33-331 Physical Mechanics I. Fall Semester, 2009
Assignment No. 2
Due Friday, Sept. 11

READING

Thornton and Marion:
Ch. 3, Secs. 3.1 through 3.6
Ch. 5, Secs. 5.1, 5.2

READING AHEAD:

Thornton and Marion
Ch. 5, Secs. 5.3 through 5.5
Tidal Forces (handout)

ANNOUNCEMENT: There will be an hour exam on Wednesday, Sept. 16 at 6:30 pm. The usual class hour at 2:30 will be devoted to review and answering questions. The exam will be closed book and closed notes and no calculators. Bring a pencil or equivalent. It you cannot come at this time and have not already contacted the instructor, please do so at once.

The exam will cover the assigned material in Chs. 2 and 3 of Thornton and Marion, as taken up in the lectures and the problem assignments. Material from Ch. 5 will *not* be on this examination.

Last year's exam will be found on the web site. Note that last year the harmonic oscillator was *not* part of the course.

EXERCISES:

1. Turn in at most one page, and not less than a third of a page, indicating what you have read, examples or exercises (apart from those assigned below) that you worked out, difficulties you encountered, questions that came to mind, etc. You may include complaints about the course. You will find a sample at the end of this problem set.

2. Thornton and Marion 2-28. Assume the marble does not hit the superball until just after the latter has bounced upwards from the floor.

3. Thornton and Marion 2-43

4. Thornton and Marion 3-10

5. An underdamped harmonic oscillator is initially at rest at the origin. At $t = 0$ a driving force $F = mA\cos\omega_0 t$ is turned out, where $\omega_0 = \sqrt{k/m}$ is the frequency of the undamped oscillator.

a) What will be the steady state motion after the transients have died out? You may make use of results in Thornton and Marion; you do not have to rederive them.

b) Find $x(t)$ for all $t > 0$, providing some explanation of your method. [Hint: What is the most general solution of the differential equation?]

6. Consider a driven damped harmonic oscillator subject to an external force $F(t)$. Let $\dot{W}_e$ be the rate at which this force does work on the oscillator, and let $\dot{W}_f$ be the rate at which the frictional force $F_f = -b\dot{x}$ does work on the oscillator. (The sign convention for doing work is such that $\dot{W}_f$ is always negative.)

a) Derive the result $dE/dt = dT/dt + dU/dt = \dot{W}_e + \dot{W}_f$ from the definition of T and U, and the differential equation of motion.

b) Assume that $F(t) = A\cos\omega t$ and the oscillator is in a steady state (transients have subsided), so that $x(t) = D_0\cos(\omega t - \delta)$. Find U, T, and $E = U + T$ as functions of time. You may express your answer using D_0 and δ (rather than expressing these in terms of A, etc.) Under what condition can E be independent of time? (Thornton and Marion use D for what is here denoted by D_0.)

c) Find the time averages of $\langle T\rangle$, $\langle U\rangle$, and $\langle E\rangle$ of the kinetic, potential, and total energy of the driven oscillator.

d) Using $x(t)$ from (b), find the time averages $\langle\dot{W}_e\rangle$ and $\langle\dot{W}_f\rangle$, and show that these are consistent with what you would expect from (a).