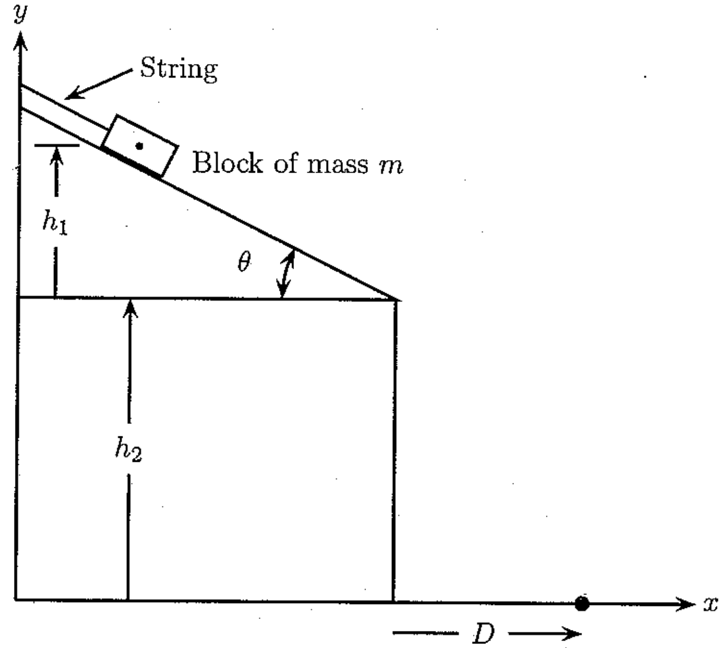
XIS / 1.

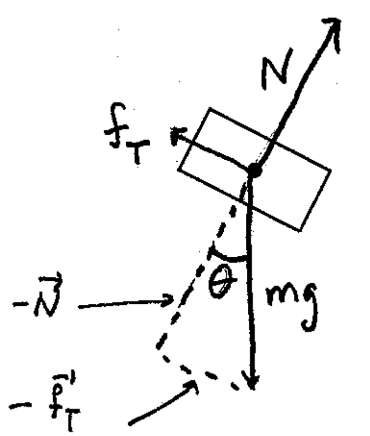
33-331 Physical Mechanics I. Fall Semester, 2008
Hour Examination No. 1, 17 September 2008

NAME________________________

INSTRUCTIONS. This examination consists of one problem with five parts, with each part worth the *same* number of points. DO NOT attempt (e) unless you are confident you have completed the other four parts correctly, as it is more difficult. As is always the case on examinations, *it is important to give a brief indication of your reasoning.* Answer in a way which shows that you *understand* the subject. Your answer should be written on the examination. If you need more room, use the back side of the page, but indicate on the front that there is material on the back.



a) A block of mass m is at rest on a frictionless plane making an angle θ with the horizontal, as shown in the figure, held in place by a string with tension f_T. What are the forces acting on the block and what is the total force? Your answer should include a diagram involving the block, which is shown below. What is the tension in the string f_T in terms of other things?



The forces acting on the block are as shown: $\vec{N}$ = normal force of plane on block, $\vec{f}_T$ = force due to string, $m\vec{g}$ = gravity

$\vec{N} + \vec{f}_T + m\vec{g}$ = <u>total force</u> $= 0$, since block is at rest, and magnitudes are:

$N = mg\cos\theta, \quad f_T = mg\sin\theta;$

since, see figure: $m\vec{g} = -\vec{N} - \vec{f}_T$ as vector sum

XIS/2.

b) The string is cut and the block slides down the frictionless plane, falls over the edge of the table and hits the floor a distance D from the edge of the table (see the figure). Find expressions for its speed v_1 as it leaves the edge of the table and v_2 just before it strikes the floor in terms of g and quantities in the figure, using a conservation law or conservation laws. You should state *what* is being conserved and how you are using the law; do not just write down a string of formulas. You may assume that the block is very small; ignore any effects due to its rotation while falling.

The conservation of energy tells us that (y = height above table)

$$E = T + U = \frac{1}{2} m v^2 + mgy = \text{kinetic} + \text{potential} = \text{constant}$$

Thus $E = 0 + mg(h_1 + h_2) = \frac{1}{2} m v_1^2 + mgh_2 = \frac{1}{2} m v_2^2 + 0$

at the times of interest. Solving:

$v_1^2 = 2gh_1 \qquad v_1 = \sqrt{2gh_1}$

$v_2^2 = 2g(h_2 + h_1) \qquad v_2 = \sqrt{2g(h_1 + h_2)}$

c) Is momentum conserved while the block is sliding down the plane or falling towards the floor? Remember that momentum is a vector, so you may wish to discuss components. You must give reasons for your answers.

