

Problems: Set 3 (due Wednesday, September 17, 2003)

11. a) If x is a function of t , use the method of separation of variables to solve the equation $\frac{dx}{dt} = -x$, with the condition $x(0) = 1$. You should get $x = e^{-t}$.
- b) Use Euler's method, with a step size of 0.1, to compute values of x for the range $t = 0$ to 1, in steps of 0.1, using Maple. Compare your values with those obtained from the exact solution in (a). Print out a list of the approximate and exact values, and the corresponding values of t , using the command `print`. What is the percent error in the Euler method result at $t = 1.0$?
- c) Repeat part (b) with a step size of 0.02, and again find the percent error at $t = 1$. Note: You need a total of 50 steps. It's OK to display them all, but you don't need to. An optional gimmick for displaying only the values you're interested in is to replace the "print" statement with

```
if type(n/5, integer) then print(n*h, x[n], exp(-n*h));
end if;
```

End all other Maple statements (including `end if` and `end do`) with colons instead of semicolons.

Translation: if $n/5$ is an integer, then n is a multiple of 5 and t is a multiple of 0.1. In that case, Maple displays ("prints") the values of $t (= n \cdot h)$ and x . Otherwise it goes on to the next step in the `do` loop. For additional explanation, consult the Maple help files for `if`, `type`, and `print`. (This extra gimmick is optional.)

12. This problem concerns ballistic trajectories with air resistance proportional to v^2 , i.e., the magnitude of the resisting force is $|F| = b v^2$.
- a) Obtain the differential equations for $x(t)$ and $y(t)$, assuming as usual that x is measured horizontally and y vertically. You will first have to express F in terms of dx/dt and dy/dt , and then obtain expressions for the x and y components of the air-resistance force. **Be careful!** F_x is *not* equal to $-b v_x^2$. Instead, the correct expression is
- $$F_x = -\frac{v_x}{v} |F| = -b v_x v = -b v_x \sqrt{v_x^2 + v_y^2}.$$
- Make sure you understand why this is correct.
- b) Write the appropriate Maple statements to define the equations and the initial conditions. For example, you could call the equations `diffeqx` and `diffeqy`, and the initial conditions `init1`, `init2`, `init3`, and `init4`. Or whatever.

(continued)

12. (continued)

- c) Assume that at time $t = 0$ the object is at the origin and has velocity components $v_{x0} = 10$ m/s and $v_{y0} = 20$ m/s. Assuming *no* air resistance, (i.e., $b = 0$), use Maple to find x and y as functions of time, and determine the maximum height, the horizontal range, and the total time in flight. Plot a graph of the trajectory, using Maple. Note: It is easiest to use the parametric form of the Maple `plot` command, which is `plot([x, y, t = 0..4])`; Check whether the graph seems consistent with your results
- d) Now suppose there is air resistance, with $b = 2$ kg/m, $m = 80$ kg, and $g = 10$ m/s². With the same initial conditions as in (b), use Maple to obtain a graph of the trajectory. The equations can't be solved in closed form, so you have to use the `numeric` option in `dsolve`. The result is a procedure for computing values of x and y for any given t .

To plot numerical solutions of differential equations, you can use the command `odeplot`. First load this by using `with (plots, odeplot)`; Then if the solution is named `sol`, use `odeplot(sol, [x(t), y(t)], 0..4)`; (Note: Maple treats $x(t)$ and $y(t)$ as *functions*, not expressions, so the range for t is `0..4`, *not* `t = 0..4`.) Once you get this working, you can experiment with different values of b .

13. Some baseball players (and fans) claim that it is easier to hit a home run in Denver than in Pittsburgh. You are asked to carry out an analysis that will support or refute this claim.

Here are a few suggestions: The ball fields may have different dimensions, but assume they are identical in shape and size. The dominant influences on the ball's flight are gravity and air resistance. For the range of speeds involved here, it is reasonable to assume that the magnitude of the air-resistance force is proportional to v^2 . The analysis in Problems 10 and 12 is directly relevant. Ignore all effects that depend on the *spin* of the ball.

Here are a few numbers. According to the official rules, the weight of a baseball is between 5 and 5 1/4 ounces, and its circumference is between 9 and 9 1/4 inches. The average density of air when the temperature is 68°F (= 20°C) is about 1.18 kg/m³ in Pittsburgh and about 0.98 kg/m³ in Denver. The drag coefficient D for a perfectly smooth sphere is 0.44; for a baseball it is probably about 0.5. Evaluate the constant b using the same analysis as in Problem 10; your result should be somewhere around $b = 0.001$ kg/m. (If you get something grossly different from this, you've made a mistake. Check your arithmetic and unit conversions.)

14. A particle with mass m moves along the positive half of the x axis; its potential energy is given as a function of x by $V(x) = V_0 \left(\frac{a^2}{x^2} - \frac{a}{x} \right)$, where V_0 and a are positive constants.
- Derive an expression for the force acting on the particle, as a function of x .
 - Show that there is one stable equilibrium point, and derive an expression for its position.
 - If $V_0 = 5.00$ J, $a = 2.00$ m, and $m = 0.500$ kg, use Maple to plot graphs of V and F as functions of x . Adjust the vertical range so you can see the behavior of the functions in the range $x = 2$ to 10 m.
 - If the total energy is -1.00 J, find the maximum and minimum values of x , using the Maple `fsolve` command. To get both values you may have to specify ranges for the values of x .
 - If the particle is at rest at the equilibrium position and is then given a velocity with magnitude v_0 in the $+x$ direction, find the minimum value of v_0 for which the particle can go indefinitely far in the $+x$ direction. Would your result be different if the particle were given a velocity with magnitude v_0 in the $-x$ direction?