33-231 **Physical Analysis** **Fall 2003**

Problems: Set 13 (due Monday, November 24, 2003)

51. a) Show that Eq. (31) satisfies the boundary condition $\partial y/\partial x = 0$ at $x = 0$.

    b) Derive Eq. (32) from Eq. (31).

    c) A rope with length L has the end at $x = 0$ attached to a ring that slides without friction on a transverse rod, as in the figure on page 13-9 of the notes. The other end, at $x = L$, is held stationary. Derive a general expression for the angular frequencies of the normal modes, and a general expression for the wave function corresponding to each.

    Hint: Equation (32) satisfies the boundary condition at $x = 0$ for *any* value of k.. The boundary condition at $x = L$ is satisfied only for certain values of k.

52. a) Derive Eqs. (38) and (39) from Eqs. (36) and (37).

    b) Express Eqs. (39) in a form that contains only μ_1, μ_2, and the amplitudes.

53. A sinusoidal wave $y(x,t) = A\cos(kx - \omega t)$ travels in the $+x$ direction on the portion of a rope to the left ($x < 0$) of the midpoint of the rope. At the midpoint (at $x = 0$) it is tied to a massless ring that slides on a frictionless rod, similar to the arrangement in the figure on page 13-9, except that there is rope on both sides of the ring. The linear mass density μ is the same on both sides, but the tension F has one value F_1 for $x < 0$ and a different value F_2 for $x > 0$.

    a) Derive expressions analogous to Eqs. (38) and (39) for the amplitudes of the transmitted and reflected waves.

    b) Express the result of (a) in a form that contains only F_1, F_2, and the amplitudes.

54. A sinusoidal wave $y(x, t) = A\cos(kx + \omega t)$ travels in the $-x$ direction on a rope, toward the point $x = 0$. If this point is held stationary, the reflected wave is inverted. If it is held as in the diagram on page 13-9, the reflected wave is *not* inverted. But it is also possible to attach the rope to a device so that there is *no* reflected wave. What characteristics should this device have?

    Hint: At the point $x = 0$ the transverse force F_y on the end of the rope, its transverse velocity v_y, and the slope M, all vary sinusoidally with time. Show that at every instant F_y is directly proportional to v_y. That is, $F_y = -bv_y$, where b is a constant that depends on the device. What sort of device has this behavior? Derive an expression for b in terms of F and μ. The constant b is called the *characteristic impedance*, for reasons that we'll discuss in class.