33-331 Physical Mechanics I. Fall Semester, 2009
Assignment No. 7
Due Friday, October 23

READING

Handout: Supplement on Lagrangian and Hamiltonian Mechanics

Thornton and Marion Secs. 7.5 to 7.12: Lagrangian with multipliers, conservation laws, Hamiltonian dynamics. We will omit Sec. 7.13, virial theorem, which properly belongs to statistical mechanics.

READING AHEAD:

Thornton and Marion Ch. 8, Central-force motion

EXERCISES

1. Turn in at most one page, and not less than a third of a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc.

2. Thornton and Marion 6-4. Solve this problem by two methods. (i) Use cylindrical coordinates (r, θ, z), and suppose that on the cylinder of radius R the unknown function is of the form $z(\theta)$. Write down the integral for the length of the curve and apply Euler. (ii) Use the length of a curve in three dimensions, but now treat z as the independent variable and handle the constraint

$$R^2 - x^2 - y^2 = 0$$

using a Lagrange multiplier $\lambda(z)$. Euler will give you differential equations, and the best method of solution is to make a reasonable guess and show that it works. What is $\lambda(z)$?

3. Consider the following Lagrangians for a single particle in polar coordinates r, θ, z. Identify all the conserved quantities—momentum, angular momentum, energy—that you can, in each case stating what the quantity is (e.g., angular momentum along some axis) and why it is obvious (or maybe not so obvious) that the quantity is conserved. Here α and τ are positive constants.

$$\text{(i) } L = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - \alpha r^2 - mgz$$
$$\text{(ii) } L = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - \alpha[1 + \sin(t/\tau)]r^2$$
$$\text{(iii) } L = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - \alpha(r^2 \sin^2\theta + z^2)$$

4. A bead of mass m slides without friction on a smooth stiff wire in the x, y plane passing through the origin and rotating with a fixed angular speed $\alpha = d\theta/dt$. Let r be the distance from the origin to the bead.

a) Write down the Lagrangian using the constraint of sliding on the wire to eliminate θ and $\dot{\theta}$ and make it a function r and $\dot{r}$. Find the equation of motion for r. What is the general solution? What is the specific solution if at $t = 0$ $r = r_0 > 0$ and $\dot{r} = 0$?

b) Next, handle the constraint $g(r, \theta, t) = \theta - \alpha t = 0$ using a Lagrange multiplier, and use this to find the generalized forces of constraint. Are they what you would have expected?

5. Thornton and Marion 7-22

6. Thornton and Marion 7-25. Check that the Hamiltonian equations of motion are consistent with the Lagrangian equations.