The Units Survival Guide

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These notes should provide a very brief introduction into the necessity and practice of proper unit handling. Using simple examples and providing some elementary tricks, they should help you to become more familiar with a subject that (unjustifiedly) receives little attention in class, is the source of much (unnecessary) frustration, and holds the potential of uncovering many (unexpected) insights resulting from nothing else but expressing the same concepts in the same units.

I. INTRODUCTION

Units belong to that part of science that has always attracted fear and loathing. There are so many of them, they can be transformed in seemingly magical ways (sometimes using the weirdest conversion factors), and one invariably tends to get points subtracted when one forgets them.

Yet, units are no invention of physicists.

You want to bake a cake. The recipe states you need 3 sugar. You are justifiably upset, because it surely matters whether it is 3 sugar cubes, 3 teaspoons, 3 tablespoons, three ounces, three gram, or – the doctor forbid! – three cups. How far away does John live? Two. Two what? Two blocks? Two miles? Two hours by car? What's your weight? 165? Say that in Europe and they consider you obese!

When we want to make precise – quantitative – statements, we aim to answer questions such as how far, how long, how heavy, how warm, etc. But there is no naturally obvious length or time or weight or temperature. So we for instance need to say so-and-so-many repeats of a pre-agreed-upon length unit. Trouble is that not everyone agrees what that unit is. Or depending on the situation some unit is more convenient than another. You might want to know the distance between Pittsburgh and Philadelphia in miles, but you'd rather measure your height in foot and inches. Not only are units inescapable, unit conversion is the price we pay for convenience in different domains of application. Today we are unimaginably better off than several hundred years ago, when some duke's elbow was the rule for length, while in some neighboring duchy it was another nobleman's foot. And they had no pocket calculators back then! So we should be glad and learn how to take care of the remaining conversions.

Richard Muller uses many different units in his book "PHYSICS AND TECHNOLOGY FOR FUTURE PRESIDENTS", but he is not particularly explicit about how to actually handle them or how to efficiently transform them. Even though "unit-fluency" is by no means a major goal of our course, being reasonably comfortable with units certainly helps to better understand a fair number of issues that we will talk about. I therefore believe that a short overview might be warranted, and this is what I aim for in these notes. Not bothering with units during calculations and then later appropriately plugging them back in is a frequent shortcut for scientists, and Muller also follows this strategy occasionally, but one already needs to understand pretty well what one is doing in order to "cheat" that way. I feel that for learners this is not the right strategy. And while I agree that quantitative calculations are neither the main aim of this book nor of this course, much insight can be obtained by very simple order of magnitude estimates, but these still require you to manage units. Moreover, surprising bits of wisdom can emerge simply from taking two expressions, which embody the same physical quantity (e.g., energy) but traditionally use different units, and convert them to the same unit—and thus making them instantly *comparable*.

II. BASIC NOTIONS

A. "Pure" units

Let's begin with a simple example. A road trip between Pittsburgh and Philadelphia is about 300 miles long. How many kilometers is this? Well, since 1 mile is approximately 1.6 kilometers, this gives about 480 kilometers.

How can we do this formally?[3] There are several ways, all essentially equivalent, but different people might prefer different approaches, so let us suggest two neat formal tricks:

1. The mile-kilometer relation can be expressed in equation form as $1\,\mathrm{mi} = 1.6\,\mathrm{km}$, so in the expression $300\,\mathrm{mi}$ we merely replace the "mi" by "$1.6\,\mathrm{km}$", leading to

   $$300\,\mathrm{mi} = 300 \times 1.6\,\mathrm{km} = 480\,\mathrm{km}\ .$$

   Notice that the numerical values multiply. If you had a little bit exposure to algebra and still remember it, you will notice that units behave in exactly the same way as the x and y in your algebra textbook.

