ME 24-731
Conduction and Radiation Heat Transfer

Final Project
April 25, 2000
Spring 2000
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Due May 16, 2000

Problem Statement
Consider participating radiation in a three-dimensional cuboidal box of side L, as shown in Figure 1. The walls are gray-diffuse and opaque; each has a known temperature T_{wi} as shown. The emissivity of all walls is the same, $\varepsilon = 0.8$. The box contains a gray participating gas with a known (constant) absorption coefficient κ. There is no scattering. The temperature of the gas is known at T_{gas}. The objective of the project is to compute the net radiant heat transfer rate from each wall using a ray tracing scheme. The problem should be done numerically by writing a computer code. The different steps you will need to take are described below.

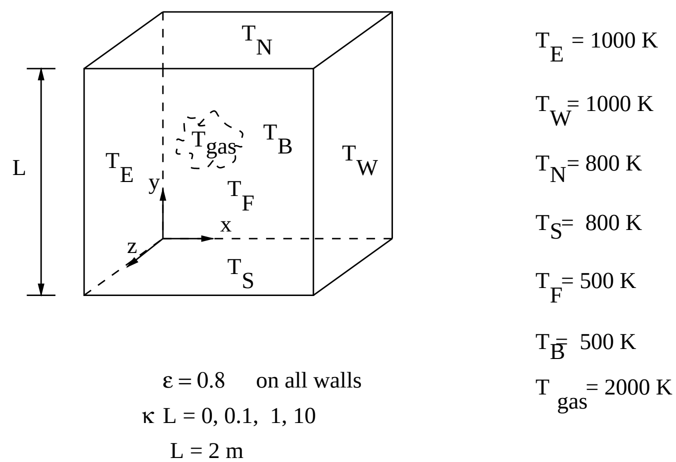

Figure 1: Box Geometry and Boundary Conditions

Calculation Procedure

1. **Surface Spatial Discretization**
   Divide each of the surfaces E,W, N, S, T, B into a 5×5 grid ($\Delta x = \Delta y = \Delta z = 0.4m$). Place a grid point at the centroid of each "control surface". The objective is to compute the heat flux at each surface grid point. The heat flux is considered constant over the associated control surface.

2. **Surface Angular Discretization**
   Consider a hemisphere associated with each grid point, spanning a solid angle of 2π. Just as we discretize space into control volumes, let us discretize the hemisphere into control angles. A typical discretization $N_\theta \times N_\phi$ for the south (S) surface is shown in Figure 2; here N_θ is the number of control angles in the θ direction and N_ϕ is the number of control angles in the ϕ direction. Choose N_θ=2 and $N_\phi = 8$. Thus, the control angle extents are:

$$\Delta\theta = \frac{\pi/2}{N_\theta}; \quad \Delta\phi = \frac{2\pi}{N_\phi}$$

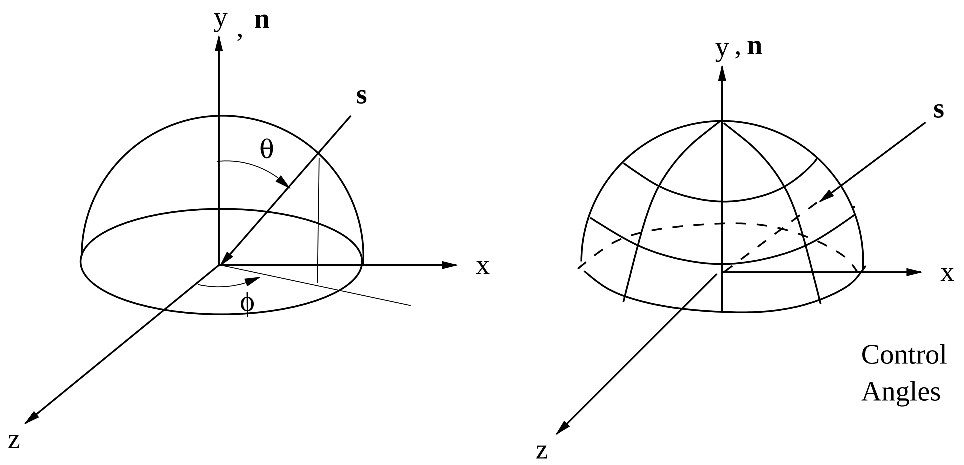

Figure 2: Angular Discretization

There are a total of 16 control angles per surface grid point. Thus for the south (S) surface, you would have a total of 25 grid points with 16 control angles per grid point. You would have a similar discretization for each of the other surfaces E,W, N, T, B. The incoming ray direction **s** for the south surface is given by

$$\mathbf{s} = -\left(\sin\theta\sin\phi\,\mathbf{i} + \cos\theta\,\mathbf{j} + \sin\theta\cos\phi\,\mathbf{k}\right) \tag{1}$$

