in the form of trig and hyperbolic functions of real quantities (e.g., $\cos 1$), obtain numerical values of u and v to a few significant figures (using a pocket calculator).

9. Kreyszig 12.7 No. 14.

10. Suppose you know the coefficients $\{a_n\}$ in the power series $\sum_n a_n(z-z_0)^n$ for $f(z)$ about the center $z = z_0$, and you wish to use them to generate the coefficients $\{b_n\}$ in the series $\sum_n b_n(z-z_1)^n$ about a different center z_1. Here is one strategy, which for simplicity is illustrated for the case $z_1 = 0$. Write

$$
\begin{aligned}
&a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \cdots \\
&= [a_0 - a_1 z_0 + a_2 z_0^2 + \cdots] + [a_1 - 2a_2 z_0 + \cdots]z + [a_2 + \cdots]z^2 + \cdots \\
&= b_0 + b_1 z + b_2 z^2.
\end{aligned}
$$

That is, expand the terms in the original series and collect the coefficients of 1, z, z^2, etc. When and why will this method work, and when will it fail? [Hint: What is the connection between the b_n and the derivatives of $f(z)$?]

11. DO NOT TURN IN. (From fall 2004 final exam.)

A complex-valued function $f(z)$ of a complex variable $z = x + iy$ can always be written in the form $f(z) = u(x, y) + iv(x, y)$, where u and v are real-valued functions.

a) What are the Cauchy-Riemann relations for u and v, and if they are satisfied in a suitable region in the complex plane, what does this tell one about the function $f(z)$? A statement will suffice; you do not have to derive the Cauchy-Riemann equations. What counts as a "suitable region" in the complex plane?

b) Suppose that $v(x, y) = 2 + e^y \cos x$. Use the Cauchy-Riemann relations to find $u(x, y)$, and thereby the function $f(z)$.