   This might seem awfully laborious, but it works completely automatically. For instance, how many miles are 280 kilometers? The conversion equation can be "solved" for kilometers to give you $1\,\mathrm{km} = \frac{1}{1.6}\,\mathrm{mi}$, and so "km" can be replaced by "$\frac{1}{1.6}\,\mathrm{mi}$", leading to

   $$280\,\mathrm{km} = 280 \times \frac{1}{1.6}\,\mathrm{mi} = 175\,\mathrm{mi}\ .$$

   We do not need to remember when to multiply and when to divide—*it works out all by itself!*

2. If $1\,\mathrm{mi} = 1.6\,\mathrm{km}$, then we can just as well say that $\frac{1\,\mathrm{mi}}{1.6\,\mathrm{km}} = 1$. Indeed, the ratio of two equal length should be 1. Now it is a well-known mathematical fact that one can multiply any expression by 1 without changing

FIG. 1: Unit conversion – taken too far?

it. Unit conversions can then be viewed as multiplication by a cleverly chosen 1, namely, the "conversion fraction". In the example from above:

$$480\,\mathrm{km} = 480\,\mathrm{km} \times 1 = 480\,\cancel{\mathrm{km}} \times \frac{1\,\mathrm{mi}}{1.6\,\cancel{\mathrm{km}}} = 300\,\mathrm{mi}\ . \quad (1)$$

cancel

Observe that we can again cancel the "km" as if it were some old x in our algebra book. You might ask how that works the other way around. If we want to convert miles into kilometers and if we use the same "conversion fraction", then we would not get a cancellation:

$$300\,\mathrm{mi} = 300\,\mathrm{mi} \times 1 = 300\,\mathrm{mi} \times \frac{1\,\mathrm{mi}}{1.6\,\mathrm{km}} = ?$$

This calculation is indeed not wrong, but it's not very helpful. However, instead of multiplying by $\frac{1\,\mathrm{mi}}{1.6\,\mathrm{km}}$, we can of course also multiply by its inverse, $\frac{1.6\,\mathrm{km}}{1\,\mathrm{mi}}$, which is also equal to 1. Then we get

$$300\,\mathrm{mi} = 300\,\mathrm{mi} \times 1 = 300\,\mathrm{mi} \times \frac{1.6\,\mathrm{km}}{1\,\mathrm{mi}} = \ 480\,\mathrm{km}\ ,$$

because now the miles cancel.

You thus see that in the second method you have to think a little bit what "version" of the "conversion fraction" you need to use. But it's not really hard: The right one is the one that helps you to cancel the unwanted unit. And if it doesn't cancel, you just need the other one. This might look slightly more complicated than the first method, but it is a bit more transparent when you want to do several unit conversions in one go, as we will soon see.

Where do we get the conversion equations from in the first place? Well, sometimes we just need to memorize them or look them up. For instance, Table I summarizes a few conversions that pertain to the change from imperial units to metric units. Sometimes we can calculate them from other bits of information. How many seconds does a year have? Approximately, a year has 365.25 days (accounting in an averaged way for leap years). A day has 24 hours, an hour has 60 minutes, a minute has 60 seconds. That makes $365.25 \times 24 \times 60 \times 60 \approx 3.16 \times 10^7\,\mathrm{s}$ in an average year. Many physicists remember this as "pi times ten to the seven seconds per year", which is only half a percent off but easier to memorize (with the necessary sense of nerdy humor).[4] How many millimeters to the yard? Here are the equations: $1\,\mathrm{yd} = 3'$, $1' = 12''$, $1'' = 25.4\,\mathrm{mm}$. Hence, $1\,\mathrm{yd} = 1 \times 3 \times 12 \times 25.4\,\mathrm{mm} = 914.4\,\mathrm{mm}$, a little less than a meter. Let's just see that this also works out very smoothly with "method 2" described above:

$$1\,\mathrm{yd} = 1\,\cancel{\mathrm{yd}} \times \frac{3\,\cancel{\mathrm{ft}}}{1\,\cancel{\mathrm{yd}}} \times \frac{12\,\cancel{\mathrm{in}}}{1\,\cancel{\mathrm{ft}}} \times \frac{25.4\,\mathrm{mm}}{1\,\cancel{\mathrm{in}}} = 914.4\,\mathrm{mm}\ .$$

Voilà! *Three* conversions in one line, and everything is completely transparent.