Since $d\vec{p}/dt = \vec{F}$, momentum $\vec{p}$ is conserved when force $\vec{F} = 0$, and if a particular component of force is 0, the corresponding component of $\vec{p}$ is conserved.

While the block slides down the plane the component of momentum perpendicular to the plane is conserved, and in fact it is 0, while the one parallel to the plane is not conserved.

While falling towards the floor, p_x is conserved — no force in the x direction, while p_y is not, due to gravitational force.

X15 / 3 •

d) Suppose the time the block spends in the air after leaving the edge of the table and before it hits the floor is τ (Greek tau). (i) Find D in the figure in terms of τ and v_1. (ii) Find an expression for τ in terms of v_1. You may leave your answer in the form of an equation, which you don't have to solve as long as it and the reasoning leading up to it are clear. .

The horizontal v_x and vertical v_y components of velocity for the block are $v_x = v_1 \cos\theta$, $v_y = -v_1 \sin\theta$ when it falls over the edge, [call this $t = 0$.] v_x is constant, so x distance traveled in time τ is $\underline{D = v_x \tau = v_1 (\cos\theta)\tau}$. However,

$v_y = -v_1 \sin\theta - gt$ due to gravity : $dv_y/dt = -g$

$y = h_2 - v_1 \sin\theta\, t - \frac{1}{2} g t^2$: $dy/dt = v_y$

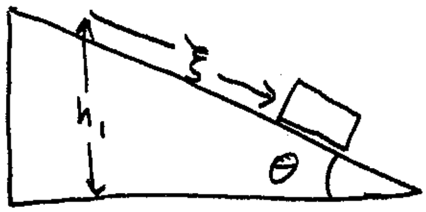
When $t = \tau$, $y = 0$, so $\frac{1}{2} g \tau^2 + v_1 \sin\theta\, \tau - h_2 = 0$

Solving: $\tau = \frac{1}{g}\left[-v_1 \sin\theta + \sqrt{v_1^2 \sin^2\theta + 2 g h_2}\right]$

X15/4.

e) Attempt this part *only* if you are confident you have answered the previous parts correctly.

Alas, the lab technician forgot to apply magic grease to make the plane frictionless, so the block slides down it with a coefficient of kinetic friction $\mu_k > 0$. Redo your calculations of v_1 and v_2 in part (b), and then discuss how this new circumstance affects your answers to (c) and (d).



Let ξ be distance measured down the plane from the starting point. Net force in this direction in the presence of friction is

$$mg\sin\theta - \mu_k N = mg(1-\epsilon)\sin\theta \, , \quad \boxed{\epsilon = \mu_k / \tan\theta}$$

Integrate $m\ddot{\xi} = mg(1-\epsilon)\sin\theta$ to get

$\dot{\xi} = g(1-\epsilon)\sin\theta\, t$ (block starts at rest at $t=0$)

$\xi = \frac{1}{2} g(1-\epsilon)\sin\theta\, t^2 = h_1/\sin\theta$ when block reaches edge.

Thus $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h_1}{g(1-\epsilon)}}$

$v_1 = \dot{\xi} = \sqrt{2gh_1(1-\epsilon)}$ when block reaches edge.

Use conservation of energy to get $v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{2g[h_2 + (1-\epsilon)h_1]}$

Alternative approach to obtaining v_1

The frictional force $f_r = \mu_k N = \mu_k mg\cos\theta$ directed up the plane is constant while the block slides through a distance $\Delta = h_1/\sin\theta$. Thus the work W done by this force on the block is

$$W = -f_r\Delta = -mgh_1\mu_k/\tan\theta = -mgh_1\epsilon$$

Add this to the initial energy = potential energy mgh_1 relative to the top of the table to obtain the kinetic energy of the block as it falls off:

$$\frac{1}{2}mv_1^2 = -mgh_1\epsilon + mgh_1 = mgh_1(1-\epsilon)$$

So $v_1 = \sqrt{2gh_1(1-\epsilon)}$

Changes to (c). Answers same as before. While sliding the component of momentum perpendicular to the plane is conserved $(=0)$; while in the air p_x is conserved.

Changes to (d). The answers expressed in terms of v_1 are exactly the same; one simply uses the v_1 value calculated above instead of that in (b).