ME 24-731
Conduction and Radiation Heat Transfer

Solution to Assignment No: 4
Due Date: March 7, 2000
Spring 2000
Instructor: J. Murthy

1. Problem 3.3 from Patankar
   Discretizing the governing equation, we have

$$
\begin{aligned}
a_P T_P &= a_E T_E + a_W T_W + b \\
\text{with} & \\
a_E &= \frac{k_P}{\Delta x^2} + \frac{dk}{dx}\frac{1}{2\Delta x} \\
a_W &= \frac{k_P}{\Delta x^2} - \frac{dk}{dx}\frac{1}{2\Delta x} \\
a_P &= \frac{2k_P}{\Delta x^2} - S_P \\
b &= S_C
\end{aligned}
$$

   If $dk/dx < 0$, then $a_E < 0$ if

$$\frac{k_P}{\Delta x^2} < |\frac{dk}{dx}|\frac{1}{2\Delta x}$$

   Similarly, if $dk/dx > 0$, then $a_W < 0$ if

$$\frac{k_P}{\Delta x^2} < \frac{dk}{dx}\frac{1}{2\Delta x}$$

   The main point being made here is that it is not good practice to expand out the $\nabla \cdot k\nabla T$ term into two terms. The former is called the "conservative" form, and finite volume schemes go to great lengths to obtain this form before discretizing.

2. Problem 4.8 from Patankar
   The best linearization is:

$$
\begin{aligned}
S &= S^* + \left(\frac{dS}{d\phi}\right)_P^* (\phi_P - \phi_P^*) \\
S_C &= A + B|\phi_P^*|\phi_P^* \\
S_P &= -2B|\phi_P^*|
\end{aligned}
$$

   (a) $S_C = A - B|\phi_P^*|\phi_P^*, S_P = 0$: This takes no advantage of the known dependence. It may be difficult to get convergence with this type of approach, depending on the sizes of A and B.
   (b) $S_C = A$, $S_P = -B|\phi_P^*|$: Here the assumed slope is less steep than the actual slope – again, there may be problems with convergence, though it is better than (a).
   (c) $S_C = A + B|\phi_P^*|\phi_P^*$, $S_P = -2B|\phi_P^*|$: This is the ideal discretization and correctly captures the tangent to the source function.
   (d) $S_C = A + 9B|\phi_P^*|\phi_P^*, S_P = -10B|\phi_P^*|$: This is a conservative discretization. A large S_P tends to slow convergence. This should only be used if there is difficulty with convergence.

3. Problem 4.20 from Patankar
   Defining

$$\theta = \frac{T - T_f}{T_0 - T_f}$$
$$X = \frac{x}{L}$$

the dimensionless governing equation is given by

$$\frac{d}{dX}\left(\frac{d\theta}{dX}\right) - \frac{hPL^2}{kA}\theta = 0$$

with boundary conditions

$$\begin{aligned} \theta(0) &= 1 \\ \frac{d\theta}{dX}(1) &= 0 \end{aligned}$$

For a uniform grid of size ΔX shown in Figure 1, the discretization equations are given by:

(a) For a general interior grid point (Point 2)

$$a_E = \frac{1}{\Delta X};\ \ a_W = \frac{1}{\Delta X};\ \ a_P = a_E + a_W + \frac{hPL^2}{kA}\Delta X;\ \ b = 0$$

(b) For the first interior grid point at the fin base (Point 1)

$$a_E = \frac{1}{\Delta X};\ \ a_W = 0.0;\ \ a_B = \frac{2}{\Delta X};\ \ a_P = a_E + a_B + \frac{hPL^2}{kA}\Delta X;\ \ b = a_B\theta(0) = a_B$$

(c) For the last interior grid point at the fin tip (Point 3)

$$a_E = 0;\ \ a_W = \frac{1}{\Delta X};\ \ a_P = a_W + \frac{hPL^2}{kA}\Delta X;\ \ b = 0$$

A base heat flux result with an accuracy greater than 1% may be predicted with 6-8 grid points.

Figure 1: Grid for Problem 3

4. Assume a uniform mesh with size $(\Delta x, \Delta y)$. The discrete equation for each of the points 1,2,3 and 4 in Figure 2 is given below.

(a) Point 1:

$$a_E = \frac{k_e \Delta y}{\Delta x};\ \ a_W = 0.0;\ \ a_N = \frac{k_n \Delta x}{\Delta y};\ \ a_S = \frac{k_s \Delta x}{\Delta y};\ \ a_B = \frac{2k_P \Delta y}{\Delta x}$$

$$a_P = a_E + a_N + a_S + a_B;\ \ b = a_B T_b$$

where T_b is the boundary temperature.

(b) Point 2:

$$a_E = \frac{k_e \Delta y}{\Delta x};\ \ a_W = 0.0;\ \ a_N = 0.0;\ \ a_S = \frac{k_s \Delta x}{\Delta y};\ \ a_{Bw} = \frac{2k_P \Delta y}{\Delta x};\ \ a_{Bn} = \frac{2k_P \Delta x}{\Delta y};$$

$$a_P = a_E + a_S + a_{Bw} + a_{Bn};\ \ b = a_{Bw}T_{bw} + a_{Bn}T_{bn}$$

where T_{bw} and T_{bn} are the west and north boundary temperatures.

(c) Point 3:

$$a_E = \frac{k_e \Delta y}{\Delta x};\ \ a_W = \frac{k_w \Delta y}{\Delta x};\ \ a_N = \frac{k_n \Delta x}{\Delta y};\ \ a_S = \frac{k_s \Delta x}{\Delta y}$$

$$a_P = a_E + a_W + a_N + a_S;\ \ b = 0$$

(d) Point 4:

$$a_E = \frac{k_e \Delta y}{\Delta x};\ \ a_W = \frac{k_w \Delta y}{\Delta x};\ \ a_N = \frac{k_n \Delta x}{\Delta y};\ \ a_S = \frac{k_s \Delta x}{\Delta y}$$

$$a_P = a_E + a_W + a_N + a_S - S_P \Delta x \Delta y;\ \ b = S_C \Delta x \Delta y;\ \ S_P = -20;\ \ S_C = 1000$$

Face values of the conductivity are evaluated using harmonic-mean averaging:

$$k_e = \frac{2k_P k_E}{k_P + k_E}$$

where k_P and k_E are evaluated using T_P and T_E respectively. At boundaries, we may either use k evaluated at the boundary temperature, or use the interior value k_P.

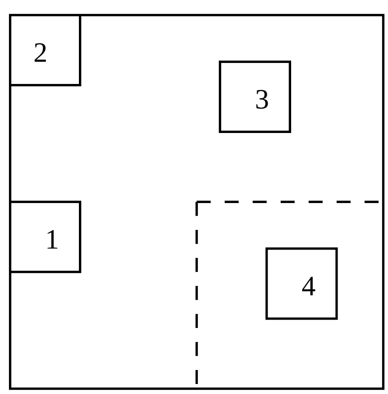

Figure 2: Grid for Problem 4