How many inches in a mile? $1\,\mathrm{mi} = 1760\,\mathrm{yd}$, $1\,\mathrm{yd} = 3'$, $1' = 12''$, hence $1\mathrm{mi} = 1 \times 1760 \times 3 \times 12'' = 63360''$.[5] You might want to check for yourself that this, again, can be written down succinctly using "method 2". How many kilometers to 8 miles – see Fig. 1.

B. Compound units

The vast majority of units emerge as the combination of other units that measure different concepts. For instance, velocity is distance traveled per time taken, so its unit must be a length unit divided by a time unit. Since there are many ways to measure distance and many ways to measure time, there are many $\times$ many ways to measure velocity: miles per hour, meters per second, yards per week, ... But the conversion works just the same. How many meters per second are 65 miles per hour? We can independently use the conversions for length and time, namely $1\,\mathrm{mi} = 1609\,\mathrm{m}$ and $1\,\mathrm{h} = 3600\,\mathrm{s}$, giving

$$65\,\frac{\mathrm{mi}}{\mathrm{h}} = 65 \times \frac{1609\,\mathrm{m}}{3600\,\mathrm{s}} = 65 \times \frac{1609}{3600}\,\frac{\mathrm{m}}{\mathrm{s}} \approx 29\,\frac{\mathrm{m}}{\mathrm{s}}\ .$$

Notice that the conventional abbreviation for "miles per hour", mph, confusingly suggests that the "miles" and "hours" are multiplied rather than divided. While "mph" is perfectly fine as an abbreviation, it is not good as an expression that can be taken seriously in a mathematical sense, for instance when working out conversions. In that case we should use $\frac{\mathrm{mi}}{\mathrm{h}}$.

Sometimes compound units get their own names. For instance, the speed unit "knot" is one *nautical*[6] mile per hour. The American and Canadian maritime authorities prefer "kn" as the abbreviation for the speed unit knot. But "kt" is also used, and "kts" is sometimes used for the plural. Since a nautical mile is exactly $1.852\,\mathrm{km}$, which is about 15% longer than the standard ("statute") mile, $1\,\mathrm{kn}$ is about 15% faster than $1\,\mathrm{mph}$. In other words, highway cruising speed is about 57 knots. Notice that the speed of *planes* is also still customarily measured in knots. This might be because if you're traveling substantial distances on the globe, a unit which maps to the globe (as the nautical-mile-derived knot does) might be convenient.

In the lecture we talk a lot about energy and power. Power is energy expended per time, hence a unit of power is a unit

concept	non-SI unit	symbol	relation to other non-SI unit	SI unit
length	inch	”	—	$2.54\,\mathrm{cm}$
length	foot	’	12”	$30.48\,\mathrm{cm}$
length	yard	yd	3’	$0.9144\,\mathrm{m}$
length	(statute) mile	mi	$1760\,\mathrm{yd}$	$\approx 1.609\,\mathrm{km}$
length	(nautical) mile	M	—	$1.852\,\mathrm{km}$
fluid volume	US fluid ounce	fl oz	—	$\approx 29.57\,\mathrm{mL}{=}29.57\,\mathrm{cm}^3$
fluid volume	US cup	cp	8 fl oz	$\approx 236.6\,\mathrm{mL}$
fluid volume	US pint	pt	2 cp	$\approx 473.2\,\mathrm{mL}$
fluid volume	US quart	qt	2 pt	$\approx 946.4\,\mathrm{mL}$
fluid volume	US gallon	US gal	4 qt	$\approx 3.785\,\mathrm{L}$
fluid volume	oil barrel	bbl	42 USgal	$\approx 159\,\mathrm{L}$
fluid volume	hogshead		63 US gal	$\approx 238.5\,\mathrm{L}$
mass	grain	gr	—	$\approx 64.8\,\mathrm{mg}$
mass	carat	CD	$\approx 3.086\,\mathrm{gr}$	$200\,\mathrm{mg}$
mass	(international avoirdupois) ounce	oz	—	$\approx 28.35\,\mathrm{g}$
mass	(international avoirdupois) pound	lb	16 oz $= 7000\,\mathrm{gr}$	$\approx 453.6\,\mathrm{g}$
mass	(international avoirdupois) stone	st	14 lb	$\approx 6.35\,\mathrm{kg}$
mass	(international avoirdupois) (short) ton	S/T	2000 lb	$\approx 907.2\,\mathrm{kg}$

