ME 24-731
Conduction and Radiation Heat Transfer

Solution to Assignment No: 6
Due Date: April 18, 2000
Spring 2000
Instructor: J. Murthy

1. Problem 3-9, Siegel and Howell.

   (a) Total hemispherical emissive power of surface:

$$\begin{aligned} E &= \int_0^1 0.2E_{b\lambda}d\lambda + \int_1^4 0.45E_{b\lambda}d\lambda + \int_4^5 0.7E_{b\lambda}d\lambda + \int_5^7 0.8E_{b\lambda}d\lambda + \int_7^{10} 0.55E_{b\lambda}d\lambda + \int_{10}^{\infty} 0.2E_{b\lambda}d\lambda \\ &= n^2\sigma T^4 \sum_{\text{i=1}}^{\text{nbands}} \varepsilon_{\lambda,i}\left(f(n\lambda_{2i}T) - f(n\lambda_{1i}T)\right) \\ &= 9800\,W/m^2 \end{aligned}$$

   Here, $\varepsilon_{\lambda,i}$ is the spectral emissivity in the band i and λ_{1i} and λ_{2i} represent the wavelength limits for each band. The blackbody fractions are obtained for the tables given in class.

   (b) Total intensity at 45° from surface normal?
   Since the surface is diffuse,

$$I = E/\pi = 3120 \;\; W/m^2$$

   (c) Total energy emitted in $4 < \lambda \le 7 \;\; \mu m$:

$$\begin{aligned} E_{\lambda_2-\lambda_1} &= \int_4^5 0.7E_{b\lambda}d\lambda + \int_5^7 0.8E_{b\lambda}d\lambda \\ &= 5316 \;\; W/m^2 = 54.25\% \text{ of total} \end{aligned}$$

   Amount of energy emitted by a gray body with $\varepsilon = 0.35$ in the same wavelength range ?

$$\begin{aligned} E &= \varepsilon n^2\sigma T^4 = 6279\,W/m^2 \\ E_{\lambda_2-\lambda_1} &= \varepsilon\left(f(n\lambda_2 T) - f(n\lambda_1 T)\right)n^2\sigma T^4 \\ &= 2451\,W/m^2 = 39.05\% \;\text{ of E} \end{aligned}$$

2. Problem 3-15, Siegel and Howell.

   (a) Hemispherical emissivity ε_λ:

$$\begin{aligned} \varepsilon_\lambda &= \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2} \varepsilon'_\lambda \cos\theta\sin\theta d\theta d\phi \\ &= \frac{1}{\pi}2\pi\left(\int_0^{\pi/6} 0.8\frac{1}{2}\sin 2\theta d\theta + \int_{\pi/6}^{\pi/2} 0.5\frac{1}{2}\sin 2\theta d\theta\right) \\ &= 0.575 \end{aligned}$$

(b) Since surface is gray

$$\varepsilon_\lambda = \varepsilon = \alpha \text{ independent of incoming } \lambda$$

Thus, a fractionn 0.575 of incoming radiation will be absorbed.

(c) Heat flux emitted by surface :

$$E = \varepsilon n^2 \sigma T^4 = 2983 \ W/m^2$$

Since the surface is surrounded by a cold medium at 0K, there is no incoming energy. Therefor the total heat flux to be supplied is 2983 W/m2.

3. Problem 6-2,Siegel and Howell.
From view-factor catalogue, page 783, #10, view factor from differential element to planar rectangle with with element's normal passing through rectangle corner (Figure 1):

$$F_{d1-2} = \frac{1}{2\pi}\left(\frac{A}{\sqrt{(1+A^2)}}tan^{-1}\frac{B}{\sqrt{(1+A^2)}} + \frac{B}{\sqrt{(1+B^2)}}tan^{-1}\frac{A}{\sqrt{(1+B^2)}}\right)$$

For our case,

$$\begin{aligned} F_{d1-2} &= \sum_i^4 F_{dA_1-A_{2i}} = 4F_{dA_1-A_{21}} \\ &= \frac{2}{\pi}\left(\frac{X}{\sqrt{(1+X^2)}}tan^{-1}\frac{Y}{\sqrt{(1+X^2)}} + \frac{Y}{\sqrt{(1+Y^2)}}tan^{-1}\frac{X}{\sqrt{(1+Y^2)}}\right) \end{aligned}$$

where $X = \frac{W}{2H}$ and $Y = \frac{L}{2H}$.

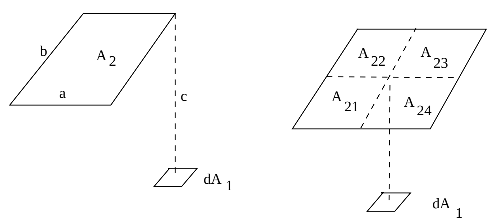

Figure 1: View Factor Algebra for Problem 3

4. Problem 6-18,Siegel and Howell.

From pages 174-176 of Modest, $F_{1+2-7+8}$, F_{1-7} and F_{2-8} can be found from knowing the view factor between perpendicular rectangular plates with a common edge. Further, Modest shows that

$$A_1 F_{1-8} = A_2 F_{2-7}$$

Therefore

$$\begin{aligned} A_{1+2}F_{(1+2)-(7+8)} &= A_1F_{1-7} + A_1F_{1-8} + A_2F_{2-7} + A_2F_{2-8} \\ &= A_1F_{1-7} + 2A_1F_{1-8} + A_2F_{2-8} \\ F_{1-8} &= \frac{1}{2A_1}\left(A_{1+2}F_{(1+2)-(7+8)} - A_1F_{1-7} - A_2F_{2-8}\right) \end{aligned}$$

