ME 24-731
Conduction and Radiation Heat Transfer

Solution to Assignment No: 7
Due Date: May 2, 2000
Spring 2000
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1. (a) The surfaces are numbered as shown in Figure 1. Using Hottel's cross strings, we find the relevant view factors as

$$\begin{aligned} F_{11} &= 0 \\ F_{12} &= F_{14} = \frac{1}{2L}(2L - (0 + \sqrt{2}L) = 0.293 \\ F_{14} &= 1 - 2F_{12} = 0.414 \end{aligned}$$

The other view factors can be deduced by symmetry.

(b) The equations governing the surface radiosities can be written as:

$$\varepsilon_i \left(E_{bi} - J_i\right) = (1-\varepsilon_i) \sum_{j=1}^{4} F_{ij} \left(J_i - J_j\right) \quad \text{for surfaces with T given} \tag{1}$$

$$q_i = \sum_{j=1}^{4} F_{ij} \left(J_i - J_j\right) \quad \text{for surfaces with } \mathrm{q_i} \text{ given} \tag{2}$$

Solving the four equations simultaneously for the J_i's yields

$$\begin{aligned} J_1 &= 7.5787 \times 10^5 \, W/m^2 \\ J_2 &= 3544 \, W/m^2 \\ J_3 &= 1.295 \times 10^5 \, W/m^2 \\ J_4 &= 3.6138 \times 10^5 \, W/m^2 \end{aligned}$$

For surfaces with temperatures given, the heat fluxes are computed from Equation 2 once the J_i's are known. For surfaces with given q_i, E_{bi} is computed from Equation 1 and the temperature computed from it. Thus

$$\begin{aligned} q_1 &= 5.9733 \times 10^5 \, W/m^2 \\ q_2 &= -4.0606 \times 10^5 \, W/m^2 \\ q_3 &= -2.9128 \times 10^5 \, W/m^2 \\ q_4 &= 10^5 \, W/m^2 \end{aligned}$$

Heat balance is satisfied. Further,

$$\begin{aligned} T_1 &= 2000 \, K \\ T_2 &= 500 \, K \\ T_3 &= 1000 \, K \\ T_4 &= 1689 \, K \end{aligned}$$

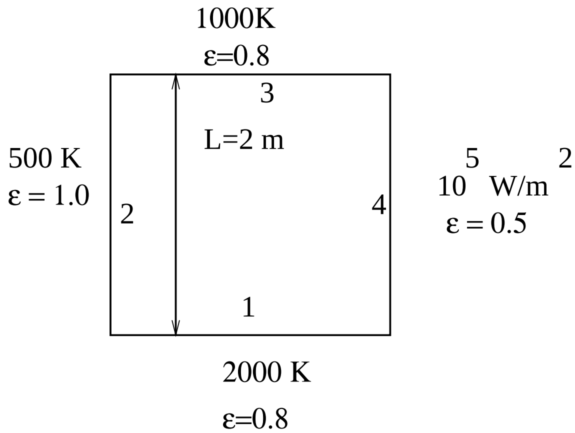

Figure 1: Domain for Problem 1

2. For this problem, E_{b1} is no longer known. The heat flux leaving surface 1 is given by:

$$q_1 = \frac{\varepsilon_1}{1-\varepsilon_1}\left(\sigma T_{s1}^4 - J_1\right) \quad (3)$$

$$= -k\frac{(T_{s1}-T_{s2})}{d} \quad (4)$$

It is possible to guess T_{s1}, solve the enclosure problem as above, find q_1 from Equation 3and update T_{s1} using Equation 4, iterating until convergence. This procedure may require some under-relaxation, depending on how you start, i.e, instead of applying the entire q_1 to Equation 4, an underrelaxed valueis applied:

$$q_1 \leftarrow (1-\alpha_u)q_1^* + \alpha_u q_1$$

where q^* is the previous iterate and α_u is an under-relaxation coefficient between zero and one.