TABLE I: Some commonly encountered non-SI units. Notice the nightmarish conversion factors between different non-SI units. (As a little side note: Why do we insist on “fluid” for the volume? Why would the “amount” of space – which is what volume measures – depend on whether it’s occupied by a fluid or a solid? Well, physically that would indeed not matter, but *historically* people have used different units for fluid and for dry volume. For instance, a “bushel” is equal to 4 “pecks” and roughly equal to 35.24 liter. But worse: A *fluid* US pint is roughly equal to $473.2\,\mathrm{mL}$, while a *dry* pint is about 16% more, namely $550.6105\,\mathrm{mL}$. Talk about certified insanity. And we remind you that this is really not the physicists fault!)

of energy divided by a unit of time. Physicists like to measure energy in “joules” (after James Prescott Joule, an English physicist and brewer), abbreviated “J”. Hence joules per second is a unit of power, called “watt” (after James Watt, a Scottish engineer who contributed major improvements to the steam engine), abbreviated “W”. You can turn this logic upside down and create a unit of energy by taking a unit of power and multiply it by a unit of time. For instance, 1000 watts is a kilowatt, kW, and multiplying this by one hour gives the kilowatthour, kWh, which therefore must be a unit of energy. The appropriate math teaches that

$$1\,\mathrm{kWh} = 1000\frac{\mathrm{J}}{\mathrm{s}} \times 3600\,\mathrm{s} = 3.6 \times 10^6\,\mathrm{J} = 3.6\,\mathrm{MJ}\ ,$$

where the last “M” stands for “mega”, which is the conventional prefix for “one million” (see Table II below). Notice again that the symbol for seconds, “s”, cancels in this little calculation. If you like, we have just discovered another conversion equation. As already mentioned, different units for the same concept (such as “energy”) abound, and we need to make peace with it.

C. Non-dimensionless conversions

We might say that Philadelphia is about 5 hours away from Pittsburgh. How can a distance be equal to a time? Of course it can not. But this statement makes sense if we can agree about a velocity with which to multiply the time such as to get a distance. Or a velocity which we divide the distance by such as to get a time. If we optimistically assume an average highway speed of 60 miles per hour, then $60\,\frac{\mathrm{mi}}{\mathrm{h}}$ multiplied by $5\,\mathrm{h}$ gives 300 miles, the distance we have mentioned earlier.

As we have already discussed, not mentioning a unit is lazy and potentially confusing. This confusion can be even bigger if one uses statements that require non-dimensional conversions, because there is likely even less agreement on the conversion factor. Is $60\,\mathrm{mph}$ a realistic highway speed? Could there be a traffic jam? What about the time it takes me to get on the highway? What if I decide to drive faster than the legal limit? To all these objections one might reply that what really matters for me is *how long it takes me to get there*, so a statement of time could be the more appropriate one. This might very well be the case, but it leaves the undeniable fact that there are two questions: ”How far is Philadelphia away from Pittsburgh?”, and ”How long does it take me (by car) to get from Pittsburgh to Philadelphia?” One question asks for a distance, the other one asks for a time. We might honor the speakers intentions by responding to the one he or she actually asked.