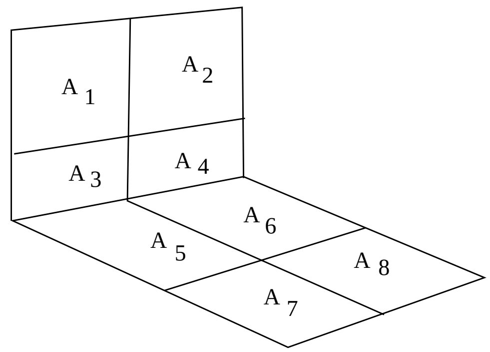

Figure 2: View Factor Algebra for Problem 4

5. Problem 7-13,Siegel and Howell.
   Create fictitious black surface 3 at environment temperature, as shown in Figure 3. Surfaces 1, 2, and 3 are black, as is the environment. The outer surface 4 has $\varepsilon = 0.4$. The different view factors are computed as:

$$\begin{aligned} F_{1-3} &= 0.17 \text{ from view factor catalogue} \\ F_{1-2} &= 1 - F_{1-3} = 0.83 \\ F_{3-1} &= \frac{A_3}{A_1} F_{1-3};\ F_{3-2} = 1 - F_{3-1} = 0.7 \\ F_{4e} &= 1 \end{aligned}$$

   From heat balance at node 2:

$$\frac{E_{b2} - E_{b3}}{\frac{1}{A_3 F_{3-2}}} + \frac{E_{b2} - E_{b1}}{\frac{1}{A_1 F_{1-2}}} = \frac{E_{be} - E_{b2}}{\frac{1}{A_4 F_{4-e}} + \frac{1-\varepsilon_4}{A_4 \varepsilon_4}}$$

   We are given $T_1 = 900K$, $T_3 = 500K$, and $T_e = 500K$. So E_{b2} can be found to be 15,118 W/m^2. Thus $T_2 = 718.6K$. The heat transfer from surface 1 is:

$$Q = \frac{E_{b1} - E_{b3}}{\frac{1}{A_1 F_{1-3}}} + \frac{E_{b1} - E_{b2}}{\frac{1}{A_1 F_{1-2}}} = 120.1\ W$$

6. Problem 7-25,Siegel and Howell.
   Heat balance at each node yields:

$$\begin{aligned} Q_1 &= \frac{J_1 - J_2}{\frac{1}{A_1 F_{1-2}}} + \frac{J_1 - J_3}{\frac{1}{A_1 F_{1-3}}} \\ 0 &= \frac{J_2 - J_1}{\frac{1}{A_1 F_{1-2}}} + \frac{J_2 - J_3}{\frac{1}{A_3 F_{3-2}}} \\ \frac{E_{b3} - J_3}{\frac{1-\varepsilon_3}{A_3 \varepsilon_3}} &= \frac{J_3 - J_1}{\frac{1}{A_1 F_{1-3}}} + \frac{J_3 - J_2}{\frac{1}{A_3 F_{3-2}}} \end{aligned}$$

   From the handout given in class, disk-to-disk view factors can be computed. We get

$$\begin{aligned} F_{1-3} &= 0.2;\ F_{1-2} = 1 - F_{1-3} = 0.8 \\ F_{3-2} &= 1 - F_{3-1} = 1 - \frac{A_1}{A_3} F_{1-3} = 0.55 \end{aligned}$$

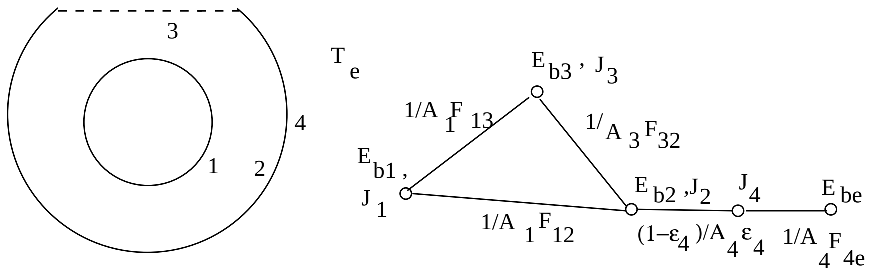

Figure 3: Surfaces for Problem 5

We solve for J_1, J_2, and J_3. The temperature T_1 is found from

$$Q_1 = \frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}}$$

Solving for E_{b1} and backing out the temperature gives $T_1 = 907\ K$.

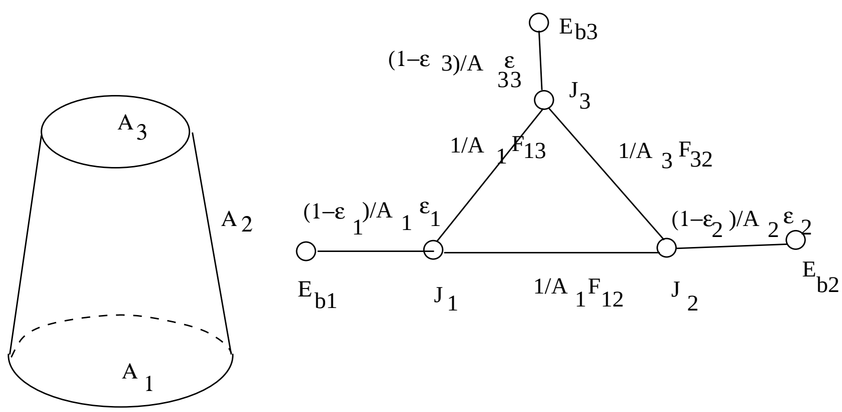

Figure 4: Surfaces for Problem 6