ME 24-731
Conduction and Radiation Heat Transfer

Mid-Term Examination
March 17, 2000
Spring 2000
Instructor: J. Murthy
Total = 25 points

This is a take-home exam. You may start the exam any time after 5:00 pm on March 17th. You must stop working on the exam at 8:00 am on March 20th. The exam is due in my office on the morning of March 20th; for those who are working, please be sure to drop it off in my office sometime during the day on March 20th. No consultation or discussion with anybody is allowed, though you may consult any textbooks and notes you like.

1. (*5 Points)* An aluminium cube of side 5 cm is initially at $T_1 = 30°C$. For $t > 0$, two of the boundaries are kept insulated, two are subjected to uniform heating with a heat flux q" = 10^4 W/m^2. The remaining two faces lose heat to surroundings with an ambient temperature of 20°C and a heat transfer coefficient of 50 W/m^{2}K. The properties of aluminium are: ρ= 2700 kg/m^3; C= 0.896 kJ/kg K; k = 204 W/mK.

   (a) Is a lumped capacitance analysis valid?

   (b) Assuming that it is, develop an expression for the temperature of the cube T(t).

   (c) Calculate the steady state temperature of the cube.

2. (*10 Points)* A two-dimensional square domain of side L has initial and boundary conditions as shown in Figure 1. The domain is initially at a uniform temperature T_i. For $t > 0$ the boundaries at $x = 0$ and $y = 0$ are subjected to a convective boundary condition with a heat transfer coefficient h and an ambient temperature T_a. The boundaries at x=L and y=L are subjected to a given-temperature condition $T = T_a$.

   (a) Write the governing equations and boundary and initial conditions in terms of a dimensionless variable θ defined as

   $$\theta = \frac{T - T_a}{T_i - T_a}$$

   (b) Postulating that

   $$\theta(x,y,t) = X(x,t)Y(y,t)$$

   find an expression for the unsteady temperature field $\theta(x,y,t)$.

   You may assume all properties to be constant. The quantities h and T_a are also constant.

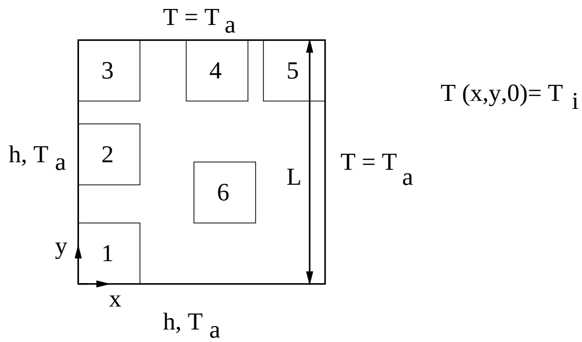

Figure 1: Geometry for Problems 2 and 3

3. *(10 Points)* Now consider the numerical solution of Problem 2. Assume a uniform mesh of size $\Delta x = \Delta y$ and a time step Δt. For the control volumes $1-6$ develop discrete equations relating the temperature at the control volume centroid to the discrete values of interior neighbor temperatures and the boundary conditions. Use the finite volume method developed in class, and the fully implicit scheme.