D. Physicist’s follies

That being said, you might guess that physicists never ever entertain the folly of using non-dimensional conversions. If so, you guessed wrong. However, physicists have an excuse that you don’t typically have in ordinary life: The laws of physics often give you conversion factors that are universally agreed upon and constant. For instance, special relativity teaches that the speed of light is a constant of nature that doesn’t even depend on the frame of reference within which you measure it. The speed of light is conventionally denoted as “c”, and for ordinary mortals the value is $c \approx 300,000\,\frac{\mathrm{km}}{\mathrm{s}} \approx 186,000\,\frac{\mathrm{mi}}{\mathrm{s}}$. Agreeing on this velocity as a universal conversion factor between time and length, we can say that the star *Proxima Centauri* is about 4.2 years away. Indeed, the distance that light travels in a year is called a "light year", abbreviated "ly".[7] And of course you can now also say that one nanosecond is about a foot long, because the speed of light is pretty close to one foot per nanosecond. But the really freaky physicists don't stop here. They just *define* $c = 1$, which means not only are they by definition measuring all speeds in units of the speed of light, they are also measuring all times in units of meter! One nanosecond *is* approximately $0.3\,\mathrm{m}$ (about one foot). And they don't stop here. They use Planck's constant $\hbar$, with units of energy times time, as a universal conversion factor between time and energy, and then agree to set $\hbar = 1$, thus measuring energy in hertz (which is a frequency unit). Don't worry: We'll not do this in our course.

Name	Prefix	$10^{\cdots}$	number	name of number
peta	P	10^{15}	1,000,000,000,000,000	quadrillion
tera	T	10^{12}	1,000,000,000,000	trillion
giga	G	10^{9}	1,000,000,000	billion
mega	M	10^{6}	1,000,000	million
kilo	k	10^{3}	1000	thousand
hecto	h	10^{2}	100	hundred
deca	da	10^{1}	10	ten
		10^{0}	1	one
deci	d	10^{-1}	0.1	tenth
centi	c	10^{-2}	0.01	hundredth
milli	m	10^{-3}	0.001	thousandth
micro	µ	10^{-6}	0.000,001	millionth
nano	n	10^{-9}	0.000,000,001	billionth
pico	p	10^{-12}	0.000,000,000,001	trillionth
femto	f	10^{-15}	0.000,000,000,000,001	quadrillionth

TABLE II: Prefixes to signify common factors-of-ten multipliers.

III. MAGNITUDE PREFIXES

A. The power of powers of ten

A kilometer is 1000 meters. By definition. No, more, because *it actually says so*. The "kilo" in front of meter is the universally accepted prefix to signify "a thousand times". It's the same "kilo" as in "kilogram", which is 1000 grams. It is therefore somewhat odd that kilometer is stressed on the "o", while kilogram is stressed on the "i". I don't know why this is so, but then I'm not a linguist.

Such prefixes are extremely useful, because they permit us to create new units which are bigger or smaller than the ones we are used to, while keeping the conversion factors trivially easy: Factors of 10 or 1000 or 1,000,000 are so easy to handle because they require us to do nothing but shifting the decimal point, or adding zeros. Once we know what a meter is, we can create a large unit such as kilometer, and any conversion between meter and kilometer is trivially easy. Compare that to the conversion between yard and mile, which involves the out-of-the-world factor 1760! Or we can create a smaller unit, such as centimeter ("centi" means $\frac{1}{100}$), and again the conversion is easy. Compare again to the conversion between yard and inch, which is a factor 36. I can't do divisions by 36 in my head, but I can easily divide by 100. Part of the beauty of the metric system relies on the agreement to use these simple-to-handle conversions.

Table II gives you a summary of the conventional prefixes that are useful to know and that we will also use in this class. Several remarks about that table seem to be in order:

1. The "µ" is the Greek letter "mu".

2. All of these units have made it into everyday language, even if you don't remember seeing them there. The very big units, for instance, regularly appear in computing contexts, such as gigabyte or petaflop (but in this context we need to say a bit more about them, see below). The smallest unit is for instance used to denote the latest generation of ultra short pulse lasers—the femtosecond laser). And deca? Isn't that esoteric? No. Ever heard of the decalog? These are the *ten* commandments (scientifically identical to ten monolog(ue)s).

3. "m" can mean "milli", but it can also mean "meter". "T" can mean "tera", but it can also be "Tesla" (a unit of magnetic field strength). Doesn't that give rise to confusions? Usually not—but indeed it pays off to pay at least a little bit of attention.

4. Notice that there exist different naming conventions for big numbers. There is a "short-scale", which goes like "million, billion, trillion, quadrillion, . . . " and a long scale, which goes like "million, milliard, billion, billiard, . . . ". The US has always used the short scale, but in British English the long scale has been in use until 1974.[1] There are many countries in the world which use the equivalent of the short scale, and many countries which use the equivalent of the long scale. Germany, for instance, uses the long scale ("Million, Milliarde, Billion, Billiarde, . . . ").

