33-231 Physical Analysis Fall 2003

Problem Solutions: Set 12 (November 19, 2003)

46. a) $$c = \lambda f = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k}\ .$$

b) Use the definitions of the various quantities:

$$T = 1/f\,, \quad \omega = 2\pi f = 2\pi/T\,, \quad k = 2\pi/\lambda,$$

and the frequency-wavelength relations: $c = \lambda f\,, \quad \omega = ck,$
to show equivalence of the various forms.

c) Take derivatives of each function and substitute into differential equation, using relations from part (a) where necessary.

d) Follow suggestion in problem statement.

47. a) $$\lambda = \frac{c}{f} = \frac{344\ \text{m/s}}{440\ \text{s}^{-1}} = 0.782\ \text{m}.$$

b) $$\lambda = \frac{c}{f} = \frac{344\ \text{m/s}}{18{,}000\ \text{s}^{-1}} = 0.0191\ \text{m} = 1.91\ \text{cm}.$$

c) $$L = \frac{\lambda}{2} = \frac{1}{2}\frac{c}{f} = \frac{344\ \text{m/s}}{2\left(262\ \text{s}^{-1}\right)} = 0.656\ \text{m} = 65.6\ \text{cm}.$$

d) $$32\ \text{ft} = \frac{32\ \text{ft}}{3.281\ \text{ft/m}} = 9.75\ \text{m}.$$

$$L = \frac{\lambda}{2} = \frac{1}{2}\frac{c}{f}, \qquad f = \frac{c}{2L} = \frac{344\ \text{m/s}}{2(9.75\ \text{m})} = 17.6\ \text{Hz}.$$

This is a severe test of the low-frequency response of a stereo system.

48. a) $$f = \frac{c}{\lambda} = \frac{36.0\ \mathrm{m/s}}{1.80\ \mathrm{m}} = 20.0\ \mathrm{Hz}; \quad \omega = 2\pi f = 126\ \mathrm{s}^{-1};$$

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} = 3.49\ \mathrm{m}^{-1}.$$

b) $y(x,t) = A\sin(kx - \omega t) = (-2.50\ \mathrm{mm})\sin\left[\left(3.49\ \mathrm{m}^{-1}\right)x - (126\ \mathrm{s}^{-1})t\right]$

c) At $x = 0$, $y(0,t) = (-2.50\ \mathrm{mm})\sin[-(126\ \mathrm{s}^{-1})t\ ]$.

d) At $x = 1.35\ \mathrm{m} = 3\lambda/4$, $kx = 3\pi/2$, and

$$y(3\lambda/4,\ t) = (2.50\ \mathrm{mm})\sin\left[\left(-126\ \mathrm{s}^{-1}\right)t + 3\pi/2\right]$$

e) $$v_y = \frac{\partial y}{\partial t} = -A\omega\cos[kx - \omega t].$$

$$[v_y]_{\max} = A\omega = (2.50\ \mathrm{mm})(126\ \mathrm{s}^{-1}) = 315\ \mathrm{mm/s} = 0.315\ \mathrm{m/s}.$$

49. a) For lowest-frequency mode, $L = \lambda/2$.

$$f_1 = \frac{c}{\lambda} = \frac{2c}{L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2(0.400\ \mathrm{m})}\sqrt{\frac{(800\ \mathrm{N})(0.400\ \mathrm{m})}{3.00\ \times 10^{-3}\ \mathrm{kg}}} = 408\ \mathrm{Hz}.$$

b) $\frac{10{,}000\ \mathrm{Hz}}{408\ \mathrm{Hz}} = 24.5$, so the 24th harmonic is heard, but not the 25th.

50. $$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta;$$

$$y = A\cos(kx + \omega t) - A\cos(kx - \omega t)$$

$$= A[\cos kx\cos\omega t - \sin kx\sin\omega t] - A[\cos kx\cos\omega t + \sin kx\sin\omega t]$$

$$= -2A\sin kx\sin\omega t\ .$$