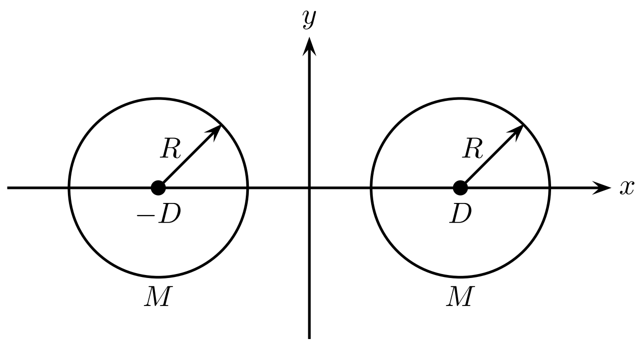
33-331 Physical Mechanics I. Fall Semester, 2008
Hour Examination No. 2, 15 October 2008

NAME______________________________

INSTRUCTIONS. This examination consists of two problems. The first, with 3 parts, is worth 60 points, and the second, with 2 parts, is worth 40 points, i.e., each part is worth 20 points. The problems are on separate sheets, to allow independent grading. Make sure your name is on each sheet. *It is important to give a brief indication of your reasoning.* Answer in a way which shows that you *understand* the subject.

1. (3 parts, total of 60 points) Two spherical objects, each of mass M, radius R, and uniform density ρ are located at $(D, 0, 0)$ and $(-D, 0, 0)$ in Cartesian coordinates, i.e., at $\pm D$ on the x axis, where D is greater than R.



a) Find the gravitational potential Φ in the $x = 0$ plane as a function of y and z. Indicate the basic principle or principles that allow you to do this without carrying out any integrals.

b) How do symmetry and other physical principles allow you to specify which *nonzero* terms arise in a power series expansion of the gravitational potential $\Phi(x, y, z)$ up to quadratic order (x^2, xy, etc.) when x, y, and z are small? You do not have to calculate values for the nonzero terms.

c) Discuss the equilibrium (is it stable or unstable?) of a particle of mass m placed at the origin and (i) constrained to move along the x axis; (ii) constrained to move along the y axis. Find the angular frequency ω of small oscillations along whichever direction is stable, in terms of M and other quantities. You may use the fact that $\omega = \sqrt{k/m}$ for an oscillator with a mass m and a potential energy $\frac{1}{2}k\Delta^2$ (force $-k\Delta$), where Δ is the displacement from some origin.

2. (2 parts, total of 40 points) A particle of mass m moves in two dimensions subject to a potential
$U = Ax^2 + By^2$, with A and B positive constants.

a) Use the Lagrangian approach to find the equations of motion.

b) Next suppose that this particle is constrained to move on the circle $x^2 + y^2 = R^2$. Express the Lagrangian for the constrained problem in terms of a *single* appropriately chosen generalized coordinate, and find the equation of motion for this coordinate. Give a brief explanation of what you are doing.