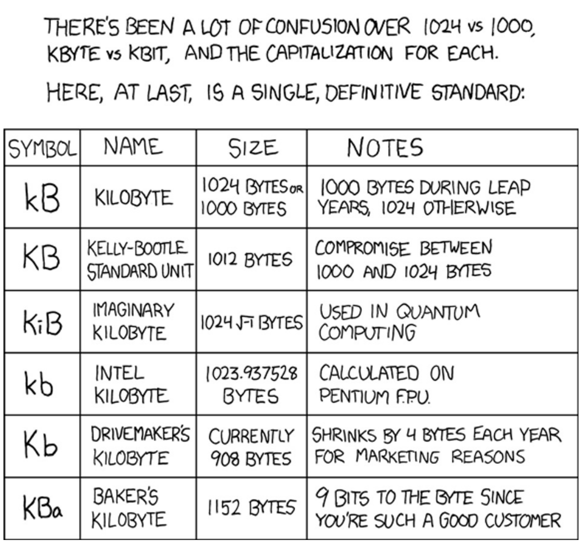
5. It is very useful in becoming fluent with these prefixes, but there is a *huge* danger involved in doing so. Sleek little prefixes can completely hide the enormity of the number they signify. For that matter, the same can happen with the name of the number itself. Time and again we can witness journalists in print and on TV to use the words "million", "billion", or "trillion" as if these are just really very big numbers. But they are not. A billion is *a thousand times bigger* than a million. And a trillion is *a thousand times bigger* than a billion. *A trillion is a million millions*! Take a break, concentrate, and then let this information completely sink in. And *then* contemplate the enormity of a "trillion dollar bailout", compared, for instance, to a "100 million dollar cut in governmental spending". In everyday language we have somehow forgotten to do the math and get the right amount of goose bumps. And it doesn't get better by speaking of "teradollars".

B. Powers of 2

As you might know, $2^{10} = 1024 \approx 1000$. Powers of 2 are very useful in computing because of the binary nature in which all our computing works (on versus off equals one "bit" of information). But since 2^{10} is approximately equal to one thousand, it has become customary to also use the prefix "kilo" when one actually means 1024. Then "kilobytes", "megabytes", or "gigabytes" really mean 2^{10} bytes, 2^{20} bytes, and 2^{30} bytes. However, when is this "kilo" really a factor of 1000, and when is it a factor of 1024? In the "kilo"-case a proposed circumvent was to say that we'll use a capital "K" as a prefix, rather than a lower-case letter "k". So there would be kB and KB. But that doesn't really help when you speak. It was also proposed to speak of "K-Bytes" or "M-Bytes", but that evidently hasn't caught on. The International Electronic Commission proposes the terms "kibibyte", "mebibyte", "gibibyte", etc. when they mean the 1024-derived scale.[2] Yet, I confess to not having seen them in use. Ever.

Despite several recommendations and bans coming from respectable organizations, the situation remains murky. I have come to believe that the following might be a good rule of thumb to figure out what might possibly be used in any given context. If a company wants to sell storage, it uses the usual power-of-10 notation. A "Gigabyte" USB stick would indeed hold a billion bytes. However, if they sell you a program that requires storage capacity on your hard disc, they tend to be inclined to use the power-of-2 scale, because then the number looks smaller. In that sense, the same "Gigabyte" would only be $\frac{10^9}{(2^{10})^3} \approx 0.93$ "G-Bytes". Notice that the difference is still about 69 power-of-ten-megabytes or 65 power-of-two-megabytes! (And you thought *physics* is confusing!)

Fig. (2) is the solution to the problem suggested by xkcd.



