33-231 **Physical Analysis** **Fall 2003**

Problems: Set 4 (due Wednesday, September 24, 2003)

15. A glider on an air track has a mass of 0.300 kg. It is attached to one end of a spring, and the other end of the spring is held stationary. When forces with magnitude 0.480 N are applied to the ends of the spring, it stretches 0.100 m.

    a) Find the force constant of the spring.

    Suppose the block is given an initial displacement of 0.0300 m in the $+x$ direction and an initial velocity of 0.160 m/s in the $+x$ direction.

    b) Find the angular frequency, frequency, and period of the resulting motion.

    c) Find the amplitude, the phase angle, and the total energy of the motion (assuming. x is given as a function of t by an equation of the form of Eq. (4) in Chapter 5 of the notes).

    d) Could you stretch this spring 0.0300 m by holding the ends in your hands and pulling? Explain briefly.

    e) Write an equation for the block's position as a function of time, giving numerical values for all constants. Use Maple to plot a graph of x as a function of t. Also plot a graph with the same amplitude and frequency but with $\varphi = 0$. Plot the two graphs together, to show the significance of the phase angle.

    f) Use Maple to take the time derivative of the first function in (e). Then make a phase plot for this motion, i.e., a graph of v as a function of x, for one cycle.

16. Consider the same potential-energy function $V(x)$ as in Problem 14.

    a) Using the Maple command taylor, obtain series expansions of $V(x)$ and $F(x)$ in powers of the quantity $(x - 2a)$ (i.e., the distance from the equilibrium position at $x = 2a$). From these, obtain an expression for the angular frequency ω_o of small oscillations about the equilibrium position. Obtain the general expression; then substitute in the numbers from Problem 14(c).

    b) Using the numbers from Problem 14(c), find the approximate values of x for which the value of F differs by 10% from the value obtained by taking only the first term in the series. This will also give you some idea of the precision of the value of ω_o you found in (a).

17. We want to consider numerical solutions of the differential equation for the situation of Problems 14 and 16, using the numerical values given in Problem 14(c).

    a) Write the differential equation (from $\Sigma F = ma$) for the motion. Take the initial position to be the equilibrium position (i.e., $x_o = 2a = 4.0$ m) and give the mass a small initial velocity (e.g., $v_o = 0.1$ m/s).

    b) Use the Maple command `dsolve({...}, numeric);` to obtain a numerical solution of the system. Then use `odeplot` to plot the solution. Remember that you first have to load `odeplot` using `with(plots, odeplot);` . Take a large enough range of values of t so you can estimate the period of the motion from the graph. Compare your estimate with the period calculated from the value of ω_o obtained in Problem 16(a), i.e., $T = 2\pi/\omega_o = 2\pi/(0.559 \text{ s}^{-1}) = 11.2$ s.

    c) Repeat part (b) with several larger values of v_o (e.g, 1.0, 1.5, 2.0 m/s), and estimate the period for each. Note that the shape of the graph of $x(t)$ changes as v_o increases. With reference to the graph of $F(x)$, discuss qualitatively the reasons for this change.

    d) If v_o exceeds the "escape velocity" found in Problem 14(e), the motion is no longer oscillatory, but instead x increases continuously. Try values of v_o slightly large and slightly smaller than this in part (c), and verify that the character of the solution changes.

18. A general expression for the period of motion T of a mass m in a potential well described by $V(x)$ is derived on p. 4-6 of Chapter 6, *Force, Work, and Potential Energy*. The result is

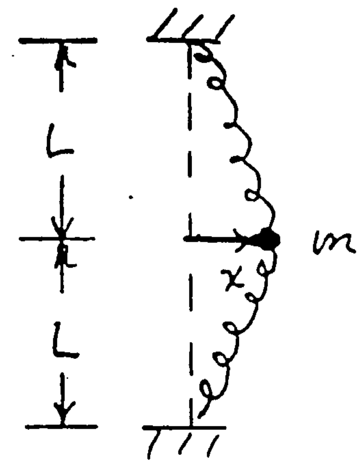
$$T = \sqrt{2m}\int_{x_{\min}}^{x_{\max}} \frac{dx}{\sqrt{E - V(x)}}.$$

    a) Apply this result to the situation of Problem 14(c,d) to find the period of this motion. The integral cannot be evaluated in terms of elementary functions. To evaluate the integral of a function $f(x)$ numerically, use the Maple command `evalf(int(f, x = xmin..xmax));` The `evalf` forces Maple to do a numerical evaluation instead of trying (and failing) to do it analytically.

    b) With the numerical values of Problem 14(c), find the numerical value of the velocity of the mass when it is at the equilibrium position, in Problem 14(d).

    c) Using the velocity you obtained in (b), carry out a numerical solution of the differential equation in Problem 17(b). Estimate the period from the graph of the solution, and compare your result to that of part (a).

19. In the system shown, mass m moves along a line perpendicular to the equilibrium position of the system. In the equilibrium position, the springs lie in a straight line, neither stretched nor compressed. There are no other forces acting on m (e.g., no gravity).



a) Assuming $V(0) = 0$, obtain an expression for the potential energy $V(x)$ of the system, as a function of x.

(Hint: The elongation of each spring is $\sqrt{L^2 + x^2} - L$.)

b) Use Maple to obtain an expression for the horizontal force $F(x)$.

c) Use Maple to obtain a Taylor series expansion of $F(x)$ about the point $x = 0$. Do you notice anything unusual about this series?

d) Approximate the series by keeping only the first non-vanishing term. Then substitute simple numerical values to obtain an equation in the form $\frac{d^2x}{dt^2} = -(\text{some power of } x)$. Then assume $x_o = 0$ and explore the numerical solutions of this equation, taking various values of v_o. Questions such as whether the motion is sinusoidal, whether the period depends on the amplitude, and what the phase plot looks like may be of interest.

Note: For some of the Maple calculations in this problem you may have to use the command `assume(L > 0);` For example, Maple can't simplify the expression $\sqrt{L^2} - L$ unless you tell it that L is a positive quantity. See the Maple help file on `assume` for further information.