33-231 **Physical Analysis** **Fall 2003**

Problems: Set 12 (due Wednesday, November 19, 2003)

46. a) Using the definitions of c, λ, k, f, T, and ω, show that the relations

$c = \lambda f$ and $\omega = ck$ are equivalent.

b) Using the relations in (a), show that the following forms for a sinusoidal wave function $y(x, t)$ are all equivalent, and that each can be written as a function of the quantity $u = x - ct$.

$$\cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right], \qquad \cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right], \qquad \cos\left[2\pi f\left(\frac{x}{c} - t\right)\right],$$

$$\cos(kx - \omega t), \qquad \cos\left[\omega\left(\frac{x}{c} - t\right)\right].$$

c) Show that each form in (b) satisfies the wave equation: $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2}$.

d) Show that *every* wave function $y = (x, t)$ that has the form $y = f(x - ct)$ satisfies the wave equation.

Suggestion: Let $u = x - ct$. Then $\dfrac{\partial f}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x}$, and so on.

47. The speed of sound in air at 20°C is 344 m/s.

a) What is the wavelength of a sound wave having a frequency of 440 Hz (corresponding to the A above middle C on the piano)?

b) If the maximum frequency that can be heard by a human ear is 18,000 Hz, what is the corresponding wavelength?

c) For an organ pipe that is open at both ends, the length of the pipe is 1/2 the wavelength of the sound wave it produces. What is the length of a pipe that sounds middle C (f = 262 Hz)?

d) The longest open pipe found in most large pipe organs is about 32 ft long. What frequency does this pipe produce? Will your stereo system reproduce this frequency?

48. A transverse sinusoidal wave with an amplitude of 2.50 mm and a wavelength of 1.80 m travels in the $+x$ direction along a horizontal stretched string; its speed is 36.0 m/s. Take the origin ($x = 0$) at the left end of the undisturbed string. At time $t = 0$ the point at the left end is at the origin and is moving upward.

    a) Find the frequency, angular frequency, and wave number of the wave.

    b) Find the wave function $\psi(x, t)$ that describes the wave.

    c) Obtain an expression for the displacement $y(0, t)$ of a point at the left end of the string, as a function of time.

    d) Obtain an expression for the displacement $y(1.35\text{ m}, t)$ of a point 1.35 m from the left end of the string, as a function of time.

    e) Find the maximum transverse velocity of a point on the string.

49. A piano tuner stretches a steel piano "string" (actually a wire) with a tension of 800 N. The wire is 0.400 m long and has a mass of 3.00 g.

    a) Derive Eq. (28) (in Chapter 13 of the notes) from Eqs. (16) and (26).

    b) What is the frequency of the wire in the fundamental (lowest-frequency) mode of vibration? (Be careful to use consistent units.)

    c) What is the number of the highest harmonic that can be heard by a person who can hear frequencies up to 10,000 Hz but no higher?

50. Derive Eq. (22) from Eq. (20).