THERE'S BEEN A LOT OF CONFUSION OVER 1024 vs 1000, KBYTE vs KBIT, AND THE CAPITALIZATION FOR EACH.
HERE, AT LAST, IS A SINGLE, DEFINITIVE STANDARD:

SYMBOL	NAME	SIZE	NOTES
kB	KILOBYTE	1024 BYTES OR 1000 BYTES	1000 BYTES DURING LEAP YEARS, 1024 OTHERWISE
KB	KELLY-BOOTLE STANDARD UNIT	1012 BYTES	COMPROMISE BETWEEN 1000 AND 1024 BYTES
KiB	IMAGINARY KILOBYTE	1024√-1 BYTES	USED IN QUANTUM COMPUTING
kb	INTEL KILOBYTE	1023.937528 BYTES	CALCULATED ON PENTIUM F.P.U.
Kb	DRIVEMAKER'S KILOBYTE	CURRENTLY 908 BYTES	SHRINKS BY 4 BYTES EACH YEAR FOR MARKETING REASONS
KBa	BAKER'S KILOBYTE	1152 BYTES	9 BITS TO THE BYTE SINCE YOU'RE SUCH A GOOD CUSTOMER

FIG. 2: Maybe this is how the 10^3 vs. 2^{10} issue is resolved in practice? Taken from xkcd.

IV. THE SI SYSTEM

The International System of Units (SI, for the French name "Le Système International d'Unités") is the modern metric system of measurement. Apart from its superiority in the sciences, it is also of considerable advantage in international commerce. An almost 100 page summary can be found in a pdf document on the web-pages of NIST, the National Institute of Standards and Technology (which, note, belongs to the Department of Commerce!).[?] As far as our unit conversions go, here's one remarkable paragraph you find in this document:

> Derived units are defined as products of powers of the base units. When the product of powers includes no numerical factor other than one, the derived units are called *coherent derived* units. The base and coherent derived units of the SI form a coherent set, designated the set of *coherent SI units*. The word coherent is used here in the following sense: when coherent units are used, equations between the numerical values of quantities take exactly the same form as the equations between the quantities themselves. Thus if only units from a coherent set are used, conversion factors between units are never required.[?, Sec.1.4]

It is this property of the SI system that Muller makes much use of in his Textbook. This is (part of) what he means when he says that physicists like the 'joule' or the 'Kelvin' because it makes their equations look simple. Table III lists the 7 base units on which the SI system rests. Table IV shows, how some other frequent concepts get their coherent SI unit. *It is not necessary that you memorize these tables!* You can use them as a convenient look-up if you wish.

The metric system is not (yet?) followed in the US, but it is at least taught. Unfortunately, the admittedly complicated conversion factors between SI units and imperial units easily make one think that the metric system is complicated. This is a regrettable delusion. Once you're *entirely within* the metric system, there are no more yucky factors and life becomes easy. In particular, any type of calculation that involves large and small quantities of the same type (say, large and small lengths) becomes much simpler than the same calculation in imperial units. But until the magic day when pigs fly, hell freezes over, and the US goes metric, you need to know the conversions. Table I summarizes a few of the most frequent ones.

concept	unit	symbol	definition
length	meter	m	The meter (m) is defined by taking the fixed numerical value of the speed of light in vacuum, c to be 299,792,458 when expressed in the unit $\mathrm{m\,s^{-1}}$, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.
mass	kilogram	kg	The kilogram (kg) is defined by taking the fixed numerical value of the Planck constant, h, to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit Js, which is equal to $\mathrm{kg\,m^2 s^{-1}}$, where the meter and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$.
time	second	s	The second (s) is defined by taking the fixed numerical value of the cesium frequency $\Delta\nu_{\mathrm{Cs}}$, the unperturbed ground-state hyperfine transition frequency of the ^{133}Cs atom, to be 9,192,631,770 when expressed in the unit Hz, which is equal to $\mathrm{s^{-1}}$.
electric current	ampere	A	The ampere (A) is defined by taking the fixed numerical value of the elementary charge, e to be $1.602\,176\,634 \times 10^{-19}$ when expressed in the unit C, which is equal to As, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.
temperature	kelvin	K	The kelvin (K) is defined by taking the fixed numerical value of the Boltzmann constant k_{B} to be $1.380\,649 \times 10^{-23}$ when expressed in the unit $\mathrm{J\,K^{-1}}$, which is equal to $\mathrm{kg\,m^2 s^{-2} K^{-1}}$, where the kilogram, meter and second are defined in terms of h, c and $\Delta\nu_{\mathrm{Cs}}$.
amount of substance	mole	mol	One mole (mol) contains exactly $6.022\,140\,76 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_{A}, when expressed in the unit $\mathrm{mol^{-1}}$ and is called the Avogadro number.
luminous intensity	candela	cd	The candela (cd) is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, Kcd, to be 683 when expressed in the unit $\mathrm{lm W^{-1}}$, which is equal to $\mathrm{cd\,sr\,W^{-1}}$, or $\mathrm{cd\,sr\,kg^{-1} m^{-2} s^3}$, where the kilogram, meter and second are defined in terms of h, c and $\Delta\nu_{\mathrm{Cs}}$.

