33-331 Physical Mechanics I. Fall Semester, 2009
Assignment No. 12
Due Monday, November 23

READING

Thornton and Marion Ch. 9, Secs. 9.1 through 9.5, 9.9, 9.11

READING AHEAD:

Thornton and Marion Ch. 9, Secs. 9.6, 9.7, 9.8, 9.10. We may or may not get to the material in Sec. 9.8

Handout: Scattering Kinematics

EXERCISES

1. Turn in at most one page, and not less than a third of a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc.

2. Show that the total external torque $\mathbf{N}$ (superscript e omitted) on a system of particles calculated about an arbitrary origin is equal to the torque $\mathbf{N}'$ evaluated about the center of mass plus $\mathbf{R}\times\mathbf{F}$, the torque which would arise if the total external force $\mathbf{F}$ were applied at the center of mass. Use this along with Thornton and Marion (9.23), written as

$$\mathbf{L} = \mathbf{R}\times\mathbf{P} + \mathbf{L}',$$

$\mathbf{L}'$ the angular momentum about the center of mass, to show that if $\dot{\mathbf{L}}' = \mathbf{N}'$ holds with origin at the center of mass, then $\dot{\mathbf{L}} = \mathbf{N}$ holds about an arbitrary origin.

3. A solid cylinder symmetrical about its axis, of total mass M and radius ρ, has an angular momentum about this axis of $cM\rho^2\dot{\theta}$ and a kinetic energy of $\frac{1}{2}cM\rho^2\dot{\theta}^2$ when rotating with an angular speed $\dot{\theta}$. Here c is a dimensionless constant whose value depends on the way mass is distributed in the cylinder. (The moment of inertia is $I = cM\rho^2$.) Now suppose the cylinder is rolling without slipping down an inclined plane making an angle γ with the horizontal, so that if it rotates by an angle $\Delta\theta$ its center of mass moves parallel to the plane by a distance $\Delta\xi = \rho\Delta\theta$.

a) Find the equation of motion $\ddot{\theta} = \cdots$ by setting $\dot{L}$ equal to N, where both angular momentum and torque are evaluated about the line of contact between the cylinder and the plane. (By L and N we mean the compoents of $\mathbf{L}$ and $\mathbf{N}$ parallel to the cylinder axis, as the others are zero. Assume a sign convention in which L and $\dot{\theta}$ increase as the cylinder rolls down the plane.)

b) The result in (a) can be used to find the tangential force which the plane exerts on the cylinder (the force that maintains the rolling constraint, so that the cylinder is not slipping) by considering either the equation relating rate of change of the total momentum to the force acting on the cylinder, or by equating torque N' about the center of mass to the rate of change of the corresponding angular momentum L'. Use one of these methods and check that it is consistent with the other.

c) Show that energy is conserved by calculating the rate at which the potential energy of the cylinder is decreasing and the rate at which its kinetic energy is increasing.

4. Thornton and Marion 9-62, modified as follows. Obtain a formula allowing fuel amounting to a fraction f of the total mass to be used, and then put in numbers for $f = 0.20$. Note that the rate $-dm/dt$ at which fuel is burned needs to vary with time.

5. According to a certain authoritative source the Commonwealth of Pennsylvania has an area of 119,283 km^2. Assuming this is roughly correct, what solid angle does it subtend (i) at the center of the earth; (ii) as observed by an astronaut on the moon when the moon is 45° above the horizon in Harrisburg?

6. a) Find the differential cross section $\sigma(\theta)$ for a hard sphere of radius R with a hole of radius a drilled through the center in the direction in which particles are arriving from the source. (I.e., some particles will pass through the hole without scattering.) Sketch it as a function of θ for the case in which $a = R/2$. Evaluate the total cross section as a function of a and R, and check that it has the value you expect.

b) Suppose the hole has been drilled in the same direction, but slightly off center. Explain why the differential cross section σ will no longer be independent of ϕ for some range of θ values. Suppose one calculates the averaged cross section

$$\sigma_A(\theta) = (2\pi)^{-1} \int_0^{2\pi} \sigma(\theta, \phi)\, d\phi.$$

Explain in a general way (do not try and do a calculation) how this differs from what you sketched in (a). What happens to the total cross section?