A better way of updating the surface temperature T_{s1} is to linearize q_1:

$$q_1 = q_1^* + \frac{\partial q_1}{\partial T_{s1}}\left(T_{s1} - T_{s1}^*\right)$$

$$= \frac{\varepsilon_1}{1-\varepsilon_1}(E_{b1}^* - J_1) + 4\sigma T_{s1}^{*3}\left(T_{s1} - T_{s1}^*\right)$$

$$= A + BT_{s1} \quad (5)$$

Substituting this linearized expression into Equation 4 yields an iterative equation for T_{s1}:

$$T_{s1} = \frac{\frac{k}{d}T_{s2} - A}{\left(B + \frac{k}{d}\right)}$$

This equation is non-linear in T_{s1}; the non-linearities are hidden in the A and B terms. (Notice how linearization causes the non-linear radiative boundary condition on the solid to appeat like a convective bounary condition). Thus, iteration is still required. Typically, convergence is obtained easily and within 10 iterations or less if you start with the solution of Problem 1.The computed radiosities, temperatures and heat fluxes are given by:

$$J_1 = 6.246 \times 10^5 \; W/m^2$$

$$J_2 = 3544 \; W/m^2$$

$$J_3 = 1.1597 \times 10^5 \; W/m^2$$

$$J_4 = 3.1838 \times 10^5 \; W/m^2$$

$$
\begin{aligned}
q_1 &= 4.8227 \times 10^5 \, W/m^2 \\
q_2 &= -3.4524 \times 10^5 \, W/m^2 \\
q_3 &= -2.3704 \times 10^5 \, W/m^2 \\
q_4 &= 10^5 \, W/m^2
\end{aligned}
$$

Heat balance is satisfied. Further,

$$
\begin{aligned}
T_1 &= 1904 \, K \\
T_2 &= 500 \, K \\
T_3 &= 1000 \, K \\
T_4 &= 1648 \, K
\end{aligned}
$$

3. For this multidimensional conduction/radiation problem. we may write the following heat balance for the discrete control volume:

$$
\begin{aligned}
q_e + q_w + q_s + q_n &= 0 \\
\frac{k_e \Delta y}{\Delta x}(T_E - T_P) + \frac{k_w \Delta y}{\Delta x}(T_W - T_P) + \frac{k_n \Delta x}{0.5 \Delta y}(T_N - T_P) + \frac{k_s \Delta x}{\Delta y}(T_S - T_P) &= 0
\end{aligned}
$$

If the T_N's were known for all the north surfaces, it would be a trivial matter to solve for the discrete solid temperatures. However, in the above equation, T_N is unknown, and differs from control surface to control surface because the view factor for each control surface to the enclosure is different. If we want to solve this problem multi-dimensionally, each of the interface control surfaces of the solid would be a surface in the enclosure calculation, and would have its radiosity and heat flux found through the enclosure calculation. The equation to update T_N is found from a surface heat balance at the north surface. Linearizing the the heat flux for the surface N using Equation 5 and equating it to the north face heat transfer rate we get:

$$
\Delta x (A + B T_N) = \frac{k_n \Delta x}{0.5 \Delta y}(T_P - T_N) \tag{6}
$$

This constitutes the iterative equation for T_N. Every control surface on the north face of the solid has a similar equation. The procedure for solving this multidimensional problem is similar to Problem 2:

(a) Guess all T_N's.

(b) Solve the enclosure problem; the enclosure has surfaces 2,3,4 as before, but surface 1 now has as many surfaces as the x-direction spatial discretization of the solid. Obtain J_i's.

(c) Find A, B for each of the north faces of the solid at the interface.

(d) Update T_N for each of the north faces using Equation 6, and the centroid temperatures for each of the control volumes in the solid.

(e) Iterate between the conduction and radiation problems until convergence.

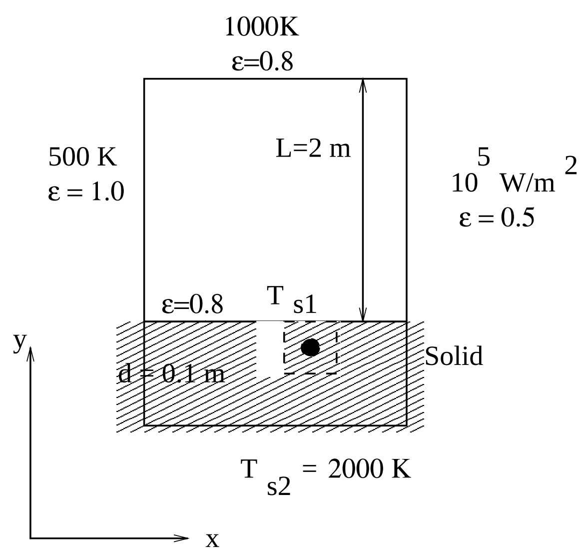

Figure 2: Domain for Problems 2 and 3