TABLE III: Fundamental SI units, from `https://www.nist.gov/pml/weights-and-measures/metric-si/si-units`.

concept	unit	symbol	coherent construction	simpler form in other SI units
force	newton	N	$\mathrm{m\,kg\,s^{-2}}$	—
energy	joule	J	$\mathrm{m^2\,kg\,s^{-2}}$	$\mathrm{N\,m}$
power	watt	W	$\mathrm{m^2\,kg\,s^{-3}}$	$\mathrm{J\,s^{-1}}$
pressure	pascal	Pa	$\mathrm{m^{-1}\,kg\,s^{-2}}$	$\mathrm{N\,m^{-2}}$
electric charge	coulomb	C	$\mathrm{s\,A}$	—
electric potential difference	volt	V	$\mathrm{m^2\,kg\,s^{-3}\,A^{-1}}$	$\mathrm{W\,A^{-1}}$
electric resistance	ohm	Ω	$\mathrm{m^2\,kg\,s^{-3}\,A^{-2}}$	$\mathrm{V\,A^{-1}}$

TABLE IV: Examples for some coherently derived SI units, see `https://www.nist.gov/pml/special-publication-811/nist-guide-si-chapter-4-two-classes-si-units-and-si-prefixes`.

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[1] see: `http://en.wikipedia.org/wiki/Short_scale`

[2] see: `http://physics.nist.gov/cuu/Units/binary.html`

[3] Don't ever underestimate the value that the automatism of a mathematical process provides. Once you have mastered any such skill, and it comes as easy as riding a bike, you can essentially "stop thinking" while you do this. Why would that be good? Because now you can focus on the parts of the problem that actually require your brain to concentrate. Ever wondered why physics professors can do all these complicated things? Here's the answer: Almost all the bits and pieces holding the key ideas together are based on automatic skills they spent years and years honing, while for the student everything – ideas, techniques, models, examples – is new.

[4] Equally nerdy, but also occasionally useful, is $1\,\mathrm{yr} \approx 10^{7.5}\,\mathrm{s}$, which is about 0.2% off. Its usefulness lies in the fact that the entire expression is a simple power of 10.

[5] Another curious observation: with about 3% accuracy the number is equal to 2^{16}.

[6] The nautical mile was derived as a convenient measure when navigating on the globe. It was originally defined as the speed at which you travel one minute of arc along any meridian per hour. Since the circumference of the earth is approximately $40\,000\,\mathrm{km}$ (a round value that is no coincidence), there are 360 degrees for the full circle, and 60 minutes of arc per degree, a straightforward calculation gives $\frac{40\,000\,\mathrm{km}}{360\times 60} \approx 1.852\,\mathrm{km}$ for the nautical mile. The SI system? has defined this value to be the *exact* conversion factor.

[7] Using our newly acquired knowledge that a year is about $\pi \times 10^7$ seconds, we see that a light year is $3 \times 10^5\,\frac{\mathrm{km}}{\mathrm{s}} \times \pi \times 10^7\,\mathrm{s} \approx 10^{13}\,\mathrm{km} = 10\,\mathrm{Pm}$, where "P" stands for "peta", which means 10^{15} (see Table II). In yet other words, a light year is approximately ten million billion meters. You would think that astronomers are happy with such a unit, but they don't seem to be. They prefer the "parsec", which is about 3.26 light years…