Similar equations may be written for the surfaces E,W,N, T and B.

3. **Computing Incoming Intensity**
For a gray medium with no scattering, the radiative transfer equation (RTE) in a ray direction **s** is given by

$$\frac{d}{ds} I\left(\mathbf{s}\right) = \kappa I_b - \kappa I \tag{2}$$

Here,

$$I_b = \frac{n^2 \sigma T_{gas}^4}{\pi}$$

is a known constant along the ray path. You may assume n=1. Integrating Equation 2 along s, we have

$$I = I(0)e^{-\kappa l} + I_b(1 - e^{-\kappa l}) \tag{3}$$

Here, I is the intensity associated with the incoming ray, I(0) is the intensity at the wall from which the ray originates, and l is the path length in the gas. I_b is the blackbody intensity associated with the gas temperature T_{gas}. Thus, there are $N_\theta \times N_\phi$ incoming rays at any grid point, each associated with a different starting intensity depending on which wall it originates from, and how far it has travelled in the gas.

4. **Updating Boundary Outgoing Intensity**
Because the wall is gray-diffuse and opaque, the radiosity associated with all outgoing directions is the same, and is given by

$$J = \varepsilon n^2 \sigma T_{wi}^4 + (1 - \varepsilon) q_{\text{incoming}}$$

where

$$q_{\text{incoming}} = \int_{\mathbf{s}\cdot\mathbf{n} \leq 0} I(\mathbf{s}) |\mathbf{s} \cdot \mathbf{n}| d\Omega$$

The intensity for all outgoing directions may be found from

$$I(\mathbf{s}) = \frac{J}{\pi} \quad \text{for } \mathbf{s}\cdot\mathbf{n} \geq 0 \tag{4}$$

In this numerical implementation, the incoming energy q_{incoming} is found by numerical integration:

$$q_{\text{incoming}} \approx - \sum_{\text{i=incoming directions}} I_i(\mathbf{s}_i)\mathbf{s}_i \cdot \mathbf{n} \sin\theta_i \Delta\theta\Delta\phi \tag{5}$$

The negative sign accounts for the fact that $\mathbf{s}_i \cdot \mathbf{n} \leq 0$.

5. **Iteration**
   The overall procedure consists of the following:
   (a) Guess the intensity for outgoing directions for each surface. Typically $I = \varepsilon n^2 \sigma T_{wi}^4/\pi$ is a good guess.
   (b) Go to each grid point, and trace rays "backwards". Find the enclosure surface intersected by each ray, and find the starting intensity for the ray I(0). Find l, the length traveled in the gas.
   (c) Find the incoming intensity in ray each direction to the surface grid point using Equation 3.
   (d) Find the incoming energy q_{incoming} at each surface grid point using Equation 5.
   (e) Hence update the outgoing intensity using Equation 4.
   (f) Repeat until the outgoing intensity stabilizes at each surface.
6. **Computing Net Wall Heat Flux**
   Once the iteration has converged, find the net heat flux (outgoing-incoming) at each grid point from:

$$q \approx \sum_{\text{i=all directions}} I_i(\mathbf{s}_i)\mathbf{s}_i \cdot \mathbf{n} \sin\theta_i \Delta\theta\Delta\phi$$

Calculations Required

1. Make calculations for four optical thicknesses κL =0, 0.1, 1, and 10.
2. Tabulate the heat transfer rate from each wall as a function of the optical thickness κL.
3. For each optical thickness, determine the radiative heat balance for the enclosure. Would you expect the wall heat transfer rates to balance?

If you wish, you may compare the results for $\kappa L = 0$ with surface-to-surface enclosure calculations using the code you wrote for Assignment 7.