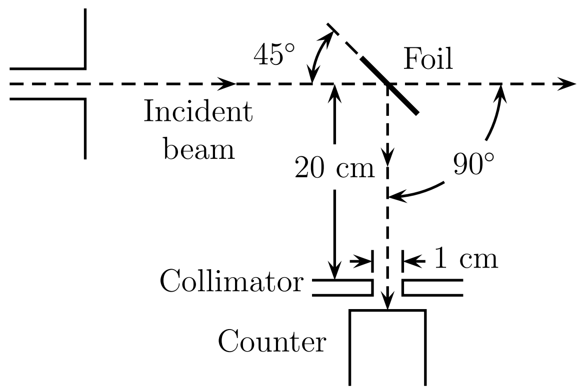
33-331 Physical Mechanics I. Fall Semester, 2009
Recitation exercise for Nov. 24

A counter is placed behind a circular collimator of diameter 1.00 cm located 20.0 cm from a scattering target which is an aluminum foil mounted at 45° to the incident beam direction, see figure. The foil has a thickness such that 1 cm^2 weights 5.00×10^{-3}g.



Suppose that the beam consists of 12 MeV protons with a current of 1.00×10^{-7} amps, and that the counter registers 250 counts/s. Calculate the differential cross section $d\sigma/d\Omega$ (in the laboratory) for scattering from the aluminum nucleus, in cm^2/steradian, at an angle of 90° (as shown in the figure).

The foil has a mass of 5.00 mg/cm^2 and thus contains

$$5.00 \times 10^{-3} \times 6.022 \times 10^{23}/26.98 = 1.116 \times 10^{20} \text{ atoms/cm}^2. \tag{1}$$

However, as it is inclined at 45° to the incident beam direction, its thickness is effectively multiplied by $1/\cos(45°) = \sqrt{2}$, thus giving

$$\rho' = 1.578 \times 10^{20} \text{ atoms/cm}^2. \tag{2}$$

The collimator subtends a solid angle of

$$\Delta\Omega = \pi(\tfrac{1}{2})^2/20^2 = \pi/1600 = 1.963 \times 10^{-3} \text{ str}, \tag{3}$$

If $\sigma'(\Omega) = d\sigma/d\Omega$ is the differential cross section, the probability that a particle in the incident beam will be scattered into the counter is

$$P = \rho'\sigma'\Delta\Omega = I_s/I_0, \tag{4}$$

where the beam current of 10^{-7}amps corresponds to

$$I_0 = 10^{-7}/1.602 \times 10^{-19} = 6.24 \times 10^{11} \text{ particles s}^{-1}, \tag{5}$$

and the current in the detector is $I_s = 250$ particles s^{-1}. Hence the differential cross section at 90° is given by

$$\begin{aligned}\sigma' =& d\sigma/d\Omega = \frac{250}{(6.24 \times 10^{11}) \cdot (1.578 \times 10^{20}) \cdot (1.963 \times 10^{-3})} \\ =& 1.293 \times 10^{-27} \text{ cm}^2 = 1.293 \times 10^{-3} \text{ barns}.\end{aligned} \tag{6}$$